A model of global instructions, from Classical to Quantum Mechanics, and its application to the Measurement Problem and Entanglement

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ABSTRACT. In this work the usual formulation of the variational methods of Classical Mechanics is slightly modified by describing space as an interface implementing instructions: these instructions, in the form of bit strings, determine the existence and the dynamics of classical systems and are global – that is, their information content is present at every point of space. These changes are then carried over to Feynman's path integral formulation of non-relativistic Quantum Mechanics by recurring to the quantum superposition principle. The information content of the instructions is expanded to include spin; it then follows an interpretation within this framework of the collapse of the wave function in terms of splitting and merging of information and, as an illustration, of Wheeler's delayed choice experiment.

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1 Introduction

The existence of non-local correlations in Quantum Theory led Einstein, Podolski and Rosen [1] to question whether the theory is indeed a fundamental picture of reality. Their objections to its completeness had profound implications for the Physics of the years to come, notably after John Bell, with his theorem [2], shed new light on this issue in 1964. Many experiments performed to test Bell's inequalities in recent years [4, 5, 6, 7, 8] have shown that local realism must be abandoned; meanwhile, concepts like holism or action at a distance, branches and many-worlds were used to explain non-locality and the measurement problem (for a review, see [12] and the literature therein), and even whether the wave function describes only our knowledge about the world (epistemic view), or whether it is real and does correspond to a given physical state (ontic view), still remains controversial. Recently, arguments in favour of the latter view have been proposed [9, 10], and the very presence of interference fringes in the double slit experiment (for its first realization faithfully following Feynman's Gedankenexperiment, see [11]) would probably induce many to support this "ontic" perspective.

In this work it is attempted, starting from this ontic perspective, to account for non-locality and the collapse of the wave function through a slight modification of the usual concept of three dimensional space and through the idea that the wave-function builds upon information present everywhere in space, information which can be put together or break apart but which, however, never gets lost.

To better introduce the point, let's imagine the following experiment. Two entangled electrons are produced, let's say a Cooper pair in the singlet state in a superconductor: let's imagine that it were possible to directly measure the spin of one of them while taking the other out from the superconductor so that it could fly against a wall with two slits sufficiently small and close to each other – in this way it would be possible to reconstruct the interference pattern of the Young experiment if we had a screen after the slits. If the electron is indeed some kind of wave passing through both slits, the information about the spin measurement on the first particle should be at both slits at the same time. Extending the argument, if we, in a new experiment, after having removed the slits measure the spin along some direction of one of the two Cooper electrons and assume that the other instantly "knows" the outcome of the experiment, then the new spin state of both particles is "ubiquitously" new; at least in the region of space where the wave function of the pair system is defined. What is then the relationship between the "information" regarding the outcome of the measurement and space? Should we think of space as a simple canvas, something to forget about once we have coordinate systems to populate with matter waves and measuring apparatuses?

To address this question, we look at the widespread idea that Physics is all about information, and that, accordingly, it is possible to derive the entire formalism of quantum mechanics starting from simple information-theoretical approaches (see, for instance, [13] and the literature therein). This idea was pushed to the degree of a modern "metaphor" of our universe, comparing it to a computing machine: our world, and the way it evolves, are the result of a quantum computation. The details of the embodiment of this idea may vary considerably, (see, for instance, [14, 15, 16, 17, 18]), but, generally, these

attempts aim, starting from quantum interactions, to get models of space for quantum gravity. The interesting point, for our approach, is that they often assign an "ontic" character to information and to the way it is processed. However, instead of starting from scratch, finding ever simpler postulates or making appealing and sophisticated computational models, thus trying to recover, from them, new and old Physics, we follow the inverse path; in other words, we try to insert this glimpse of "ontic information" already in Classical Mechanics, and then follow the trail down to Quantum Physics.

2 Global instructions

2.1 The Sketch of a classical model

An interesting feature of Analytical Mechanics is the possibility to describe a system, no matter how complicated, by means of a single independent variable τ: in this way, time too becomes function of τ, that is, it becomes a coordinate of the system on the same footing as the other generalized variables (See, for instance, [19]).

In other words, instead of a given minimum problem described by some Lagrangian L:

$$\delta \int_{t_1}^{t_2} L\left(q_1(t), ..., q_n(t), t; \dot{q}_1(t), ..., \dot{q}_n(t)\right) dt = 0 \tag{1}$$

we can turn to:

$$\delta \int_{\tau_1}^{\tau_2} L\left(q_1(\tau), ..., q_n(\tau), q_{n+1}(\tau); \frac{q_1'(\tau)}{q_{n+1}'(\tau)}, ..., \frac{q_n'(\tau)}{q_{n+1}'(\tau)}\right) q_{n+1}' d\tau = 0, \tag{2}$$

where $q_{n+1} = t$ and $q_i' = \frac{dq_i}{d\tau}$.

