The formula of number of prime pair in Goldbach's conjecture

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Abstract

If $\pi_g(N)$ is the number of cases that even number N could be expressed as the sum of the two primes of $6n \pm 1$ type then the formula of $\pi_g(N)$ is below

$$\pi_g(6n+0) = n-1-\frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)-\frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)}{m}$$

$$\text{where}, \beta_g(6k-1) = \tau(6k-1) - 2 + \tau(6(n-k)+1) - 2, \ldots$$

But,the formula of $\pi_g(6n+2), \pi_g(6n-2)$ is omitted in abstract.

1. Introduction

As well known, Goldbach's conjecture is “All of even number could be expressed expressed as the sum of the two primes”. We redefined $\beta_g(N), \rho_g(N)$ using by $\beta(N), \rho(N)$ defined in paper “The formula of $\pi(N)$” [1] of myself. By using this, we build the formula of $\pi_g(N)$ as the number of cases that even number N could be expressed as the sum of the two primes of $6n \pm 1$ type.
In addition, we express the formula of $\pi_g(N)$ with $\pi(6n+1)$ by using apple box principle in the paper “theorem 6 in The number of twin prime” [4] of myself.

2. The formula of number of prime pair in Goldbach's conjecture

Definition 1. We apply same definition in paper "The formula of $\pi(N)$" [1] of myself.

Definition 2. For an arbitrary even number N and $P = 6p \pm 1, T = 6t \pm 1$, when $N = P + T$, for arbitrary d

Let us define $\beta_g(P) = \begin{Bmatrix} 0,\ if \text{ all of } P, N-P \text{ is prime} \\ d, if \text{ one or more of } P, N-P \text{ is composite number} \end{Bmatrix}$

Definition 3. Let us define $\rho_g(P) = \begin{Bmatrix} 0,\ if \text{ all of } P, N-P \text{ is prime} \\ 1, if \text{ one or more of } P, N-P \text{ is composite number} \end{Bmatrix}$

Definition 4. Let us define $\pi_g(N)$ as the number of cases that all of $P, N-P$ is prime. But, we exclude duplication. That is, $\pi_g(N)$ means the number of cases except duplication that N could be expressed as the sum of the two primes of $6n \pm 1$ type..

Theorem 1. $\boldsymbol{\beta_g(P)}$

If we define $N(N > 2)$ as an even number and $P = 6p \pm 1, T = 6t \pm 1, N = P + T$ then

$$\beta_g(P) = \beta(P) + \beta(N-P)$$
$$\beta_g(P) = \rho(P) + \rho(N-P)$$

Proof 1.

Let us define $N(N > 2)$ as an even number and $P = 6p \pm 1, T = 6t \pm 1, N = P + T$.

According to "definition 4 in paper The formula of $\pi(N)$" [1] of myself and $T = N - P$,

let us define as if P is composite then $\beta(P) = a$,

if $T = N - P$ is composite then $\beta(T) = \beta(N-P) = b$.

If all of $P, N-P$ is composite then $\beta(P) + \beta(N-P) = a + b,\ \rho(P) + \rho(N-P) = 2$

If one of $P, N-P$ is composite then $\beta(P) + \beta(N-P) = a \text{ or } b,\ \rho(P) + \rho(N-P) = 1$

If all of $P, N-P$ is prime then $\beta(P) + \beta(N-P) = 0,\ \rho(P) + \rho(N-P) = 0$

Therefore, $\beta_g(P) = \beta(P) + \beta(N-P), \beta_g(P) = \rho(P) + \rho(N-P)$

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Theorem 2. $\boldsymbol{\rho_g(P)}$

If we define $N(N > 2)$ as an even number and $P = 6p \pm 1, T = 6t \pm 1, N = P + T$ then

$$\rho_g(P) = \left[\frac{\beta_g(P) + 1}{2}\right] = \left[\frac{\rho(P) + \rho(N - P) + 1}{2}\right]$$

$$\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P) - w}\right], 0 < w < \frac{1}{2}, w \in \overline{\mathbb{R}}, w = \frac{1}{e}, \frac{1}{\pi}, \frac{1}{N}(N > 2), \ldots$$

If one or more of $P, N - P$ is composite then

$$\rho_g(P) = \frac{\beta_g(P)}{\beta_g(P) - w} - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - w}\right)}{k}$$

If all of $P, N - P$ is prime then

$$\rho_g(P) = \left\{\frac{\beta_g(P)}{\beta_g(P) - w} - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - w}\right)}{k}\right\} + \frac{1}{2}$$

Especially, if $= \frac{1}{\pi}$,

If one or more of $P, N - P$ is composite then

$$\rho_g(P) = \frac{\pi\beta_g(P)}{\pi\beta_g(P) - 1} - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin\left(\frac{2k\pi^2\beta_g(P)}{\pi\beta_g(P) - 1}\right)}{k}$$

If all of $P, N - P$ is prime then

$$\rho_g(P) = \left\{\frac{\pi\beta_g(P)}{\pi\beta_g(P) - 1} - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin\left(\frac{2k\pi^2\beta_g(P)}{\pi\beta_g(P) - 1}\right)}{k}\right\} + \frac{1}{2}$$

Proof 2.

