The Universal Uncertainty Principle

Rodolfo A. Frino – August 2014 (v1) - January 2015 (v3)
Electronics Engineer – Degree from the National University of Mar del Plata - Argentina

Abstract

This paper is concerned with a generalization of the Heisenberg's uncertainty principle which I developed in 2012 and that I called the universal uncertainty principle. This principle takes into account the quantized nature of space-time (granularity) and the quantum fluctuations of the empty space. I have successfully applied the special version of these relations to calculate the thermodynamic properties of black holes, the approximate size of the electron and to derive the Einstein's formula of equivalence of mass and energy. This formulation can change the standard model of particle physics by introducing gravity into the model through the Planck length and the Planck time.

Keywords: *quantum fluctuations, zero point momentum, zero point energy, entropy, Planck length, Planck time, GUUP, SUUP.*

1. Introduction

In 1927 the German physicist Werner Heisenberg discovered a principle known as the Heisenberg uncertainty principle [1] which are normally written as

Heisenberg momentum-position uncertainty relations

$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \tag{1-1a}$$

$$\Delta p_y \Delta y \geq \frac{\hbar}{2} \tag{1-1b}$$

$$\Delta p_z \Delta z \geq \frac{\hbar}{2} \tag{1-1c}$$

Heisenberg energy-time uncertainty relation

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{1-2}$$

In the article entitled *"The Special Quantum Gravitational Theory of Black Holes"* [2], which I published online in 2014, I applied the special universal uncertainty principle to black holes to derive their thermodynamics properties: temperature and entropy. The surprising result of that research was that the equation for the black hole entropy was part of the equation for the temperature of the black hole. This meant that the black hole entropy emerged naturally from this theory as there was no need to introduce any additional physical concepts or postulates. In a second paper entitled *"The Size of Fundamental Particles"* [3], I applied the same principle again to calculate the diameter of the electron. As a result the size of the electron turned out to be smaller than 10.166

times the Planck length. Despite of not including the quantum fluctuations of the empty space, the special version of the universal uncertainty principle turned out to be an invaluable quantum mechanical principle.

2. Abbreviations

In order to refer to these two principles I shall use the following abbreviations:

(a) GUUP or "general UUP": stands for *General Universal Uncertainty Principle*. This is the most general uncertainty principle, and

(b) SUUP or "special UUP": stands for *Special Universal Uncertainty Principle* [2]. This is a special version of the GUUP principle.

3. Rationale

The universal uncertainty principle has to satisfy the following conditions

1) The principle must be quadratic in $\Delta p_x \Delta x$

2) When $P_Z = 0$ and $L_Z = 0$ the principle will reduce to $\Delta p_x \Delta x \geq \frac{\hbar}{2}$

3) When $\Delta p_x = 0$ and $\Delta x = 0$ the principle will reduce to $P_Z L_Z \geq \frac{\hbar}{2}$

Let's consider these three conditions separately

1) We shall adopt a second order uncertainty principle. We want this principle to be as general as possible to ensure an accurate description of nature for all phenomena including black holes. One of the most exciting aspects of this principle is that the entropy of the black hole emerges naturally without introducing any additional postulates. This is not possible using a linear principle. Based on this observation we can postulate that the universal principle will have the following form

$$(\Delta p_x \Delta x)^2 \geq \left(\frac{\hbar}{2}\right)^2 + other\, terms \tag{3-1}$$

Hence taking the square root on both sides yields the following relation

$$\Delta p_x \Delta x \geq \sqrt{\left(\frac{\hbar}{2}\right)^2 + other\ terms} \tag{3-2}$$

2) When the effects of quantum fluctuations of space-time are neglected. (mathematically means that $P_Z = 0$ and $L_Z = 0$), the relation will be identical to the Heisenberg uncertainty principle (HUP). Thus, under these conditions, the relation will reduce to

