

TITLE: PEEKING UNDER THE EVENT HORIZON

AUTHOR: Francois Zinserling (23 July 2014); francois@designtech.co.za

Abstract:

It is shown mathematically with Newtonian physics that a particle or photon can exit from underneath the event horizon of a singularity or black hole, where it can be observed, even though it may never fully escape the gravitational field. The radius of the event horizon is an observed effect, dependent on the distance of the observer from the singularity, where R_s (Schwarzschild radius) is the event horizon observed from a distance approaching infinity. We will show that it is possible for matter to exit from the event horizon, and be observed outside the event horizon, as has been observed in many instances. While we may not be able to observe objects under the event horizon, we may observe how objects nearer the event horizon are affected from within.

Introduction:

Upon initial discussions regarding black holes, these were presented as stellar objects unable to reveal information about themselves, that any events occurring underneath the event horizon, or Schwarzschild radius (R_s) can never be revealed to the outside world. [6][7][8]. This model became problematic when numerous observations were made of activity immediately surrounding black holes [9][10][11][12][13], indicating that there was more there than mere radiation of objects outside of R_s . More models followed, including various models suggesting interaction of matter with the event horizon and accretion disk [14] and Hawking's popular radiation model. [4][5]

It is the purpose of this document to show that a singularity is not surrounded by a "bubble of invisibility" under the event horizon, but that more of it can be observed, as an observer gets closer to what was initially the event horizon; that the event horizon is not a physical construct to be encountered in space, but merely an observational effect.

In 1916, Schwarzschild [1] provided a solution to Einstein's General Relativity equations [2] around black holes, resulting in the spacetime metric as below [3]

$$ds^2 = g_{ab} dx^a dx^b = - \left(1 - \frac{G}{c^2} \frac{2M}{r}\right) dt^2 + \left(1 - \frac{G}{c^2} \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (1)$$

, which becomes singular at a radius of:

$$r = \frac{2 * G * M}{c^2} \quad (2)$$

This radius became known as the Schwarzschild radius R_s

The same solution can be achieved using Newtonian physics. To start, we will derive, with explanations, the following known equations below from Newtonian physics. The purpose of this exercise is to show that the same result is achieved as with General Relativity and Schwarzschild's solution, thus rendering simple Newtonian physics quite valid for our further consideration in this document:

- a. Escape velocity

$$V_e = \sqrt{\frac{2 * G * M}{r}} \quad (3)$$

b. Event horizon, Schwarzschild radius

$$R_s = \frac{2 * G * M}{c^2} \quad (4)$$

Where

M = mass of the black hole or singularity

$$G = 6.67 * 10^{-11} \frac{m^3}{kg * s^2} \text{ (Newton's gravitational constant)} \quad (5)$$

$$c \cong 3 * 10^8 m/s \text{ (speed of light in vacuum)} \quad (6)$$

These equations are valid for a black hole under ideal conditions, but have become misleading when used in conversation, or even in science papers. The prime example of this, pertaining to this document, is that an “escape velocity” is needed in order to reach an infinite distance from the black hole. This has become more loosely (but still scientifically correct), termed as “nothing can escape from a black hole”. Unfortunately it has further deteriorated to “nothing can escape through the event horizon” as if the event horizon is an impenetrable barrier.

This erroneous mental image was further strengthened by application of general relativity around a singularity in empty space [15] showing all paths of light from the singularity lead back to the singularity[16]. Although mathematically correct, the assumption of this model remains that the black hole is in empty space. An object breaching (from our observation point) the event horizon, will necessarily disturb this model when it adds its own space-time to the scene.