We would like to focus on the new free variable τ in eq. (2). Before that, however, let's first consider a one-particle system like the one depicted in fig.1. We can compare the points of a discretized x-axis to a series of aligned light bulbs turning on and off and, accordingly, identify a material point moving in uniform motion over a discretized time to the only bulb turned on. That is, at time t some light bulb turns on, and there lies the material point for some time interval Δt; at $t + \Delta t$ it turns off and the next on, and all this has to happen simultaneously. Each signal will be made up by a series of 0's and 1's. More specifically, 0 everywhere except where the light is on: there the signal is 1. For example, if at t the light bulb in the picture turns on and stays on until $t + \Delta t$, this means that at t it received the signal 1; at $t + \Delta t$ the next input will arrive: the light bulb that now is on will be getting a 0, so it will turn off, but simultaneously the next one will be getting 1, so it will turn on.

Now we would like to identify this series of 0's and 1's – one for each light bulb – with the old variable τ of eq. (2):

$$\vec{\tau} = ... \boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{0} ...$$

From the one dimensional variable τ of eq. (2) we thus turn to a vector $\vec{\tau}$, with as many components as the number of light bulbs. Subsequently, to the i-th light bulb we assign its own "interpreting" function $q_i = q_i(\vec{\tau})$. The reason for

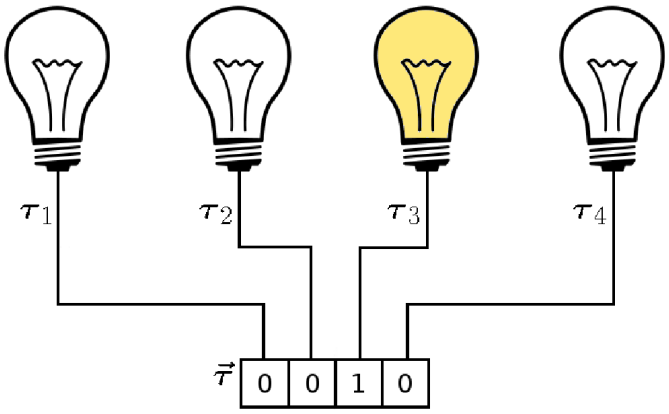

Figure 1: In the present framework the points of space are compared to light bulbs turning on and off according to an underlying "ontic instruction" $\vec{\tau}$, a vector with as many components as light bulbs, in this case four. The material point is located where the light is on, corresponding to the slot in $\vec{\tau}$ containing 1; as soon as one light turns off the next turns on: in this way, a moving particle keeps existing because of the implementation of successive instructions on space.

this name is that, since in eq. (2) every q_i depends on the same variable τ, it seems reasonable to call the points of space in the same way, as they too depend on $\vec{\tau}$.

Each q_i reads the content of the i-th slot: if it is 0, $q_i(\vec{\tau})$ turns or leaves the light off, if it's 1 then $q_i(\vec{\tau})$ turns or leaves the light on. More specifically, if 2 denotes the two-element boolean ring, $2 = \{0, 1\}$, we define:

$$\begin{aligned} q_i : 2^N &\rightarrow \mathbb{N} \times 2 \\ \vec{\tau} &\mapsto (i, \tau_i), \end{aligned} \tag{3}$$

where τ_i denotes the i-th component of $\vec{\tau}$. In the following, we will be adopting the lighter notation $i * \tau_i$ instead of (i, τ_i).

We could think that not only information about position is being sent, but also information about other quantities identifying the system we want to describe, the spin of a particle for instance.

The next step is to consider the continuum limit for the x-axis. That is, as far as space is concerned, we can turn from the set of N points and functions $\{q_i, i = 0, ..., N-1\}$ to an infinite set of points and functions when $N \rightarrow \infty$. We will then switch from the $q_i(\vec{\tau})$ to $\phi(x)[\tau]$, where now τ is a vector with an infinite number of components; x refers to the x-th "slot" of the "vector" τ, and the function $\phi(x)$ then looks at its content, $\tau(x)$. In analogy with definition (3) of q_i, we can write:

$$\begin{aligned} \phi(x) : 2^{\mathbb{R}} &\rightarrow \mathbb{R} \times 2 \\ \tau &\mapsto x * \tau(x). \end{aligned} \tag{4}$$