$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \tag{3-3}$$

The reason of this is that a wave packet representing the wave function *ψ(x,y,z,t)* of the particle is formed by the addition of a number of different wavelengths that produce interference (the superposition principle in quantum mechanics gives rise to interference). The more wavelengths we add the more localized the wave function will be and therefore the probability of finding the particle in a cubic box of volume $dV = dx\ dy\ dz$ will be higher. This is so because the square of the wave function $|\psi(x,y,z,t)|^2$ is the probability density of a measurement of the finding the particle in the cubic volume dV. Thus the probability $P_{x1,x2,y1,y2,z1,z2}(t)$ of finding the particle in a cubic volume defined as

$$x \in [x1, x2]\ and\ y \in [y1, y2]\ and\ z \in [z1, z2])$$
$$\text{where } x1 < x2;\ y1 < y2;\ z1 < z2$$

at time t will be

$$P_{x1,x2,y1,y2,z1,z2}(t) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} |\psi\ (x, y, z, t)|^2\ dxdydz \qquad (3\text{-}4)$$

For the special case one spatial dimension and a time-independent wave function, the above expression reduces to

$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} |\Psi(x)|^2 dx \qquad (3\text{-}5)$$

The integral (3-4) shows that the more localized the wave function the higher the probability of finding the particle in a given volume. However, this mechanism will make the momentum of the particle more uncertain. The reason is that, according to de Broglie, each individual wavelength has a momentum associated with it which is given by

$$p = \frac{h}{\lambda} \qquad (3\text{-}6)$$

Because the wave function of the particle is composed of a large number of different wavelengths of different amplitudes (only the de Broglie relationships are shown here):

$$p_1 = \frac{h}{\lambda_1};\quad p_2 = \frac{h}{\lambda_2};\quad p_3 = \frac{h}{\lambda_3};\quad p_4 = \frac{h}{\lambda_4};\quad \ldots\ ;\quad p_n = \frac{h}{\lambda_n} \qquad (3\text{-}7)$$

the momentum of the particle becomes more uncertain (which is the momentum of the particle $p_1, p_2, p_3, p_4, \ldots or\ p_n$?) From this approximate analysis we see that the HUP relates to the wave nature of the wave packet and not to the quantum fluctuations of the vacuum (See Fig 1).

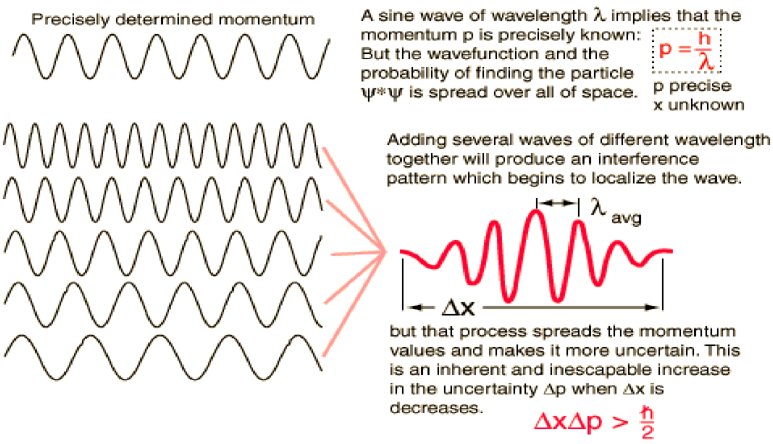

Fig 1: A wave packet is made of an infinite number of sine waves. *Graphics credits: Copyright R. Nave, Hyperphysics (http://hyperphysics.phy-astr.gsu.edu/hbase/uncer.html)*

3) When the effects of the uncertainties due to the wave nature of the wave packet describing the particle are neglected (mathematically means that $\Delta p = 0$ and $\Delta x = 0$), the principle will reduce to

$$P_Z L_Z \geq \frac{h}{4\pi} \tag{3-8}$$

4. The General Momentum-Position Universal Uncertainty Relations (Momentum-Position GUUP)

The only way of satisfying all three conditions given in the previous section simultaneously is to define the general momentum-position universal uncertainty relations as follows:

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z \left(\Delta x + L_Z\right) - \frac{\hbar}{4}\left(\Delta p_x + P_Z\right) L_Z} \tag{4-1a}$$

$$\Delta p_y \Delta y \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z \left(\Delta y + L_Z\right) - \frac{\hbar}{4}\left(\Delta p_y + P_Z\right) L_Z} \tag{4-1b}$$

$$\Delta p_z \Delta z \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z \left(\Delta z + L_Z\right) - \frac{\hbar}{4}\left(\Delta p_z + P_Z\right) L_Z} \tag{4-1c}$$