Because $\beta_g(P) = \rho(P) + \rho(N-P)$ according to theorem 1,

if all of $P, N-P$ is composite then

$$\beta_g(P) = \rho(P) + \rho(N-P) = 2 \rightarrow \left[\frac{\beta_g(P)+1}{2}\right] = \left[\frac{2+1}{2}\right] = 1$$

if one of $P, N-P$ is composite then

$$\beta_g(P) = \rho(P) + \rho(N-P) = 1 \rightarrow \left[\frac{\beta_g(P)+1}{2}\right] = \left[\frac{1+1}{2}\right] = 1$$

if all of $P, N-P$ is prime then

$$\beta_g(P) = \rho(P) + \rho(N-P) = 0 \rightarrow \left[\frac{\beta_g(P)+1}{2}\right] = \left[\frac{0+1}{2}\right] = 0$$

So, $\left[\frac{\beta_g(P)+1}{2}\right]$ is satisfied with the definition of $\rho_g(P)$

Therefore, $\rho_g(P) = \left[\frac{\beta_g(P)+1}{2}\right] = \left[\frac{\rho(P)+\rho(N-P)+1}{2}\right]$

And, if we define $\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P)-w}\right]$ like as "theorem 3 in paper The formula of $\pi(N)$" [1] of myself then for $0 < w < \frac{1}{2}, w \in \mathbb{R}$

when all of $P, N-P$ is composite, $\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P)-w}\right] = \left[\frac{2}{2-w}\right] = 1$

when one of $P, N-P$ is composite , $\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P)-w}\right] = \left[\frac{1}{1-w}\right] = 1$

when all of $P, N-P$ is prime , $\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P)-w}\right] = \left[\frac{0}{0-w}\right] = 0$

so, $\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P)-w}\right]$ and $w = \frac{1}{e}, \frac{1}{\pi}, \frac{1}{N} (N > 2), \ldots$ (detail proof is omitted)

And, when one of more of $P, N-P$ is composite, because $\beta_g(P) > 0, 0 < w < \frac{1}{2}, w \in \mathbb{R}$

$1 < \frac{\beta_g(P)}{\beta_g(P)-w} < 2$, that is, $\frac{\beta_g(P)}{\beta_g(P)-w} \in \mathbb{R}$ and

for arbitrary $x \in \mathbb{R}$ $[x] = x - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin(2k\pi x)}{k}$

[3]

$$\rho_g(P) = \left[\frac{\beta_g(P)}{\beta_g(P) - w} \right] = \frac{\beta_g(P)}{\beta_g(P) - w} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - w} \right)}{k}$$

When all of $P, N - P$ is prime, because $\beta_g(P) = 0$

$$\frac{\beta_g(P)}{\beta_g(P) - w} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - w} \right)}{k} = \frac{0}{0 - w} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(2k\pi \frac{0}{0 - w} \right)}{k} = -\frac{1}{2},$$

And, because $\rho_g(P) = 0$

$$\rho_g(P) = 0 = -\frac{1}{2} + \frac{1}{2} = \left\{ \frac{\beta_g(P)}{\beta_g(P) - w} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - w} \right)}{k} \right\} + \frac{1}{2}$$

Especially, if $= \frac{1}{\pi}$,

When one or more of $P, N - P$ is composite,

$$\rho_g(P) = \frac{\beta_g(P)}{\beta_g(P) - \frac{1}{\pi}} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi\beta_g(P)}{\beta_g(P) - \frac{1}{\pi}} \right)}{k}$$

$$= \frac{\pi\beta_g(P)}{\pi\beta_g(P) - 1} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi^2\beta_g(P)}{\pi\beta_g(P) - 1} \right)}{k}$$

When all of $P, N - P$ is prime,

$$\rho_g(P) = 0 = -\frac{1}{2} + \frac{1}{2} = \left\{ \frac{\pi\beta_g(P)}{\pi\beta_g(P) - 1} - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{sin\left(\frac{2k\pi^2\beta_g(P)}{\pi\beta_g(P) - 1} \right)}{k} \right\} + \frac{1}{2}$$

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Theorem 3. $\boldsymbol{\pi_g(N)}$

If we define $N(N>2)$ as an even number and $P=6p\pm 1, T=6t\pm 1, N=P+T$ then for $n\geq 2$

1) When $N=6n-2=P+T=(6p-1)+(6t-1)$

$$\pi_g(6n-2)=\left[\frac{n}{2}\right]-\sum_{k=1}^{[n/2]}\rho_g(6k-1)$$

$$=\left[\frac{n}{2}\right]-\frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)-\frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)}{m}$$

$$=\frac{1}{2}\left(n-1-\sum_{k=1}^{n-1}\rho_g(6k-1)+a(n)\right)$$

$$=\frac{n-1+a(n)}{2}-\frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)-\frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)}{m}$$

where,

$$\beta_g(6k-1)=\beta(6k-1)+\beta(6(n-k)-1)=\tau(6k-1)-2+\tau(6(n-k)-1)-2$$

$$=\sum_{p=1}^{6k-1}\left(\left[\frac{6k-1}{p}\right]-\left[\frac{6k-2}{p}\right]\right)+\sum_{p=1}^{6(n-k)-1}\left(\left[\frac{6(n-k)-1}{p}\right]-\left[\frac{6(n-k)-2}{p}\right]\right)-4$$

$$a(u)=\begin{cases}1-\rho_g(6[u/2]-1), if\ u \text{ is even number}\\ 0, if\ u \text{ is odd number}\end{cases}$$

2) When $N = 6n + 2 = P + T = (6p + 1) + (6t + 1)$

$$\pi_g(6n+2) = \left[\frac{n}{2}\right] - \sum_{k=1}^{[n/2]} \rho_g(6k+1)$$

$$= \left[\frac{n}{2}\right] - \frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right) - \frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)}{m}$$

$$= \frac{1}{2}\left(n - 1 - \sum_{k=1}^{n-1}\rho_g(6k+1) + b(n)\right)$$

$$= \frac{n-1+b(n)}{2} - \frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right) - \frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)}{m}$$

Where,

$$\beta_g(6k+1) = \beta(6k+1) + \beta(6(n-k)+1) = \tau(6k+1) - 2 + \tau(6(n-k)+1) - 2$$

$$= \sum_{p=1}^{6k+1}\left(\left[\frac{6k+1}{p}\right] - \left[\frac{6k}{p}\right]\right) + \sum_{p=1}^{6(n-k)+1}\left(\left[\frac{6(n-k)+1}{p}\right] - \left[\frac{6(n-k)}{p}\right]\right) - 4$$

$$b(u) = \begin{cases} 1 - \rho_g(6[u/2]+1), if\ u \text{ is even number} \\ 0, if\ u \text{ is odd number} \end{cases}$$