Let's now define ϕ and τ for the three-dimensional case, the generalization

to other dimensions being straightforward. For $\mathbf{x} = (x, y, z)$ we set

$$\boldsymbol{\tau}(\mathbf{x}) = (\tau_x(x), \tau_y(y), \tau_z(z))$$

and:

$$\begin{aligned} \boldsymbol{\phi}(\mathbf{x})[\boldsymbol{\tau}] &= (\phi(x), \phi(y), \phi(z))[(\tau_x, \tau_y, \tau_z)] \\ &= (x * \tau_x(x), y * \tau_y(y), z * \tau_z(z)) := \mathbf{x} * \boldsymbol{\tau}(\mathbf{x}). \end{aligned} \tag{5}$$

Let's consider now a system of n particles in some region $R \subset \mathbb{R}^3$. We impose, completely arbitrarily, that:

$$\boldsymbol{\tau}(\mathbf{x}) = \boldsymbol{\tau}_1(\mathbf{x}) + \boldsymbol{\tau}_2(\mathbf{x}) + \cdots + \boldsymbol{\tau}_n(\mathbf{x}).$$

We can then say that $\boldsymbol{\tau} \in (2^R \oplus \cdots \oplus 2^R)$, where the direct sum is taken n times, generalizing definition (5) accordingly:

$$\begin{aligned} \boldsymbol{\phi}(\mathbf{x}) : (2^R \oplus ... \oplus 2^R) &\rightarrow R \times (2 \oplus ... \oplus 2) \\ \boldsymbol{\tau} &\mapsto \mathbf{x} * \boldsymbol{\tau}(\mathbf{x}). \end{aligned} \tag{6}$$

Furthermore, from definition (6) we can write:

$$\begin{aligned} \boldsymbol{\phi}(\mathbf{x})[\boldsymbol{\tau}] &= \boldsymbol{\phi}(\mathbf{x})[\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \cdots + \boldsymbol{\tau}_n] \\ &= \mathbf{x} * \Big(\boldsymbol{\tau}_1(\mathbf{x}) + \boldsymbol{\tau}_2(\mathbf{x}) + ... + \boldsymbol{\tau}_n(\mathbf{x})\Big) \\ &= \boldsymbol{\phi}_{1+2+...+n}(\mathbf{x})[\boldsymbol{\tau}], \end{aligned}$$

where

$$\boldsymbol{\phi}_{i+...+j}(\mathbf{x})[\boldsymbol{\tau}] = \phi_{i+...+j}(\mathbf{x})[\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + ... + \boldsymbol{\tau}_n] := \mathbf{x} * [\boldsymbol{\tau}_i + ... + \boldsymbol{\tau}_j](\mathbf{x}), \tag{7}$$

$i = 1, ..., n$, $j > i$. In other words, in this framework space is different depending on the content of the instructions, and to each particle that exists it can be assigned its own "layer" of space. We could push the game a little further and speculate about the very nature of emptiness: let's turn to the discrete case in one dimension for a pictorial image; looking at (7), one is tempted to turn from

$$\vec{\tau}_i = ... \boxed{0 \mid 0 \mid 0 \mid 1_i \mid 0 \mid 0 \mid 0} ...$$

to

$$\vec{\tau}_i = ... \boxed{0_i \mid 0_i \mid 0_i \mid 1_i \mid 0_i \mid 0_i \mid 0_i} ...$$

This, however, entails a peculiar definition of emptiness, if we exclude those $\vec{\tau}$ whose entries are all 0. In fact, if the "world", or a part of it, the region R, is composed by n particles, the sentence "nothing is at point x" means that "particle 1 is not there, nor particle 2,..., nor particle n". This is conceptually much different from saying: "There is nothing there", because in the first case one knows exactly and a priori what the world contains, whereas in the second this knowledge is not assumed. To visualize the shift, we consider a given $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$, which turns out to be:

$$... \boxed{0_1 + 0_2 \mid 1_1 + 0_2 \mid 0_1 + 0_2 \mid 0_1 + 0_2 \mid 0_1 + 1_2 \mid 0_1 + 0_2 \mid 0_1 + 0_2} ...$$

After some work shown in the appendix, we come back to our starting point, eq. (2), which now can be rewritten, for a system of n interacting particles, as:

$$\delta \int_{\tau_{c,1}}^{\tau_{c,2}} L\left(\Phi_1[\tau_1(t[\tau_c])],\ldots,\Phi_n[\tau_n(t[\tau_c])];\frac{\Phi_1'[\tau_1]}{t'}[\tau_c],\ldots,\frac{\Phi_n'[\tau_n]}{t'}[\tau_c]\right)\cdot \frac{dt}{d\Phi_c[d\tau_c]}d\Phi_c[d\tau_c]=0, \qquad (8)$$

where $\Phi_i[\tau_i(t[\tau_c)], i=1,...,n$, gives the coordinate of particle i at time t, t being given by the position of some other particle c: $\Phi_i[\tau_i(t[\tau_c)] = x_i(t[\tau_c]) * 1_i$ (for a careful explanation see sec. A.2 in the appendix).