Where

h = Planck's constant

$\hbar$ = reduced Planck's constant ($\hbar = \dfrac{h}{2\pi}$)

Δp_x = Uncertainty in the momentum of a particle along the *x* axis due to its wave nature (wave packet representing the particle). This uncertainty does not include the uncertainty in the zero point momentum, P_{ZPM} or P_Z , due to the quantum fluctuations of space-time.

Δp_y = Uncertainty in the momentum of a particle along the *y* axis due to its wave nature (wave packet representing the particle). This uncertainty does not include the uncertainty in the zero point momentum, P_{ZPM} or P_Z , due to the quantum fluctuations of space-time.

Δp_z = Uncertainty in the momentum of a particle along the *z* axis due to its wave nature (wave packet representing the particle). This uncertainty does not include the uncertainty in the zero point momentum, P_{ZPM} or P_Z , due to the quantum fluctuations of space-time.

Δx = Uncertainty in the position of the particle along the *x* axis due to the wave packet representing the particle. This uncertainty does not include the uncertainty in the position, L_Z , due to the granularity of space.

Δy = Uncertainty in the position of the particle along the *y* axis due to the wave packet representing the particle. This uncertainty does not include the uncertainty in the position, L_Z , due to the granularity of space.

Δz = Uncertainty in the position of the particle along the *z* axis due to the wave packet representing the particle. This uncertainty does not include the uncertainty in the position, L_Z , due to the granularity of space.

P_{ZPM} or P_Z = Uncertainty in the momentum of the particle in the direction of the movement of the wave packet representing the particle (the direction could be either the x-axis, the y-axis or the z-axis). This momentum uncertainty is due to the quantum fluctuations of space-time and does not include any of the uncertainties in the momentum: Δp_x, Δp_y or Δp_z , defined above. This momentum is also know as *zero point momentum*.

L_Z = Uncertainty in the position of the particle due to the granularity of space. This uncertainty does not include the uncertainty in the position along any of the three Cartesian coordinate axes (this means that it does not include neither Δx , nor Δy nor Δz). It seems logical to assume that L_Z is identical to the Planck length, L_P. However, these two lengths could be slightly different. We assume that the Planck length is the minimum length with physical meaning. It is worthy to remark that the minimum value of this length uncertainty cannot be measured experimentally with the present technology.

See **Appendix 1** for the theoretical verification.

5. The General Energy-Time Universal Uncertainty Relation (Energy-Time GUUP)

In a similar way as we did when we defined relations (4-1a), (4-1b) and (4-1c), we can define the general energy-time universal uncertainty relation as follows

$$\Delta E \, \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} E_Z \left(\Delta t + T_Z \right) - \frac{\hbar}{4} \left(\Delta E + E_Z \right) T_Z} \qquad (5\text{-}1)$$

where

ΔE = Uncertainty in the energy of a particle or energy of a particle whose lifetime is Δt. This energy uncertainty does not include the zero point energy uncertainty, $E_Z = E_{ZPE}$, due to the quantum fluctuations of space-time.

Δt = Time uncertainty. We can interpret Δt in several ways:
a) Time uncertainty in the duration of a given energy state of the particle; or the lifetime of a particle of a given energy, ΔE.
b) Time needed to measure the energy of a system to within an uncertainty of ΔE.
c) Time taken by the particle to travel a distance equal to the uncertainty, Δx (or Δy or Δz), in its position. The nature of a given phenomenon determines the interpretation to adopt.

E_{ZPE} or E_Z = Uncertainty in the energy of a particle due to the quantum fluctuations of space-time. This uncertainty does not include the energy uncertainty, ΔE, defined above. E_{ZPE} is also known as *zero point energy*.

T_Z = Time uncertainty of the particle due to the granularity of time. This uncertainty does not include the time uncertainty, Δt, defined above. It is worthy to remark that the value of this time granularity cannot be measured experimentally with the present technology. However, it seems logical to assume that this uncertainty is identical to the Planck time, T_P. However, these two time uncertainties could be slightly different. We assume that the Planck time is the minimum time with physical meaning.