3) When $N = 6n + 0 = P + T = (6p - 1) + (6t + 1)\ or\ (6p + 1) + (6t - 1)$

$$\pi_g(6n+0) = n - 1 - \sum_{k=1}^{n-1} \rho_g(6k-1)$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)}{m}$$

$$= n - 1 - \sum_{k=1}^{n-1} \rho_g(6k+1)$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)}{m}$$

Where,

$$\beta_g(6k-1) = \beta(6k-1) + \beta(6(n-k)+1) = \tau(6k-1) - 2 + \tau(6(n-k)+1) - 2$$

$$= \sum_{p=1}^{6k-1}\left(\left[\frac{6k-1}{p}\right] - \left[\frac{6k-2}{p}\right]\right) + \sum_{p=1}^{6(n-k)+1}\left(\left[\frac{6(n-k)+1}{p}\right] - \left[\frac{6(n-k)}{p}\right]\right) - 4$$

$$\beta_g(6k+1) = \beta(6k+1) + \beta(6(n-k)-1) = \tau(6k+1) - 2 + \tau(6(n-k)-1) - 2$$

$$= \sum_{p=1}^{6k+1}\left(\left[\frac{6k+1}{p}\right] - \left[\frac{6k}{p}\right]\right) + \sum_{p=1}^{6(n-k)-1}\left(\left[\frac{6(n-k)-1}{p}\right] - \left[\frac{6(n-k)-2}{p}\right]\right) - 4$$

Proof 3.

Let us define $N(N>2)$ as an even number and $P=6p\pm 1, T=6t\pm 1, N=P+T$.

If we list up all cases of $N=P+T$ by using P_k then the following equation is satisfied.

But, we exclude the duplicated cases of $P_k=T_m$ and $P_m=T_k$.

$$N=\begin{Bmatrix} P_1+T_1=P_1+(N-P_1) \\ P_2+T_2=P_2+(N-P_2) \\ \cdots \\ P_r+T_r=P_r+(N-P_r) \end{Bmatrix} \text{---------- (3.1)}$$

If all of $P_k, N-P_k$ is prime then $1-\rho_g(P_k)=1-0=1$,

If one or more of $P_k, N-P_k$ is composite then $1-\rho_g(P_k)=1-1=0$. So,

$$\pi_g(N)=\sum_{k=1}^{r}\{1-\rho_g(P_k)\}=\sum_{k=1}^{r}1-\sum_{k=1}^{r}\rho_g(P_k)=r-\sum_{k=1}^{r}\rho_g(P_k) \text{ -------- (3.2)}$$

And, let us define $\mathbb{P}$ as a set which all of $P_k, N-P_k$ is prime,

and let us define $\mathbb{C}$ as a set which one or more of $P_k, N-P_k$ is composite

and let us define $A=\dfrac{\beta_g(P_k)}{\beta_g(P_k)-w}-\dfrac{1}{2}+\dfrac{1}{\pi}\displaystyle\sum_{m=1}^{\infty}\dfrac{sin\left(\dfrac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-w}\right)}{m}$

According to theorem 2, if one or more of $P_k, N-P_k$ is composite then $\rho_g(P_k)=A$,

if all of $P_k, N-P_k$ is prime then $\rho_g(P_k)=A+\frac{1}{2}$,and

let us express $\displaystyle\sum_{\mathbb{Z}}^{n}u(k)$ with the sum of $u(k)$, only if $u(k)\in\mathbb{Z}$ in $1\le k\le n$ for a certain $u(k)$, $\mathbb{Z}$

because $\mathbb{P}\cap\mathbb{C}=\emptyset$,

$$\sum_{k=1}^{r}\rho_g(P_k)=\sum_{\mathbb{P}}^{r}\rho_g(P_k)+\sum_{\mathbb{C}}^{r}\rho_g(P_k) \text{ -------- (3.3)}$$

So, if we apply (3.3) to (3.2) then

$$\pi_g(N)=r-\left(\sum_{\mathbb{P}}^{r}\rho_g(P_k)+\sum_{\mathbb{C}}^{r}\rho_g(P_k)\right)=r-\left(\sum_{\mathbb{P}}^{r}\left(A+\frac{1}{2}\right)+\sum_{\mathbb{C}}^{r}A\right)$$

$$=r-\left(\sum_{\mathbb{P}}^{r}\left(\frac{1}{2}\right)+\sum_{\mathbb{P}}^{r}A+\sum_{\mathbb{C}}^{r}A\right) \text{ -------- (3.4)}$$

$$\sum_{\mathbb{P}}^{r}\left(\frac{1}{2}\right)=\frac{1}{2}\sum_{\mathbb{P}}^{r}1=\frac{\pi_g(N)}{2},\sum_{\mathbb{P}}^{r}A+\sum_{\mathbb{C}}^{r}A=\sum_{k=1}^{r}A$$

So,if we apply this to (3.4) then

$$\pi_g(N)=r-\left(\frac{\pi_g(N)}{2}+\sum_{k=1}^{r}A\right)=r-\frac{\pi_g(N)}{2}-\sum_{k=1}^{r}A \text{-------- (3.5)}$$