Solving the Euler-Lagrange equations implies virtual displacements of the functions Φ_i, but this, in turn, entails virtual variations of the τ_i themselves.

2.2 Superposition of instructions in Feynman's formulation of Quantum Mechanics

In 1948 Richard Feynman reformulated Quantum Mechanics by introducing the path-integral approach[22] and by writing the wave function $\psi(x,t)$ as:

$$\psi(x_k,t)=\lim_{\epsilon\to 0}\int_R \exp\left[\frac{i}{\hbar}\sum_{i=-\infty}^{k-1}S(x_{i+1},x_i)\right]\frac{dx_{k-1}}{C}\frac{dx_{k-2}}{C}..., \qquad (9)$$

being

$$S(x_{i+1},x_i)=\min\int_{t_i}^{t_{i+1}}L(\dot{x}(t),x(t))dt. \qquad (10)$$

C is a constant, $C=(2\pi\hbar\epsilon i/m)^{\frac{1}{2}}$, and ϵ follows from a "sampling" of positions along the x-axis and from a similar sampling in time, so that $t_{i+1}=t_i+\epsilon$; R is, in turn, some region of space. In the argument of the matter wave (9) the old paths of Analytical Mechanics show up, and we simply need to rewrite them in terms of global instructions, eq. (8). In Quantum Mechanics, however, we deal with a superposition of paths, so that we will be assuming a superposition of instructions. As an example, let's come back to the four points of fig. 1 and let's forget the other points of space where our clock is located: time will become therefore the independent variable; for particle a we will have:

$$\mathcal{T}_a(t) := \boxed{1_a}\boxed{0}\boxed{0}\boxed{0}_t+\boxed{0}\boxed{1_a}\boxed{0}\boxed{0}_t+\boxed{0}\boxed{0}\boxed{1_a}\boxed{0}_t+\boxed{0}\boxed{0}\boxed{0}\boxed{1_a}_t\equiv\boxed{1_a}\boxed{1_a}\boxed{1_a}\boxed{1_a}_t .$$

Let's now consider a superposition of paths. In our framework, this will entail a superposition of $\Delta\tau$. For our four points, we will have:

$$\{\Delta\tau_a(\Delta t)\} = \Big\{ \boxed{0}\boxed{0}\boxed{0}\boxed{0}_{\Delta t}, \boxed{1_a}\boxed{1_a}\boxed{0}\boxed{0}_{\Delta t}, \boxed{1_a}\boxed{0}\boxed{1_a}\boxed{0}_{\Delta t}, \boxed{1_a}\boxed{0}\boxed{0}\boxed{1_a}_{\Delta t}, \boxed{0}\boxed{1_a}\boxed{1_a}\boxed{0}_{\Delta t}, \boxed{0}\boxed{1_a}\boxed{0}\boxed{1_a}_{\Delta t}, \boxed{0}\boxed{0}\boxed{1_a}\boxed{1_a}_{\Delta t} \Big\} := \Delta\mathcal{T}_a(\Delta t).$$

Through the action of the elements of $\Delta\mathcal{T}_a(\Delta t)$ on the elements of $\mathcal{T}_a(t)$ we can now imagine to get a new superposition $\mathcal{T}_a(t+\Delta t)=\mathcal{T}_a(t)+\Delta\mathcal{T}_a(\Delta t)$. For

instance, $\boxed{1_a \mid 0 \mid 0 \mid 0}_{t+\Delta t}$ is the output of:

$$\boxed{1_a \mid 0 \mid 0 \mid 0}_t + \boxed{0 \mid 0 \mid 0 \mid 0}_{\Delta t}\ , \boxed{0 \mid 1_a \mid 0 \mid 0}_t + \boxed{1_a \mid 1_a \mid 0 \mid 0}_{\Delta t},$$
$$\boxed{0 \mid 0 \mid 1_a \mid 0}_t + \boxed{1_a \mid 0 \mid 1_a \mid 0}_{\Delta t}, \boxed{0 \mid 0 \mid 0 \mid 1_a}_t + \boxed{1_a \mid 0 \mid 0 \mid 1_a}_{\Delta t}.$$