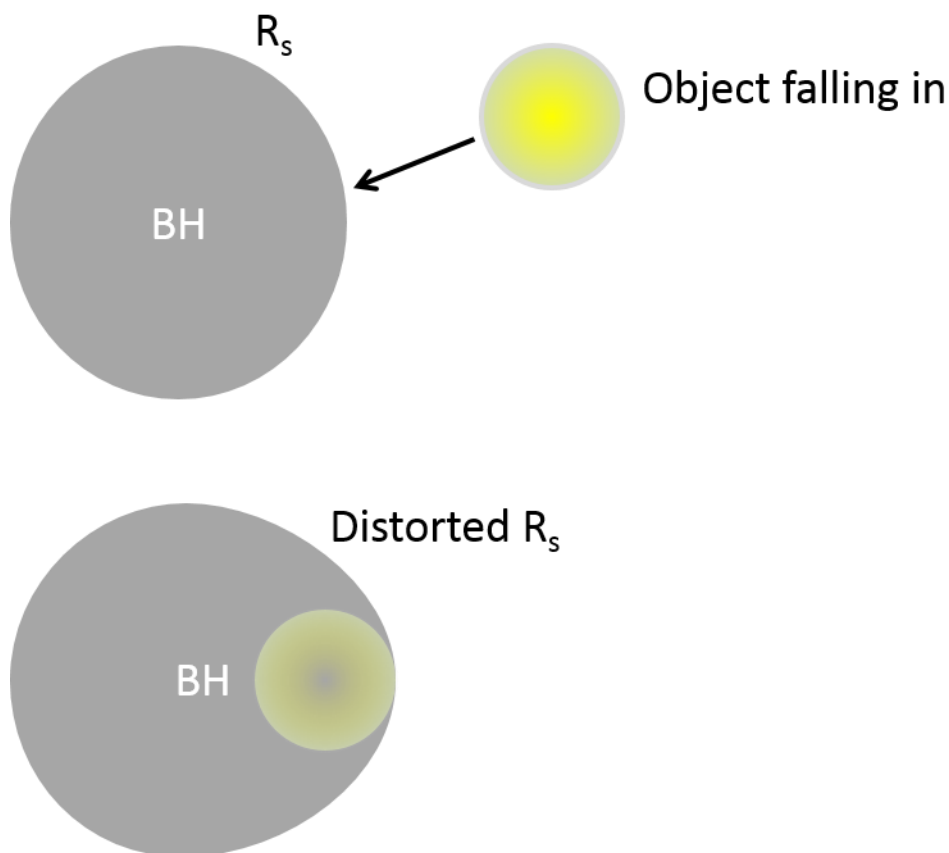


Figure 1: In the top figure the event horizon (R_s) of the black hole is undisturbed. In the bottom figure the space-time of the Event Horizon is distorted by the in-falling object.

It can for instance be shown that the required escape velocity for our own solar system (from SGR-A*) is greater than our actual orbiting velocity around it, yet it doesn't mean we're doomed to soon become accretion material. Said differently: We've not actually managed to escape the gravitational field of SGR-A*. We are, however, "outside" the event horizon. The point to note here: If it hasn't escaped, does not mean it cannot be observed.

We will follow on, using the same methods of calculation, to show that it is possible to "escape" past the Schwarzschild radius, even from well within the radius.

Discussion:

1. ESCAPE VELOCITY

Definition: To escape from the gravitational pull of a massive object, with only an initial velocity, and no further forces acting to accelerate or decelerate it, the escaping object tries to reach an infinite distance away from the gravitational pull at a remaining velocity of 0 or greater. Since gravitational force is proportional to $F \propto \frac{1}{r^2}$, where r is the distance between the objects; at an infinite distance the force tends to 0.

Assume the escaping object starts with velocity V_e and ends with velocity $V = 0$
It also starts with a local potential energy of

$$PE = \frac{-G*M*m}{r} \quad (7)$$

, and ends with a potential energy of $PE = 0$, since the potential energy increases as the object moves away.

For any particle, total energy remains constant (rem: no other external forces)

$$KE_1 + PE_1 = KE_2 + PE_2 \quad (8)$$

$$\frac{1}{2} * m * V_e^2 - \frac{G*M*m}{r} = 0 + 0 \quad (9)$$

$$V_e = \sqrt{\frac{2*G*M}{r}} \quad (10)$$

This is the initial velocity required to escape the gravitational field, or escape velocity.

Or another way of determining this same velocity:

Gravitational force between two masses M and m , distance r apart:

$$F = \frac{G*M*m}{r^2} \quad (11)$$

Work done to move the mass m over a distance dr

$$dW = \frac{G*M*m}{r^2} dr \quad (12)$$

Total work done from starting distance r_i to infinity

$$W = \int_{r_i}^{\infty} \frac{G*M*m}{r^2} dr \quad (13)$$

$$W = \frac{-G*M*m}{r_i} \quad (14)$$

Kinetic energy required to do this work:

$$\frac{1}{2} * m * V_e^2 = \frac{G * M * m}{r_i} \quad (15)$$

$$V_e = \sqrt{\frac{2 * G * M}{r_i}} \quad (16)$$

2. SCHWARZSCHILD RADIUS

From the above equation we see that the required escape velocity increases as the mass M increases, and it also requires a greater escape velocity for smaller values of r.