See **Appendix 2** for the theoretical verification.

6. The Special Universal Uncertainty Principle

The special universal uncertainty relations can be derived from the general universal uncertainty relations by making $P_Z = 0$ in the momentum-position relations; and taking $E_Z = 0$ in the energy-time relation as shown in the following two subsections.

6a) The special momentum-position universal uncertainty relations

Let us consider the GUUP principle given by relation (4-1a)

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z (\Delta x + L_Z) - \frac{\hbar}{4} (\Delta p_x + P_Z) L_Z} \qquad (6\text{-}1)$$

Let us make $P_Z = 0$. This yields

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} 0 (\Delta x + L_Z) - \frac{\hbar}{4} (\Delta p_x + 0) L_Z} \qquad (6\text{-}2)$$

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_x L_Z} \qquad (6\text{-}3)$$

This is the special momentum-position universal uncertainty principle (momentum-position SUUP). The same analysis can be applied to relations (4-1b) and (4-1c) to obtain the corresponding special relations. Thus the three special momentum-position universal uncertainty relations turn out to be

Momentum-position SUUP

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_x L_Z} \qquad (6\text{-}4a)$$

$$\Delta p_y \Delta y \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_y L_Z} \qquad (6\text{-}4b)$$

$$\Delta p_z \Delta z \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_z L_Z} \qquad (6\text{-}4c)$$

6b) The special energy-time universal uncertainty relation

Let us consider the GUUP principle given by relation (5-1)

$$\Delta E \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} E_Z (\Delta t + T_Z) - \frac{\hbar}{4} (\Delta E + E_Z) T_Z} \qquad (6\text{-}5)$$

Let us make $E_Z = 0$. This yields

$$\Delta E \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} 0 (\Delta t + T_Z) - \frac{\hbar}{4} (\Delta E + 0) T_Z} \qquad (6\text{-}6)$$

Energy-time SUUP

$$\Delta E \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta E T_Z} \qquad (6\text{-}7)$$

This is the special energy-time universal uncertainty principle (energy-time SUUP).

7. Comparison

In this section we compare, side by side, the Heisenberg uncertainty relations with the corresponding special universal uncertainty counterparts.

Description of the uncertainty	Heisenberg uncertainty relations	Special universal uncertainty relations
Momentum-position uncertainty relation (x axis)	$\Delta p_x \Delta x \geq \frac{\hbar}{2}$	$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_x L_P}$
Momentum-position uncertainty relation (y axis)	$\Delta p_y \Delta y \geq \frac{\hbar}{2}$	$\Delta p_y \Delta y \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_y L_P}$
Momentum-position uncertainty relation (z axis)	$\Delta p_z \Delta z \geq \frac{\hbar}{2}$	$\Delta p_z \Delta z \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta p_z L_P}$
Energy-time uncertainty relation	$\Delta E \Delta t \geq \frac{\hbar}{2}$	$\Delta E \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} \Delta E T_P}$

Table 1: Comparison of the Heisenberg uncertainty relations with the special universal uncertainty relations. Note that L_Z has been replaced by L_P and T_Z by T_P .

8. Conclusions

In summary, this paper introduced the General Universal Uncertainty Principle (GUUP) which is an extension to the legendary Heisenberg Uncertainty Principle (HUP). The special universal uncertainty principles help us to make the following three predictions:

1) **The law for the temperature of the black hole**. This law turned out to be [2]

Black Hole temperature formula derived from the SUUP

$$T = \frac{1}{8\sqrt{\pi}} \frac{h c^3}{16 \pi^2 k_B G M} \left(\sqrt{\frac{L_P^2}{\pi R^2} + 64 \pi} - \frac{L_P}{\sqrt{\pi} R} \right) \tag{8-1}$$

Thus the universal principle, in its special version, has already shown that the Hawking formula for the black hole temperature is a special case of equation (8-1) when we make the Planck length, L_P , equal to zero, as shown below:

$$T = \frac{1}{8\sqrt{\pi}} \frac{h c^3}{16 \pi^2 k_B G M} \sqrt{64 \pi} \qquad (8\text{-}2)$$

After simplification the above equation transforms into the Hawking formula

Hawking formula for the temperature of a black hole

$$T = T_H = \frac{h c^3}{16 \pi^2 k_B G M} \qquad (8\text{-}3)$$

Now we can predict the temperature of black holes with higher accuracy.