All of them belong to $\mathcal{T}_a(t+\Delta t)$, so that the instruction $\boxed{1_a \mid 0 \mid 0 \mid 0}_{t+\Delta t}$ will be present four times, as the result of four different actions on different instructions belonging to $\mathcal{T}_a(t)$. Therefore we rewrite $\mathcal{T}_a(t)$ as:

$$\begin{aligned}\mathcal{T}_a(t) &= \left(\left\{ \boxed{1_a \mid 0 \mid 0 \mid 0} \right\}_t + \left\{ \boxed{0 \mid 1_a \mid 0 \mid 0} \right\}_t + \right. \\ &\quad \left. \left\{ \boxed{0 \mid 0 \mid 1_a \mid 0} \right\}_t + \left\{ \boxed{0 \mid 0 \mid 0 \mid 1_a} \right\}_t \right) \\ &\equiv \left\{ \boxed{1_a \mid 1_a \mid 1_a \mid 1_a} \right\}_t\end{aligned}$$

where the four sets represent the sets of instructions with different histories such that at time t they are at points 1,2,3 and 4, respectively. For example, if l denotes a particular path and Δx_i the i-th step of the l-th path, then $\left\{ \boxed{1_a \mid 0 \mid 0 \mid 0} \right\}_{t_n}$ is the set:

$$\left\{ \vec{\tau}_{a,0} + \sum_{i=1}^{n} \Delta\tau_{li}^{a}(\Delta t_i), \quad 1 \leq l \leq 4^{n-1} \right\},$$

such that

$$\vec{\tau}_{a,0} + \sum_{i=1}^{n} \Delta\tau_{li}^{a}(\Delta t_i) := \boxed{1_{a,l} \mid 0 \mid 0 \mid 0} := \vec{\tau}_{a,l}(t_n)\ ,$$

$\forall\ l$. In this way we are assigning to each path l of particle a its own classical "particle" labelled by the subscripts (a,l) and, accordingly, its own set of instructions, whose members are the single $\vec{\tau}_{a,l}$.

Turning to the continuum, we switch from $\vec{\tau}_{a,l}$ for path l to $\tau_{(x_i,x_{i-1},...)}$ for the particular path of particle a – the subscript a has been omitted – passing through the points $x_i, x_{i-1}, ...$ at times $t_i, t_{i-1}, ...$ respectively. Therefore we rewrite now eq. (9) as:

$$\begin{aligned}\psi(x_k,t_k) &= \lim_{\epsilon\to 0} \int_R \cdots \int_R \exp\left\{ \frac{i}{\hbar} \sum_{i=-\infty}^{k-1} \int_{\tau_{c,i}}^{\tau_{c,i+1}} L\left(\Phi[\tau_{(x_i,x_{i-1},...)}(t)], \right.\right. \\ &\quad \left.\left. \frac{\Phi'[\tau_{(x_i,x_{i-1},...)}(t)]}{t'} \right) \frac{dt}{d\Phi_c[d\tau_c]} d\Phi_c[d\tau_c] \right\} \frac{dx_{k-1}}{C} \frac{dx_{k-2}}{C} ... \qquad (11)\end{aligned}$$

In this framework, a particle a is a composite entity formed by many fictitious "sub-particles", one for each path ending at the points of region R: if to each path labelled by the points $(x_k, x_{k-1}, x_{k-2}, ...)$ of its trajectory it corresponds

in this framework a specific instruction $\tau_{a,(x_k,x_{k-1},x_{k-2},...)}$, then the evaluation of the whole of the instructions at some points will yield:

$$\begin{aligned}\phi(x)[\tau_a] &= \phi(x)\left[\int_R \tau_{a,(x_k,x_{k-1},x_{k-2},...)} dx_k dx_{k-1} dx_{k-2}...\right] \\ &= x * \left[\int_R \tau_{a,(x_k,x_{k-1},x_{k-2},...)} dx_k dx_{k-1} dx_{k-2}...\right](x) \\ &= x * (\{1_a\} + \{0_a\}),\end{aligned}$$

so that at point $\phi(x)[\mathcal{T}(t)]$ there is a superposition of states.

One last remark: thanks to the position of some particle c we are able to identify each single τ as $\tau = \tau(t)$. In this way it is implicit, however, that there are two particles involved, a quantum one, described by ψ along with an uncountable set of τ's, and a classical one, which is being continuously spotted and whose position is given by Φ_c. If it is correct to say that the notion of time developed from witnessing classical movements, it thus seems that quantum physics needs a classical world with classical clocks and classical trajectories in order for the Schrödinger Equation to account for the time evolution of the wave function. Therefore, in the present framework time implies continuous position measurements and collapses of the wave function of particle c.