No object can exceed the speed of light [Einstein 1905], so we can place an upper limit on V_e

$$V_e \leq c$$

Replacing V_e (max) with c in the above equation:

$$c^2 = \frac{2 * G * M}{r_i} \quad (17)$$

$$r_i = \frac{2 * G * M}{c^2} \quad (18)$$

This is known as the **Schwarzschild radius** from within nothing, not even light can escape.

$$R_s = \frac{2 * G * M}{c^2} \quad (19)$$

3. EVENT HORIZON

Classic definition: The term event horizon defines the visible limit of the Schwarzschild radius around a black hole. If nothing can escape from this radius, not even light, then it is not possible to observe anything “under” this radius. Above the radius events can be seen, below the radius events cannot be seen, and no information can be obtained about what goes on below the event horizon.

4. REVISITING OUR ASSUMPTIONS

Following the above paragraphs, it seems understandable that no information can escape a black hole. However we have to go back to these paragraphs and analyse a few of the assumptions we made:

“To escape from the gravitational pull of a massive object, with only an initial velocity, and **no further forces** acting to accelerate or decelerate it, the escaping object tries to reach **an infinite distance** away from the gravitational pull at a velocity of 0 or greater.”

NOTE:

- Black holes don't exist in empty space. There are stars, dust, and other galaxies that may influence a particle or photon that wishes to escape from a black hole. Any of these may exert a gravitational pull on a particle that may alter its conditions of escape. Considering this argument from a general relativity perspective, it can be argued that any object that enters R_s , is surrounded by its own space-time, and will hence affect the idealistic GR space-time model of the black hole.

- While a photon or particle may not be able to escape to an infinite distance, it may well be able to escape a reasonable distance beyond the event horizon, where it may interact with another object, hence it can be observed!

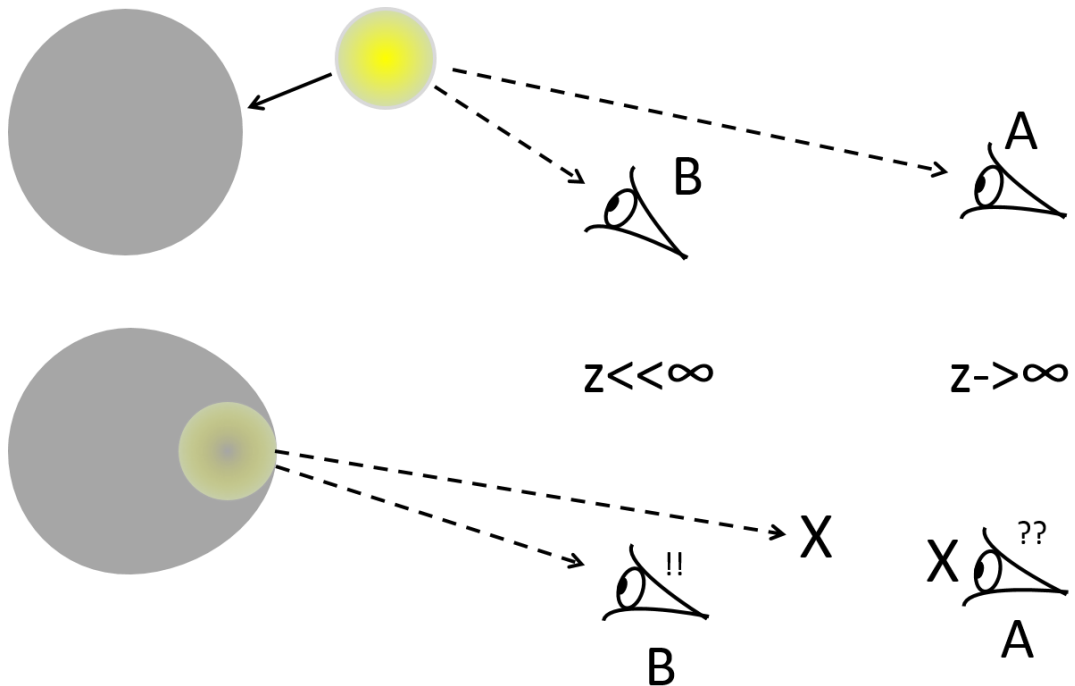


Figure 2: In the top figure both observer A and B can observe the object, either by its emitted light or particles ejected at very high velocity. In the bottom figure observer A does not see the object under the event horizon, or the particle does not have enough energy to reach observer A. Observer B can still see the object, or observe the particles emitted.