If we substitute A to (3.5) and arrange then

$$\pi_g(N)+\frac{\pi_g(N)}{2}=r-\sum_{k=1}^{r}A\rightarrow\frac{3\pi_g(N)}{2}=r-\sum_{k=1}^{r}A\rightarrow$$

$$\pi_g(N)=\frac{2}{3}\left\{r-\sum_{k=1}^{r}\left(\frac{\beta_g(P_k)}{\beta_g(P_k)-w}-\frac{1}{2}+\frac{1}{\pi}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-w}\right)}{m}\right)\right\}$$

$$=\frac{2}{3}\left\{r-\sum_{k=1}^{r}\left(-\frac{1}{2}\right)-\sum_{k=1}^{r}\left(\frac{\beta_g(P_k)}{\beta_g(P_k)-w}\right)-\frac{1}{\pi}\sum_{k=1}^{r}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-w}\right)}{m}\right\}$$

$$=\frac{2}{3}\left\{\frac{3r}{2}-\sum_{k=1}^{r}\left(\frac{\beta_g(P_k)}{\beta_g(P_k)-w}\right)-\frac{1}{\pi}\sum_{k=1}^{r}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-w}\right)}{m}\right\}$$

$$=r-\frac{2}{3}\sum_{k=1}^{r}\left(\frac{\beta_g(P_k)}{\beta_g(P_k)-w}\right)-\frac{2}{3\pi}\sum_{k=1}^{r}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-w}\right)}{m} \text{-------- (3.6)}$$

If we substitute $w=\frac{1}{\pi}$ to (3.6) especially then

$$\pi_g(N)=r-\frac{2}{3}\sum_{k=1}^{r}\left(\frac{\beta_g(P_k)}{\beta_g(P_k)-\frac{1}{\pi}}\right)-\frac{2}{3\pi}\sum_{k=1}^{r}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(P_k)}{\beta_g(P_k)-\frac{1}{\pi}}\right)}{m}$$

$$=r-\frac{2}{3}\sum_{k=1}^{r}\left(\frac{\pi\beta_g(P_k)}{\pi\beta_g(P_k)-1}\right)-\frac{2}{3\pi}\sum_{k=1}^{r}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(P_k)}{\pi\beta_g(P_k)-1}\right)}{m} \text{-------- (3.7)}$$

N is one of $N = 6n - 2, N = 6n + 0, N = 6n + 2$ and the cases that we could express $N = P + T$ type is following equation.

$$\left.\begin{cases} N = 6n - 2 = P + T = (6p - 1) + (6t - 1) \\ N = 6n + 0 = P + T = (6p + 1) + (6t - 1)\ or\ (6p - 1) + (6t + 1) \\ N = 6n + 2 = P + T = (6p + 1) + (6t + 1) \end{cases}\right\} \text{------------- (3.8)}$$

1) When $N = 6n - 2 = P + T = (6p - 1) + (6t - 1)$

If we define $p_1 = 1, p_2 = 2, \ldots,$ that is, $p_k = k$ then $P_k = 6p_k - 1 = 6k - 1$ and if we express (3.1) by using this then the following equation is satisfied.

$N = 6n - 2$

$$= \left\{\begin{array}{c} P_1 + T_1 = (6p_1 - 1) + (6t_1 - 1) = (6 \times 1 - 1) + (6(n-1) - 1) \\ P_2 + T_2 = (6p_2 - 1) + (6t_2 - 1) = (6 \times 2 - 1) + (6(n-2) - 1) \\ \cdots \\ P_r + T_r = (6p_r - 1) + (6t_r - 1) = (6r - 1) + (6(n-r) - 1) \\ \cdots \\ P_{n-2} + T_{n-2} = (6p_{n-2} - 1) + (6t_{n-2} - 1) = (6(n-2) - 1) + (6 \times 2 - 1) \\ P_{n-1} + T_{n-1} = (6p_{n-1} - 1) + (6t_{n-1} - 1) = (6(n-1) - 1) + (6 \times 1 - 1) \end{array}\right\} \text{---------- (3.1.1)}$$

All of the cases that we could express $N = (6p - 1) + (6t - 1)$ is until $N = (6p_{n-1} - 1) + (6t_{n-1} - 1)$, but, this duplication that $6k - 1 + 6(n-k) - 1 = (6p_k - 1) + (6t_k - 1)$ is same as $6(n-k) - 1 + 6k - 1 = (6p_{n-k} - 1) + (6t_{n-k} - 1)$ is occurred because $t_k = n - p_k = n - k$.

Because this duplication occurs from $k = n - k$, if we define $p_r = t_r$ then $r = n - r \rightarrow r = n/2$, and $r = [n/2]$ because r has to be an natural number

When n is an odd number, $r = [n/2] \neq n/2$, so, $p_r = t_r$ is not exist.

But, when n is an even number $r = [n/2] = n/2$, so, $p_r = t_r$ is occurred. But, $r = n/2$ term is allowed because “one prime is allowed to use twice” in Goldbach's conjecture.

That is, $1 \leq p_k \leq [n/2]$ term and $[n/2] + 1 \leq p_k \leq n - 1$ term is duplicated.

And, it should be $p_k \geq 1, t_k \geq 1$ because $= p_k + t_k$, so, $n \geq 2$.

So, if we apply the above contents to (3.2),(3.6),(3.7) then

$$\pi_g(6n-2) = \left[\frac{n}{2}\right] - \sum_{k=1}^{[n/2]} \rho_g(6k-1)$$

$$= \left[\frac{n}{2}\right] - \frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\beta_g(6k-1)}{\beta_g(6k-1) - w}\right) - \frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k-1)}{\beta_g(6k-1) - w}\right)}{m}$$

$$= \left[\frac{n}{2}\right] - \frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1) - 1}\right) - \frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1) - 1}\right)}{m}$$