3 Applications

3.1 Collapse of the wave function

In this framework we link the very existence of the wave function to the presence of global instructions: only when these are present can the wave function exist. We propose then the following interpretation of the collapse. Our supposition is that the wave function doesn't simply represent a state of knowledge but that it really corresponds to a physical state,that it is "ontic". The electron really is at both slits in a Young experiment: how do we then explain that we find it only in one place when we measure its position, even though we witness interference fringes? Or, how do we explain the outcomes of the Stern-Gerlach experiment if the particle really is in a superposition of two spin configurations?

The idea is that a particle is made up of a set of global instructions, upon which a wave, or, more generally, a quantum state in its Hilbert space, originates and evolves in time. We could therefore imagine that the measurement process entails that part of these instructions gets detached from the set of original instructions pertaining to the particle described by the wave function. Symbolically, for any particle a we could turn from position instructions of the form:

$$\left\{\boxed{1_a}\boxed{1_a}\boxed{1_a}\boxed{1_a}\right\}_{t_n} \text{ to } \left\{\boxed{a}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\right\}_{t_n},$$

meaning that the set of instructions

$$\left\{\left\{\boxed{1}\boxed{0}\boxed{0}\boxed{0}\right\}_{t_n}, \left\{\boxed{0}\boxed{1}\boxed{0}\boxed{0}\right\}_{t_n}, \left\{\boxed{0}\boxed{0}\boxed{1}\boxed{0}\right\}_{t_n}, \left\{\boxed{0}\boxed{0}\boxed{0}\boxed{1}\right\}_{t_n}\right\}$$

all belong to a. The collapse at point 1 then would mean:

$$\Big\{\boxed{a}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\Big\}_{t_n} \longrightarrow \Big\{\boxed{a}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\Big\}_{t_{n+1}} + \Big\{\boxed{}\boxed{0}\boxed{1}\boxed{1}\boxed{1}\Big\}_{t_{n+1}} . \quad (12)$$

In words, the information splits between the one still attached to the particle and three other instructions left "free" in an "information environment" underneath space. Particle a becomes then localized at point 1, but the other instructions don't get lost. The idea is that, as they can be separated from one "particle", they can conversely became part of other ones. For instance, omitting for clarity the brackets $\{\}_{t_n}$, one could have:

$$\boxed{b}\boxed{}\boxed{}\boxed{}\boxed{} + \boxed{c}\boxed{}\boxed{}\boxed{}\boxed{} + \boxed{d}\boxed{}\boxed{}\boxed{}\boxed{} + \boxed{}\boxed{0}\boxed{1}\boxed{1}\boxed{1} \longrightarrow$$
$$\longrightarrow \boxed{b}\boxed{0}\boxed{1}\boxed{0}\boxed{0} + \boxed{c}\boxed{0}\boxed{0}\boxed{1}\boxed{0} + \boxed{d}\boxed{0}\boxed{0}\boxed{0}\boxed{1} .$$

In this case, this would mean having three particles, b, c and d localized, respectively, at points 2, 3 and 4. This way of writing "information" simply symbolizes the fact that one can add and subtract information by filling or emptying a string of bits and, depending on the amount of information present, one can have a particle or less than a particle but, still, more than nothing. The "slots" $\boxed{b}$, $\boxed{c}$, $\boxed{d}$ represent information like mass, charge, spin, etc. In other words, if we suppose that not only the matter wave, but information too exists, then it can make sense to have instructions about position at some place without anything (a mass, a charge...) being there; or, conversely, existing masses or charges but without any specific position. In this way, it would be conceivable to imagine an environment, a "thermal bath" of information, that particles use to propagate as waves and to which they leave part of the information when it is no longer needed. A similar mechanism could help explain why the measurement process breaks unitarity. The idea here is that it is not unitarity in itself which must be preserved but, rather, the total amount of information upon which the wave function lives.

This could have interesting consequences in the case of non commuting observables, like spin values in different directions. Let's take the case, for instance, of repeated Stern-Gerlach experiments on a beam of Ag atoms where the spin is measured along different directions. If one measures the spin, say, along direction x and then along direction z, the information about the value of the spin in the x direction gets destroyed, and one remains with only information about the spin along z. If we write the information corresponding to spin up and down along $i = x, y, z$ as

$$\boxed{\uparrow}_i = \boxed{1}\boxed{0}_i \text{ and } \boxed{\downarrow}_i = \boxed{0}\boxed{1}_i$$

for a given particle we will have a specific instruction for the three directions. For instance, in case of spin up along x we can write:

$$\boxed{1}\boxed{0}_x \boxed{1}\boxed{1}_y \boxed{1}\boxed{1}_z .$$

In other words, the spin along y and z is in a superposition of ups and downs: if the spin along x is in a specific configuration, we lose information about the spin in the other directions. However, if we think that in the double slit experiment the particle has passed through both slits and that it really is in a