5. OBSERVATION HORIZON

We will now revisit the initial equations, where the photon (or particle) is not required to travel an infinite distance to escape a black hole, but only a finite distance. If the photon or particle doesn't "escape" the black hole (to infinite distance), but only manages to travel as far as distance z , where it finds its kinetic energy depleted, and its velocity is zero. Up until this point an observer can still interact with - or observe - the particle.

For a distance $z > R_s$

Total work done from any starting distance r_z to distance z

$$W = \int_{r_z}^z \frac{G*M*m}{r^2} dr \quad (20)$$

$$W = \frac{-G*M*m}{z} + \frac{G*M*m}{r_z} \quad (21)$$

To reach this distance z , we will need to equip the particle with an initial velocity V_z such that: (showing algebra steps)

$$\frac{1}{2} * m * V_z^2 = \frac{-G*M*m}{z} + \frac{G*M*m}{r_z} \quad (22)$$

$$V_z^2 = \frac{-2*G*M}{z} + \frac{2*G*M}{r_z} \quad (23)$$

$$z * r_z * V_z^2 = -2 * r_z * G * M + 2 * z * G * M \quad (24)$$

$$r_z * (z * V_z^2 + 2 * G * M) = 2 * z * G * M \quad (25)$$

$$r_z = \frac{2 * z * G * M}{z * V_z^2 + 2 * G * M} \quad (26)$$

In the case of a particle approaching $V_z \rightarrow c$, or for a light particle:

$$r_z = \frac{2 * z * G * M}{z * c^2 + 2 * G * M} \quad (27)$$

$$r_z = \frac{z * \frac{2 * G * M}{c^2}}{z + \frac{2 * G * M}{c^2}} \quad (28)$$

But the **Schwarzschild radius**

$$R_s = \frac{2 * G * M}{c^2} \quad (29)$$

$$r_z = \frac{z * R_s}{z + R_s} \quad (30)$$

This will be the event horizon to an observer at distance z .

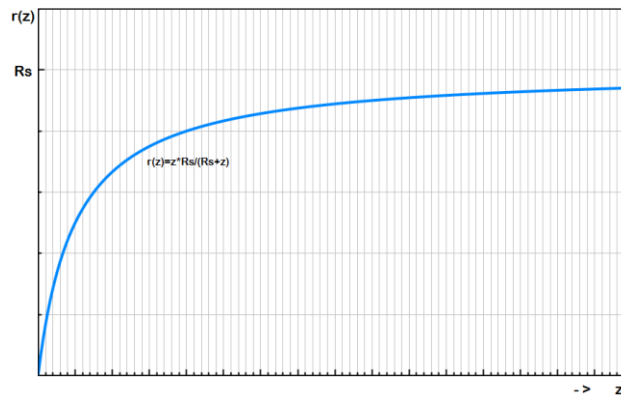


Figure 3: Observable Event Horizon, as a function of distance z from the black hole

$$r_z = \frac{z * R_s}{z + R_s} \quad (31)$$

NOTE:

- $r_z \leq R_s$ for all $z \geq 0$ This observed radius is equal or smaller than the **Schwarzschild radius**
- $r_z \leq z$ for all $z \geq 0$ This radius gets smaller as the observer gets closer to the black hole, and is always closer to the black hole than the current position.
- Validity check: $r_z = R_s$ for $z \rightarrow \infty$; (d/dx rule)
- $r_z \rightarrow 0$ for $z \rightarrow 0$; in fact $r_z = 0$ for $z = 0$
- $r_z \leq z$ for all $z \geq 0$

6. BUT NOTHING CAN ESCAPE THE EVENT HORIZON, RIGHT!?

We will show that a particle, from within the **Schwarzschild radius**, can have enough energy to reach the event horizon, and go beyond, where it may be observed.

Let's say the particle starts at $z < R_s$ (under the event horizon) and needs enough energy to travel to r_{ii} , somewhere before or past R_s .

(r_{ii} is the distance to which the particle or photon can travel, given enough energy, and where it can be observed.)