If we define A as the number of pair of that P, T are all prime which includes duplication,

if we define $a(u) = \begin{Bmatrix} 1-\rho_g(6[u/2]-1), if\ u \text{ is even number} \\ 0, if\ u \text{ is odd number} \end{Bmatrix}$ then

$$A = n - 1 - \sum_{k=1}^{n-1} \rho_g(6k-1)$$

When $a(u) = 1$, $[n/2]$ term of $N = (6[n/2]-1) + (6(n-[n/2])-1)$ is not duplicated but the one is only exist ,so,

$$\pi_g(6n-2) = \frac{1}{2}\big(A + a(n)\big)$$

Therefore,

$$\pi_g(6n-2) = \frac{1}{2}\left(n - 1 - \sum_{k=1}^{n-1} \rho_g(6k-1) + a(n)\right)$$

$$= \frac{n-1+a(n)}{2} - \frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\beta_g(6k-1)}{\beta_g(6k-1)-w}\right) - \frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k-1)}{\beta_g(6k-1)-w}\right)}{m}$$

$$= \frac{n-1+a(n)}{2} - \frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right) - \frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1)-1}\right)}{m}$$

And, according to theorem 1 $\beta_g(P_k) = \beta(P_k) + \beta(N - P_k)$,so,

$$\beta_g(6k-1) = \beta(6k-1) + \beta(6n-2-(6k-1)) = \beta(6k-1) + \beta(6(n-k)-1)$$

In the above formula, if we express $\beta_g(6k-1)$ by using only $\beta(N) = \tau(N) - 2$ which is the most simple function in "theorem 2 in paper The formula of $\pi(N)$" [1] of myself then

$\beta_g(6k-1) = \beta(6k-1) + \beta(6(n-k)-1) = \tau(6k-1) - 2 + \tau(6(n-k)-1) - 2$

$$= \sum_{p=1}^{6k-1}\left(\left[\frac{6k-1}{p}\right] - \left[\frac{6k-2}{p}\right]\right) - 2 + \sum_{p=1}^{6(n-k)-1}\left(\left[\frac{6(n-k)-1}{p}\right] - \left[\frac{6(n-k)-2}{p}\right]\right) - 2$$

$$= \sum_{p=1}^{6k-1}\left(\left[\frac{6k-1}{p}\right] - \left[\frac{6k-2}{p}\right]\right) + \sum_{p=1}^{6(n-k)-1}\left(\left[\frac{6(n-k)-1}{p}\right] - \left[\frac{6(n-k)-2}{p}\right]\right) - 4$$

2) When $N=6n+2=P+T=(6p+1)+(6t+1)$

If we define $p_1=1, p_2=2, \dots$, that is, $p_k=k$ then $P_k=6p_k+1=6k+1$ and if we express (3.1) by using this then the following equation is satisfied.

$N=6n+2$

$$=\left\{\begin{matrix} P_1+T_1=(6p_1+1)+(6t_1+1)=(6\times 1+1)+(6(n-1)+1) \\ P_2+T_2=(6p_2+1)+(6t_2+1)=(6\times 2+1)+(6(n-2)+1) \\ \dots \\ P_r+T_r=(6p_r+1)+(6t_r+1)=(6r+1)+(6(n-r)+1) \\ \dots \\ P_{n-2}+T_{n-2}=(6p_{n-2}+1)+(6t_{n-2}+1)=(6(n-2)+1)+(6\times 2+1) \\ P_{n-1}+T_{n-1}=(6p_{n-1}+1)+(6t_{n-1}+1)=(6(n-1)+1)+(6\times 1+1) \end{matrix}\right\} \text{---------- (3.2.1)}$$

As the same with the above 1), $1\le p_k\le[n/2]$ term and $[n/2]+1\le p_k\le n-1$ term is all duplicated. And, it should be $p_k\ge 1, t_k\ge 1$ because $=p_k+t_k$, so, $n\ge 2$.

If we apply the above contents to (3.2),(3.6),(3.7) then

$$\pi_g(6n+2)=\left[\frac{n}{2}\right]-\sum_{k=1}^{[n/2]}\rho_g(6k+1)$$

$$=\left[\frac{n}{2}\right]-\frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\beta_g(6k+1)}{\beta_g(6k+1)-w}\right)-\frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k+1)}{\beta_g(6k+1)-w}\right)}{m}$$

$$=\left[\frac{n}{2}\right]-\frac{2}{3}\sum_{k=1}^{[n/2]}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)-\frac{2}{3\pi}\sum_{k=1}^{[n/2]}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)}{m}$$

If we define $b(u)=\left\{\begin{matrix} 1-\rho_g(6[u/2]+1), if\ u \text{ is even number} \\ 0, if\ u \text{ is odd number}\end{matrix}\right\}$

and arrange by using same method with 1) then (detail proof is omitted)

$$\pi_g(6n-2)=\frac{1}{2}\left(n-1-\sum_{k=1}^{n-1}\rho_g(6k+1)+b(n)\right)$$

$$=\frac{n-1+b(n)}{2}-\frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\beta_g(6k+1)}{\beta_g(6k+1)-w}\right)-\frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k+1)}{\beta_g(6k+1)-w}\right)}{m}$$

$$=\frac{n-1+b(n)}{2}-\frac{1}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)-\frac{1}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1)-1}\right)}{m}$$

And, according to theorem 1 $\beta_g(P_k) = \beta(P_k) + \beta(N - P_k)$,so,

$$\beta_g(6k+1) = \beta(6k+1) + \beta(6n+2-(6k+1)) = \beta(6k+1) + \beta(6(n-k)+1)$$

Therefore, according to "theorem 2 in paper The formula of $\pi(N)$" [1] of myself

$$\beta_g(6k+1) = \beta(6k+1) + \beta(6(n-k)+1) = \tau(6k+1) - 2 + \tau(6(n-k)+1) - 2$$

$$= \sum_{p=1}^{6k+1}\left(\left\lceil\frac{6k+1}{p}\right\rceil - \left\lceil\frac{6k}{p}\right\rceil\right) + \sum_{p=1}^{6(n-k)+1}\left(\left\lceil\frac{6(n-k)+1}{p}\right\rceil - \left\lceil\frac{6(n-k)}{p}\right\rceil\right) - 4$$

3) When $N = 6n + 0 = P + T = (6p-1) + (6t+1)\ or\ (6p+1) + (6t-1)$

If we change p, t in $(6p+1) + (6t-1)$ then it is same as $(6p-1) + (6t+1)$.

