Two conjectures, on the primes of the form 6k plus 1 respectively of the form 6k minus 1

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Abstract. In this paper I make two conjectures, one about how could be expressed a prime of the form 6k + 1 and one about how could be expressed a prime of the form 6k – 1.

Conjecture 1:

Any prime p of the form 6*k + 1 greater than or equal to 13 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 229)

: 13*3 – 20 = 19, prime, so [p, q, r] = [13, 5, 19];
: 19*3 – 20 = 37, prime, so [p, q, r] = [19, 5, 37];
: 31*3 – 20 = 73, prime, so [p, q, r] = [31, 5, 73];
: 37*3 – 110 = 1, so [p, q, r] = [37, 11, 1];
: 43*3 – 20 = 109, prime, so [p, q, r] = [43, 5, 109] and also 43*3 – 110 = 29, prime, so [p, q, r] = [43, 11, 29];
: 61*3 – 20 = 163, prime, so [p, q, r] = [61, 5, 163] and also 61*3 – 110 = 73, prime, so [p, q, r] = [61, 11, 73];

We also found the following triplets [p, q, r]: [67, 5, 181], [73, 5, 199], [73, 11, 109], [79, 11, 127], [97, 5, 271], [97, 11, 181], [97, 17, 19], [103, 11, 199], [109, 5, 307], [127, 11, 271], [127, 17, 109], [139, 5, 397], [139, 11, 307], [151, 5, 433], [151, 17, 71], [157, 17, 199], [163, 1, 379], [181, 5, 523], [181, 11, 433], [181, 17, 271], [181, 23, 37], [193, 17, 307], [193, 23, 73], [199,5, 577], [199, 11, 487], [211, 5, 613], [211, 11, 523], [211, 17, 19^2], [211, 23, 127], [223, 17, 397], [223, 506, 163], [229, 11, 577], [229, 23, 181], so the conjecture is verified up to p = 229.

Conjecture 2:

Any prime p of the form 6*k - 1 greater than or equal to 11 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 179)

: 11*3 – 20 = 13, prime, so [p, q, r] = [11, 5, 13];
: 17*3 – 20 = 31, prime, so [p, q, r] = [17, 5, 31];
We also found the following triplets [p, q, r]: [29, 5, 67], [41, 5, 103}, [47, 11, 31], [53, 5, 129], [59, 5, 157], [59, 11, 67], [71, 5, 193], [71, 11, 103], [83, 5, 229], [83, 11, 139], [891, 11, 157], [101, 5, 283], [101, 11, 193], [101, 17, 31], [107, 11, 211], [113, 11, 229], [113, 17, 67], [131, 5, 373], [131, 11, 283], [137, 17, 139], [149, 11, 337], [167, 17, 229], [173, 5, 449], [173, 11, 409], [173, 23, 13], [179, 23, 31] so the conjecture is verified up to p = 179.

Comment:

In the case that the conjectures are invalidated, still remain two open problems:

(1) Which are the smallest primes that don't satisfy each from the two conjectures?;
(2) Which is the maximum length of a chain formed in the following way: p_2 = 3*p_1 – (q^2 – q), p_3 = 3*p_2 – (q^2 – q), ..., p_n = 3*p_{n-1} – (q^2 – q)? For instance, such a chain of length 3 is [43, 109, 307] for q = 5.