Total work done from any starting distance z to some distance r_{ii} where $r_{ii} > z$.

$$W = \int_z^{r_{ii}} \frac{G*M*m}{r^2} dr \quad (32)$$

$$W = \frac{-G*M*m}{r_{ii}} + \frac{G*M*m}{z} \text{ for } 0 < z < R_s; r_{ii} > z \quad (33)$$

Note: $z > 0!$ (This possibly implies we cannot escape from within the singularity, or we have undefined physics in the region $z=0!$)

To reach r_{ii} from z , we will need to equip the particle with an initial velocity V_{ii} such that: (not showing algebra steps)

$$\frac{1}{2} * m * V_{ii}^2 = \frac{-G*M*m}{r_{ii}} + \frac{G*M*m}{z} \quad (34)$$

$$r_{ii} = \frac{z*R_s}{R_s - z} \quad (35)$$

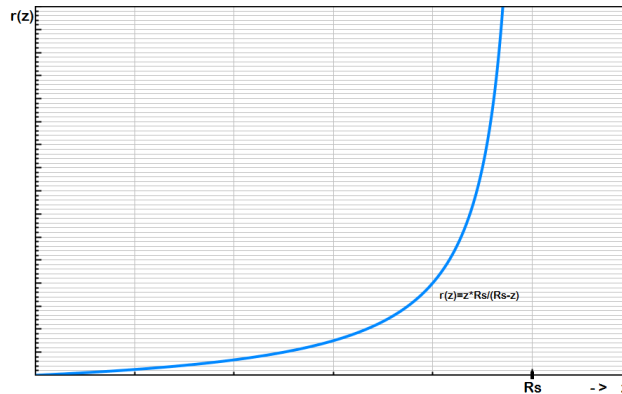


Figure 4: Particle's view of escape radius, as a function of distance z from the black hole

$$r_{ii} = \frac{z*R_s}{R_s - z} \quad (36)$$

Note:

- $r_{ii} \rightarrow \infty$ as $z \rightarrow R_s$ (the photon sees it can travel up to infinity)
- $r_{ii} \rightarrow 0$ as $z \rightarrow 0$ (the photon sees no escape from within the singularity)
- and $r_{ii} > z$ for all ($z > 0; r_{ii} < R_s$) (the particle or photon can get enough energy to travel outward from its current position)

If the particle or photon had to escape as far as R_s

$$W = \frac{-G*M*m}{R_s} + \frac{G*M*m}{z} \text{ for } 0 < x < R_s \quad (37)$$

But the Schwarzschild radius

$$R_s = \frac{2 * G * M}{c^2} \quad (38)$$

$$W = \frac{-m * c^2}{2} + \frac{G * M * m}{z} \text{ for } 0 < z < R_s \quad (39)$$

Kinetic energy required to reach R_s

$$\frac{1}{2} * m * V_z^2 = \frac{-m * c^2}{2} + \frac{G * M * m}{z} \text{ for } 0 < z < R_s \quad (40)$$

$$V_z = \sqrt{\frac{2 * G * M}{z} - c^2} \text{ for } 0 < z < R_s \quad (41)$$

We can test some limits:

Test 1: for $V_z > 0$

$$\sqrt{\frac{2 * G * M}{z} - c^2} > 0 \text{ for } 0 < z < R_s \quad (42)$$

$$z < \frac{2 * G * M}{c^2} \quad (43)$$

Which just means $z < R_s$ which was one of our original assumptions. Thus for any $0 < z < R_s$, V_z is a real and thus a valid number.

Test2: for $V_z < c$

$$\sqrt{\frac{2 * G * M}{z} - c^2} < c \text{ for } 0 < z < R_s \quad (44)$$

$$z > \frac{G * M}{c^2} \quad (45)$$

Which equates $\frac{1}{2} R_s$.

Thus a particle from $R_s/2$ can escape to R_s , with an initial velocity $V_z \rightarrow c$ (approaching c)

In other words, a particle or photon in the band between $R_s/2$ and R_s can possibly travel far enough and be observed at $z > R_s$

7. CONCLUSIONS

- It was shown that the event horizon and/or **Schwarzschild radius** is a mathematical radius, only applicable to observers who are at an infinite distance from a black hole.
- It was shown (eq 30) that for any observer at a finite distance 'z', the observed event horizon would be smaller than R_s :

$$r_z = \frac{z * R_s}{z + R_s} \quad (46)$$

- It was shown (eq 35) that any particle at position ($z > 0$; $r_{ii} < R_s$) can gain energy to allow it to move away from the singularity, and the freedom of movement increases as it approaches R_s .