So,we study only $(6p-1) + (6t+1)$.

If we define $p_1 = 1, p_2 = 2, \ldots,$ that is, $p_k = k$ then $P_k = 6p_k - 1 = 6k - 1$ and if we express (3.1) by using this then the following equation is satisfied.

$$N = 6n + 0$$

$$= \left\{\begin{array}{c} P_1 + T_1 = (6p_1 - 1) + (6t_1 + 1) = (6\times 1 - 1) + (6(n-1)+1) \\ P_2 + T_2 = (6p_2 - 1) + (6t_2 + 1) = (6\times 2 - 1) + (6(n-2)+1) \\ \ldots \\ P_{n-2} + T_{n-2} = (6p_{n-2} - 1) + (6t_{n-2} + 1) = (6(n-2) - 1) + (6\times 2 + 1) \\ P_{n-1} + T_{n-1} = (6p_{n-1} - 1) + (6t_{n-1} + 1) = (6(n-1) - 1) + (6\times 1 + 1) \end{array}\right\} \text{----------}\ (3.3.1)$$

As not same with the above 1) and 2), the duplicated term of $P_k = T_m$ and $P_m = T_k$ is not exist because $P_k \equiv -1 (mod\ 6), T_k \equiv 1 (mod\ 6)$.

And, it should be $p_k \geq 1, t_k \geq 1$ because $= p_k + t_k$, so, $n \geq 2$.

If we apply the above contents to (3.2),(3.6),(3.7) then

$$\pi_g(6n+0) = n - 1 - \sum_{k=1}^{n-1}\rho_g(6k-1)$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\beta_g(6k-1)}{\beta_g(6k-1) - w}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k-1)}{\beta_g(6k-1) - w}\right)}{m}$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k-1)}{\pi\beta_g(6k-1) - 1}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k-1)}{\pi\beta_g(6k-1) - 1}\right)}{m}$$

And, if we apply (3.3.1) to the case of $P_k \equiv 1(mod\ 6), T_k \equiv -1(mod\ 6)$ then the following equation is satisfied.

$$N = 6n + 0$$

$$= \left\{\begin{array}{c} P_1 + T_1 = (6p_1 + 1) + (6t_1 - 1) = (6 \times 1 + 1) + (6(n-1) - 1) \\ P_2 + T_2 = (6p_2 + 1) + (6t_2 - 1) = (6 \times 2 + 1) + (6(n-2) - 1) \\ \dots \\ P_{n-2} + T_{n-2} = (6p_{n-2} + 1) + (6t_{n-2} - 1) = (6(n-2) + 1) + (6 \times 2 - 1) \\ P_{n-1} + T_{n-1} = (6p_{n-1} + 1) + (6t_{n-1} - 1) = (6(n-1) + 1) + (6 \times 1 - 1) \end{array}\right\} \text{----------} \ (3.3.2)$$

Because the above contents of (3.3.2) is same as the contents of (3.3.1) (detail proof is omitted)

$$\pi_g(6n+0) = n - 1 - \sum_{k=1}^{n-1} \rho_g(6k+1)$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\beta_g(6k+1)}{\beta_g(6k+1) - w}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi\beta_g(6k+1)}{\beta_g(6k+1) - w}\right)}{m}$$

$$= n - 1 - \frac{2}{3}\sum_{k=1}^{n-1}\left(\frac{\pi\beta_g(6k+1)}{\pi\beta_g(6k+1) - 1}\right) - \frac{2}{3\pi}\sum_{k=1}^{n-1}\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta_g(6k+1)}{\pi\beta_g(6k+1) - 1}\right)}{m}$$

And, according to theorem 1, $\beta_g(P_k) = \beta(P_k) + \beta(N - P_k)$, so,

$$\beta_g(6k-1) = \beta(6k-1) + \beta(6n + 0 - (6k-1)) = \beta(6k-1) + \beta(6(n-k)+1)$$

Therefore, according to "theorem 2 in paper The formula of $\pi(N)$" [1] of myself

$$\beta_g(6k-1) = \beta(6k-1) + \beta(6(n-k)+1) = \tau(6k-1) - 2 + \tau(6(n-k)+1) - 2$$

$$= \sum_{p=1}^{6k-1}\left(\left[\frac{6k-1}{p}\right] - \left[\frac{6k-2}{p}\right]\right) + \sum_{p=1}^{6(n-k)+1}\left(\left[\frac{6(n-k)+1}{p}\right] - \left[\frac{6(n-k)}{p}\right]\right) - 4$$

And, $\beta_g(6k+1) = \beta(6k+1) + \beta\big(6n + 0 - (6k+1)\big) = \beta(6k+1) + \beta(6(n-k)-1)$

$$\beta_g(6k+1) = \beta(6k+1) + \beta(6(n-k)-1) = \tau(6k+1) - 2 + \tau(6(n-k)-1) - 2$$

$$= \sum_{p=1}^{6k+1}\left(\left[\frac{6k+1}{p}\right] - \left[\frac{6k}{p}\right]\right) + \sum_{p=1}^{6(n-k)-1}\left(\left[\frac{6(n-k)-1}{p}\right] - \left[\frac{6(n-k)-2}{p}\right]\right) - 4$$

■

Theorem 4. $\pi_g(N)$ expression by using $\pi(6n+1)$

Let us define $N(N>2)$ as an even number $P=6p\pm1, T=6t\pm1$, $N=P+T, n\geq 2$.

Let us define $\pi_-(M)$ as the number of $6m-1$ type prime number of M or less,

$\pi_+(M)$ as the number of $6m+1$ type prime number of M or less.

Let us define $\mathbb{M}$ be a set of that only one of P,T is prime .

$a(n), b(n)$ is same with function defined in theorem 3.