superposition of two states, then we are forced to think that the particle really is in a superposition of two spin states along directions y and z as well. If one subsequently measures the spin along z, and gets, say, up, information about spin along x gets lost, so that we have

$$\left(\boxed{1}\boxed{0}_x \boxed{1}\boxed{1}_y \boxed{1}\boxed{1}_z\right)_a + \boxed{0}\boxed{1}_x \longrightarrow \left(\boxed{1}\boxed{1}_x \boxed{1}\boxed{1}_y \boxed{1}\boxed{0}_z\right)_a + \boxed{0}\boxed{1}_z .$$

However, we can't say that information is lost or destroyed, but, rather, the contrary: the particle has acquired new information and the spin configuration along x grows to its maximal information content.

This line of thought entails the following. Let's consider four points of space and the collapse of a particle at point 1. We will have the situation described by eq. (12), so that

$$\begin{aligned} q_1\left(\left\{\boxed{a}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\right\}_{t_n}\right) &= 1 * \{1_a\}_{t_n}, \\ q_{i\neq 1}\left(\left\{\boxed{a}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\right\}_{t_n}\right) &= i * \{0_a\}_{t_n}. \end{aligned}$$

That is, particle a is at point 1, because all information needed for the particle to be there is at hand, and this is symbolized by the subscript a of $\{1_a\}_{t_n}$. However,

$$\begin{aligned} q_{i\neq 1}\left(\left\{\boxed{\ }\boxed{0}\boxed{1}\boxed{1}\boxed{1}\right\}_{t_n}\right) &= i * \{1\}_{t_n} , \\ q_1\left(\left\{\boxed{\ }\boxed{0}\boxed{1}\boxed{1}\boxed{1}\right\}_{t_n}\right) &= 1 * \{0\}_{t_n}. \end{aligned}$$

This will entail that there is something at points $i \neq 1$, namely, instructions about position, but this something has not enough information as to constitute an interacting particle. In other words, a full-fledged wave function, eq. (11) for the continuum, cannot exist, as no information regarding mass, spin, charge or momentum is present there.

We can now try to fit the present framework into the usual description of the measurement process [23]. An open system interacts not only with the apparatus performing the experiment, but also with the environment surrounding the two. Roughly, we can divide everything into two steps: in the first the system we want to measure entangles with the apparatus, in what is called a premeasurement:

$$\left(\sum_s \alpha_s |s\rangle\right)|A_0\rangle \longrightarrow \sum_s \alpha_s |s\rangle|A_s\rangle \equiv |\Psi_{pre}\rangle \tag{13}$$

so that we can write the density matrix of the whole system as $\rho = |\Psi_{pre}\rangle\langle\Psi_{pre}|$. Decoherence subsequently forces the destruction of all but a small subset of possible states of the combined system and apparatus, the so-called einselected states:

$$\rho_{\mathcal{SA}} = \sum_{s,s'} \alpha_s \alpha^*_{s'} |s\rangle\langle s'||A_s\rangle\langle A_{s'}| \longrightarrow \rho^D_{\mathcal{SA}} = \sum_s |\alpha_s|^2 |s\rangle\langle s||A_s\rangle\langle A_s|.$$

First, in order to recover eq. (13) we need to turn from tensor products to global instructions, at first for the position of two particles a and b in a 4-point space. We set:

$$|1\rangle_a \otimes |3\rangle_b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_a \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}_b \longleftrightarrow \boxed{ab}\boxed{10}\boxed{00}\boxed{01}\boxed{00} \tag{14}$$

We can accordingly define ϕ_{ab} as:

$$\phi_{ab}(x)[\tau_{ab}] = x * \tau_{ab}(x).$$

For instance, if one takes instruction (14), we get:

$$q_{ab,1}[\vec{\tau}_{ab}] = 1 * (1_a\, 0_b).$$

Alternatively, instruction (14) implies that, if one finds particle a at position 1, b must be at position 3.