2) **The size of the electron**. The SUUP predicts that there is an upper limit to the diameter of the electron [3]. The prediction is that the diameter of the electron is smaller than 10.166 times the Planck length.

3) **The Einstein's formula of equivalence of mass and energy**. I have derived the famous Einstein's formula, $E = m c^2$, from the special universal uncertainty relations [4]. This means that quantum physics encompasses not only all of classical physics but also all of relativistic physics.

Thus part of the potential of this formulation has already been proven as it passed the above three tests. However, it is too early to envisage the extent of the implications of the present formulation.

Appendix 1
Theoretical Verification of the Spatial General Universal Uncertainty Relations

Let us consider the momentum-position relation (4-1a)

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z \left(\Delta x + L_Z \right) - \frac{\hbar}{4} \left(\Delta p_x + P_Z \right) L_Z}$$

(a) where we make $P_Z = 0$ and $L_Z = 0$

$$\Delta p_x \Delta x \geq \sqrt{\frac{\hbar^2}{4} - 0 - 0}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

This is the momentum-position Heisenberg uncertainty relation along the x-axis.
A similar analysis proves the other two momentum-position uncertainty relations:

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

This is the momentum-position Heisenberg uncertainty relation along the y-axis.

$$\Delta p_z \Delta z \geq \frac{\hbar}{2}$$

This is the momentum-position Heisenberg uncertainty relation along the z-axis.

(b) where we make $\Delta p_x = 0$ and $\Delta x = 0$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z \left(0 + L_Z\right) - \frac{\hbar}{4} \left(0 + P_Z\right) L_Z}$$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} P_Z L_Z - \frac{\hbar}{4} P_Z L_Z}$$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{2} P_Z L_Z}$$

$$0 \geq \frac{\hbar^2}{4} - \frac{\hbar}{2} P_Z L_Z$$

$$\frac{\hbar}{2} P_Z L_Z \geq \frac{\hbar^2}{4}$$

$$P_Z L_Z \geq \frac{\hbar}{2}$$

Because Heisenberg did not include neither P_Z nor L_Z in his momentum-position uncertainty relations, we could call the last expression the "*Zero Point Momentum-Planck length uncertainty relation*", just to refer to it in the future.

Appendix 2
Theoretical Verification of the Temporal General Universal Uncertainty Relation

Let us consider the general energy-time relation (5-1)

$$\Delta E \Delta t \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} E_Z \left(\Delta t + T_Z\right) - \frac{\hbar}{4} \left(\Delta E + E_Z\right) T_Z}$$

(a) where we make $E_Z = 0$ and $T_Z = 0$

$$\Delta E\,\Delta t \geq \sqrt{\frac{\hbar^2}{4} - 0 - 0}$$

$$\Delta E\,\Delta t \geq \frac{\hbar}{2}$$

This is the energy-time Heisenberg uncertainty relation.

(b) where we make $\Delta E = 0$ and $\Delta T = 0$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} E_Z (0 + T_Z) - \frac{\hbar}{4} (0 + E_Z) T_Z}$$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{4} E_Z T_Z - \frac{\hbar}{4} E_Z T_Z}$$

$$0 \geq \sqrt{\frac{\hbar^2}{4} - \frac{\hbar}{2} E_Z T_Z}$$

$$0 \geq \frac{\hbar^2}{4} - \frac{\hbar}{2} E_Z T_Z$$

$$\frac{\hbar}{2} E_Z T_Z \geq \frac{\hbar^2}{4}$$

$$E_Z T_Z \geq \frac{\hbar}{2}$$

Because Heisenberg did not include neither E_Z nor T_Z in his energy-time uncertainty relation, we could call the last expression the "*Zero Point Energy-Planck time uncertainty relation*", just to refer to it in the future.

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