1) When $N=6n-2=P+T=(6p-1)+(6t-1)$

$$\pi_g(6n-2)=\frac{\pi_-(6(n-1)-1)+a(n)}{2}-\frac{1}{4}\sum_{\mathbb{M}}^{n-1}\big(\rho(6k-1)+\rho(6(n-k)-1)\big)$$

$$=\frac{\pi_-(6(n-1)-1)+a(n)}{2}-\frac{1}{6}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k-1)}{\pi\beta(6k-1)-1}+\frac{\pi\beta(6(n-k)-1)}{\pi\beta(6(n-k)-1)-1}\right)$$

$$-\frac{1}{6\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta(6k-1)}{\pi\beta(6k-1)-1}\right)+sin\left(\frac{2m\pi^2\beta(6(n-k)-1)}{\pi\beta(6(n-k)-1)-1}\right)}{m}\right)$$

2) When $N=6n+2=P+T=(6p+1)+(6t+1)$

$$\pi_g(6n+2)=\frac{\pi_+(6(n-1)+1)+b(n)}{2}-\frac{1}{4}\sum_{\mathbb{M}}^{n-1}\big(\rho(6k+1)+\rho(6(n-k)+1)\big)$$

$$=\frac{\pi_+(6(n-1)+1)+b(n)}{2}-\frac{1}{6}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k+1)}{\pi\beta(6k+1)-1}+\frac{\pi\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)$$

$$-\frac{1}{6\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta(6k+1)}{\pi\beta(6k+1)-1}\right)+sin\left(\frac{2m\pi^2\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)}{m}\right)$$

3) When $N=6n+0=P+T=(6p-1)+(6t+1)\ or\ (6p+1)+(6t-1)$

$$\pi_g(6n+0)=\frac{1}{2}\left(\pi(6(n-1)+1)-2-\sum_{\mathbb{M}}^{n-1}\big(\rho(6k-1)+\rho(6(n-k)+1)\big)\right)$$

$$=\frac{\pi(6(n-1)+1)-2}{2}-\frac{1}{3}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k-1)}{\pi\beta(6k-1)-1}+\frac{\pi\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)$$

$$-\frac{1}{3\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta(6k-1)}{\pi\beta(6k-1)-1}\right)+sin\left(\frac{2m\pi^2\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)}{m}\right)$$

Proof 4.

Let us define $N(N > 2)$ as an even number and $P = 6p \pm 1, T = 6t \pm 1$, $N = P + T, n \geq 2$.

Let us define $\pi_-(M)$ as the number of $6m - 1$ type prime number of M or less,

$\pi_+(M)$ as the number of $6m + 1$ type prime number of M or less. Let us $\mathbb{M}$ be a set of that only one of P, T is prime . $a(n), b(n)$ is same with function defined in theorem 3.

1) When $N = 6n - 2 = P + T = (6p - 1) + (6t - 1)$

Let us define A as the number of pair of that P, T be all prime included duplication and if we regard prime as red apple and regard composite as green apple in "theorem 6 in The number of twin prime" [4] then

$$A = \frac{1}{2}\left(2\pi_-(6(n-1) - 1) - \sum_{\mathbb{M}}^{n-1} 1\right) \text{---------- (4.1.1)}$$

If we exclude duplication in (4.1.1) according to theorem 3 then

$$\pi_g(6n - 2) = \frac{1}{2}\big(A + a(n)\big) = \frac{1}{2}\left(\frac{1}{2}\left(2\pi_-(6(n-1) - 1) - \sum_{\mathbb{M}}^{n-1} 1\right) + a(n)\right) \rightarrow$$

$$\pi_g(6n - 2) = \frac{\pi_-(6(n-1) - 1) + a(n)}{2} - \frac{1}{4}\sum_{\mathbb{M}}^{n-1} 1 \rightarrow$$

$$\pi_g(6n - 2) = \frac{\pi_-(6(n-1) - 1) + a(n)}{2} - \frac{1}{4}\sum_{\mathbb{M}}^{n-1} \rho_g(6k - 1) \text{---------- (4.1.2)}$$

When $6k - 1, 6(n - k) - 1 \in \mathbb{M}$,

$\rho_g(6k - 1) = 1 = 1 + 0 \; or \; 0 + 1 = \rho(6k - 1) + \rho(6(n - k) - 1)$,so, If we apply to (4.1.2) then

$$\pi_g(6n - 2) = \frac{\pi_-(6(n-1) - 1) + a(n)}{2}$$

$$- \frac{1}{4}\sum_{\mathbb{M}}^{n-1}\big(\rho(6k - 1) + \rho(6(n - k) - 1)\big) \text{------------- (4.1.3)}$$

If we arrange (4.1.3) then (detail proof is omitted)

$$\pi_g(6n - 2) = \frac{\pi_-(6(n-1) - 1) + a(n)}{2} - \frac{1}{6}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k - 1)}{\pi\beta(6k - 1) - 1} + \frac{\pi\beta(6(n - k) - 1)}{\pi\beta(6(n - k) - 1) - 1}\right)$$

$$- \frac{1}{6\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty} \frac{sin\left(\frac{2m\pi^2\beta(6k - 1)}{\pi\beta(6k - 1) - 1}\right) + sin\left(\frac{2m\pi^2\beta(6(n - k) - 1)}{\pi\beta(6(n - k) - 1) - 1}\right)}{m}\right)$$

2) When $N = 6n + 2 = P + T = (6p + 1) + (6t + 1)$

Let us define A as the number of pair of that P, T be all prime included duplication and if we regard prime as red apple and regard composite as green apple in "theorem 6 in The number of twin prime" [4] then

$$A = \frac{1}{2}\left(2\pi_+(6(n-1)+1) - \sum_{\mathbb{M}}^{n-1} 1\right) \text{---------- } (4.2.1)$$