$$r_{ii} = \frac{z * R_s}{R_s - z} \quad (47)$$

- It was shown that it is possible for a particle or photon from within the classic **Schwarzschild radius** to be observed outside the **Schwarzschild radius**. We cannot observe under the observable event horizon, but we can observe what comes out of the observable event horizon.
- A black hole reveals more information as an observer gets closer.
- From the distance we are to SGR-A*, we may not be able to distinguish anything under R_s , but we are possibly able to observe what comes out of R_s and interacts with the surrounding material.
- We need to revise our understanding of observations around black holes.
- This solution could possibly be improved by a student of General Relativity.
- We can look closer at further evidence that there is activity observed from underneath R_s . Outflow[9], bow-shock[10], outflows[11], flares[12], jets[13]. These are but a few of many observations that have been made that would provide evidence toward this hypothesis.

8. EXAMPLE CALCULATION and AUTHOR NOTES

Estimated and rough figures taken for example calculation

- Mass of Milky Way SMBH SGR-A* $3.6 * 10^6 * \text{sol}$
- Mass of sol $2 * 10^{30} \text{ kg}$
- $G = 6.67 * 10^{-11} \frac{\text{m}^3}{\text{kg} * \text{s}^2}$

Calculate:

$$R_s(\text{SGR-A}^*) \approx 1.066 * 10^{10} \text{ m}$$

Calculate: Observed from $z = 10 * R_s$ from SGR-A*:

$$R_z(\text{SGR-A}^*) \approx 0.969 * 10^{10} \text{ m}$$

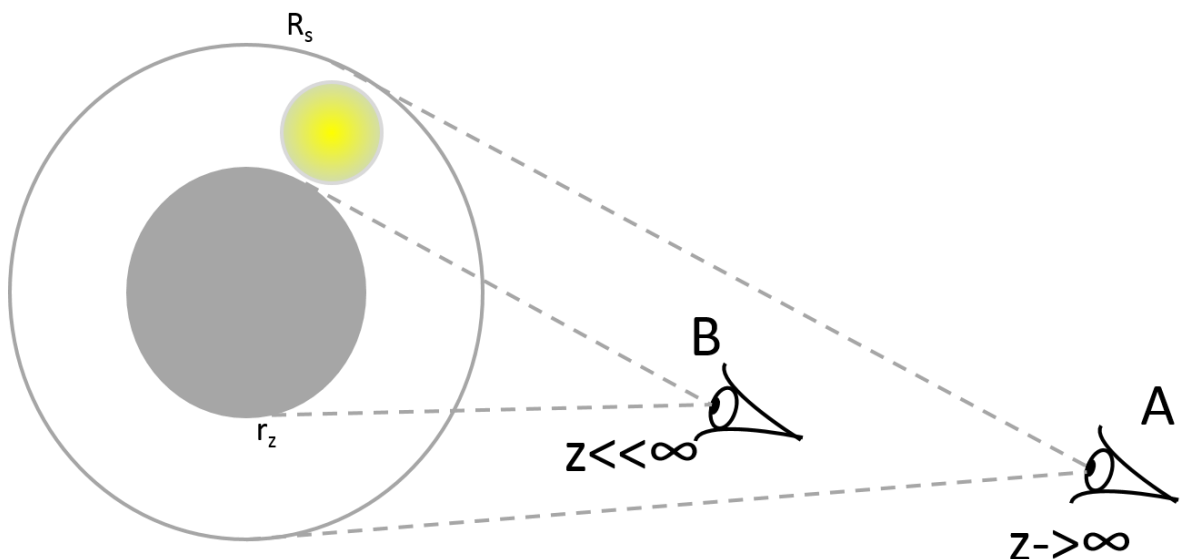


Figure 5: Observer A can only see what's outside the classic event horizon, or R_s . Observer B can see under R_s , as far as $r(z)$. If observer B is affected by what it observes, observer A can observe the effect on B.

How is this significant?

This is significant because what we see as the event horizon, is not what objects close-up see as the event horizon. Interactions we see in that vicinity is therefore NOT because of the object's interaction with the event horizon.

While we may not be able to see (from earth) what's underneath R_s , we can see objects outside and close to R_s , and if we can possibly detect what these objects observe, then we are getting information from within R_s !

For example, let an orbiting body be one observer at $z \ll \infty$. We can look for an object at $R_s < z \ll \infty$ that shows signs of being radiated or pushed or affected outward from SGR-A*, or we can see an object with a bow-shock as it passes through the radiation exiting R_s

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10. FIGURES

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