If we exclude duplication in (4.2.1) according to theorem 3 then

$$\pi_g(6n+2) = \frac{1}{2}\big(A + b(n)\big) = \frac{1}{2}\left(\frac{1}{2}\left(2\pi_+(6(n-1)+1) - \sum_{\mathbb{M}}^{n-1} 1\right) + b(n)\right) \rightarrow$$

$$\pi_g(6n+2) = \frac{\pi_+(6(n-1)+1) + b(n)}{2} - \frac{1}{4}\sum_{\mathbb{M}}^{n-1} 1 \rightarrow$$

$$\pi_g(6n+2) = \frac{\pi_+(6(n-1)+1) + b(n)}{2} - \frac{1}{4}\sum_{\mathbb{M}}^{n-1} \rho_g(6k+1) \text{---------- } (4.2.2)$$

When $6k+1, 6(n-k)+1 \in \mathbb{M}$

$\rho_g(6k+1) = 1 = 1 + 0 \; or \; 0 + 1 = \rho(6k+1) + \rho(6(n-k)+1)$,so, if we apply to (4.2.2) then

$$\pi_g(6n+2) = \frac{\pi_+(6(n-1)+1) + b(n)}{2}$$

$$-\frac{1}{4}\sum_{\mathbb{M}}^{n-1}\big(\rho(6k+1) + \rho(6(n-k)+1)\big) \text{------------- } (4.2.3)$$

If we arrange (4.2.3) then (detail proof is omitted)

$$\pi_g(6n+2) = \frac{\pi_+(6(n-1)+1) + b(n)}{2} - \frac{1}{6}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k+1)}{\pi\beta(6k+1)-1} + \frac{\pi\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)$$

$$-\frac{1}{6\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{sin\left(\frac{2m\pi^2\beta(6k+1)}{\pi\beta(6k+1)-1}\right) + sin\left(\frac{2m\pi^2\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)}{m}\right)$$

3) When $N = 6n + 0 = P + T = (6p - 1) + (6t + 1)\ or\ (6p + 1) + (6t - 1)$

Duplication is not exist unlike 1),2) and if we regard prime as red apple and regard composite as green apple in "theorem 6 in The number of twin prime" [4] then

$$\pi_g(6n+0) = \frac{1}{2}\left(\left(\pi_-(6(n-1)-1) + \pi_+(6(n-1)+1)\right) - \sum_{\mathbb{M}}^{n-1} 1\right) \text{---------- (4.3.1)}$$

$\left(\pi_-(6(n-1)-1) + \pi_+(6(n-1)+1)\right) = \pi(6(n-1)+1) - 2$,so,if we arrange (4.3.1) then

$$\pi_g(6n+0) = \frac{1}{2}\left(\pi(6(n-1)+1) - 2 - \sum_{\mathbb{M}}^{n-1} 1\right) \rightarrow$$

$$\pi_g(6n+0) = \frac{1}{2}\left(\pi(6(n-1)+1) - 2 - \sum_{\mathbb{M}}^{n-1} \rho_g(6k-1)\right) \text{---------- (4.3.2)}$$

When $6k-1, 6(n-k)+1 \in \mathbb{M}$

$\rho_g(6k-1) = 1 = 1 + 0\ or\ 0 + 1 = \rho(6k-1) + \rho(6(n-k)+1)$,so,if we apply to (4.3.2) then

$$\pi_g(6n+0) = \frac{1}{2}\left(\pi(6(n-1)+1) - 2 - \sum_{\mathbb{M}}^{n-1}\left(\rho(6k-1) + \rho(6(n-k)+1)\right)\right) \text{---------- (4.3.3)}$$

If we arrange (4.3.3) then (detail proof is omitted)

$$\pi_g(6n+0) = \frac{\pi(6(n-1)+1) - 2}{2} - \frac{1}{3}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k-1)}{\pi\beta(6k-1)-1} + \frac{\pi\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)$$
$$-\frac{1}{3\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{\sin\left(\frac{2m\pi^2\beta(6k-1)}{\pi\beta(6k-1)-1}\right) + \sin\left(\frac{2m\pi^2\beta(6(n-k)+1)}{\pi\beta(6(n-k)+1)-1}\right)}{m}\right)$$

$N = (6p+1) + (6t-1)$ is also same with the above contents,so,

$$\pi_g(6n+0) = \frac{1}{2}\left(\pi(6(n-1)+1) - 2 - \sum_{\mathbb{M}}^{n-1}\left(\rho(6k+1) + \rho(6(n-k)-1)\right)\right)$$

$$= \frac{\pi(6(n-1)+1) - 2}{2} - \frac{1}{3}\sum_{\mathbb{M}}^{n}\left(\frac{\pi\beta(6k+1)}{\pi\beta(6k+1)-1} + \frac{\pi\beta(6(n-k)-1)}{\pi\beta(6(n-k)-1)-1}\right)$$
$$-\frac{1}{3\pi}\sum_{\mathbb{M}}^{n}\left(\sum_{m=1}^{\infty}\frac{\sin\left(\frac{2m\pi^2\beta(6k+1)}{\pi\beta(6k+1)-1}\right) + \sin\left(\frac{2m\pi^2\beta(6(n-k)-1)}{\pi\beta(6(n-k)-1)-1}\right)}{m}\right)$$

■

References

[1] Oh Jung Uk, "The formula of $\pi(N)$", http://vixra.org/pdf/1408.0041v1.pdf

[2] wikipedia, *Goldbach's conjecture*,
http://ko.wikipedia.org/wiki/%EA%B3%A8%EB%93%9C%EB%B0%94%ED%9D%90%EC%9D%98_%EC%B6%94%EC%B8%A1

[3] wikipedia, *floor functions*,
http://ko.wikipedia.org/wiki/%EB%B0%94%EB%8B%A5_%ED%95%A8%EC%88%98

[4] Oh Jung Uk, "The formula of number of twin prime", http://vixra.org/pdf/1408.0044v1.pdf

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