In case of superposition of instructions, collapse would entail the following process:

$$\boxed{ab}\left(\boxed{10}\boxed{01}\boxed{00}\boxed{00}_t + \boxed{01}\boxed{10}\boxed{00}\boxed{00}_t + \dots\right) \longrightarrow$$
$$\longrightarrow \boxed{ab}\boxed{10}\boxed{01}\boxed{00}\boxed{00}_{t+\Delta t} + \boxed{\ }\boxed{01}\boxed{10}\boxed{00}\boxed{00}_{t+\Delta t} + \dots$$

It wouldn't be inconceivable to further suppose the subsequent splitting of the freed instructions:

$$\begin{aligned} \boxed{\ }\boxed{01}\boxed{10}\boxed{00}\boxed{00} &= \boxed{\ }\boxed{10}\boxed{10}\boxed{00}\boxed{00} \\ &\equiv \boxed{\ }\boxed{1}\boxed{1}\boxed{0}\boxed{0} \\ &= \boxed{\ }\boxed{1+0}\boxed{0+1}\boxed{0+0}\boxed{0+0} \\ &= \boxed{\ }\boxed{1}\boxed{0}\boxed{0}\boxed{0} + \boxed{\ }\boxed{0}\boxed{1}\boxed{0}\boxed{0}. \end{aligned}$$

Starting from our initial question, namely how can the information of the result of the measurement be ubiquitous, we can then make the following hypothesis about the process of collapse of two entangled electrons in a singlet state:

$$\boxed{e_1\, e_2}\boxed{\uparrow\downarrow + \downarrow\uparrow}\boxed{10}\boxed{...}\boxed{01}_t \longrightarrow \boxed{e_1\, e_2}\boxed{\uparrow\downarrow}\boxed{10}\boxed{...}\boxed{01}_{t+\Delta t} +$$
$$\boxed{\ }\boxed{\downarrow\uparrow}\boxed{\ }\boxed{...}\boxed{\ }_{t+\Delta t}.$$

These instructions can be interpreted as the information, present at all points of space, regarding two entangled electrons, e_1 and e_2. At first their spin configuration is made up of two superposed instructions, $\boxed{\uparrow\downarrow}$ and $\boxed{\downarrow\uparrow}$; due to the measurement, only $\boxed{\uparrow\downarrow}$ remains attached to the rest of the information, and at all points only the result of the measure will be known.

It shouldn't be difficult, proceeding in the same way, to describe the state $|\Psi_{pre}\rangle$, eq. (13). A generalized instruction for particle s and for detector A can

be summed up as:

$$\boxed{sA}\boxed{s_1A_1}\boxed{s_2A_2}\boxed{...}\boxed{s_nA_n} = \boxed{sA}\Big(\boxed{11}\boxed{00}\boxed{...}\boxed{00}+ \boxed{00}\boxed{11}\boxed{...}\boxed{00}+\dots\Big),$$

where the two-bit slots can be interpreted as a semiclassical approximation of a quantum particle and a classical state made up by billions of quantum particles. The collapse would then, as before, correspond to the detachment of pieces of information:

$$\boxed{sA}\boxed{s_1A_1}\boxed{s_2A_2}\boxed{...}\boxed{s_nA_n} \longrightarrow \boxed{sA}\boxed{11}\boxed{00}\boxed{...}\boxed{00}+ \boxed{}\boxed{00}\boxed{11}\boxed{...}\boxed{00}+\dots$$

which, in our framework, will end up in the information bath and subsequently reused.

Now let's come back to eq. (11). One thing still remains to be specified. In fact, in the exponent a boolean part is present: it amounts to 0_a if particle a didn't collapse. However, when its position is measured, the wave function "shrinks" to some spot, stopping being in all other regions of space. In our framework we can relate this to two phenomena: on the one hand, part of the position information broke apart (see eq. (12) for the discrete case), thus that from a superposition $\tau_a(x) = 1_a + 0_a$ only those τ_a remain such that $\tau_a(x) = 0_a$; subsequently $\tau'_a(x) = 1$, symbolizing that there the particle stopped existing: the wave function is not present, and we can express this by requiring that the exponential in (11) be defined as:

$$\exp\Big((f_\phi * f_B)[\tau](x)\Big) = \begin{cases} e^{f_\phi(x)} & \text{if } f_B(\tau(x)) = 0 \\ 0 & \text{if } f_B(\tau(x)) = 1 \end{cases},$$

where $f_B(\tau(x)) = \tau'(x)$, eq. (B.5).

3.2 An example: Wheeler's delayed choice experiment

Let's consider now the delayed choice experiment first envisaged by J. A. Wheeler in 1978 [24]. In his variation of the double slit experiment, only after the photon passed through the slits it is decided what to observe, either a particle flown through one of the two slits, or a wave diffracting on the screen. Fig. **??** shows the setting of Wheeler's experiment, as realized by Jacques, Wu, Grosshans, Treussart, Grangier, Aspect and Roch in 2006 [25].

In their setting, a single linearly polarized photon is sent to the interferometer: a first polarization beam splitter, BS_{input}, splits the path of the incoming photon into two, Path 1 and Path 2, along which the photon has orthogonal polarizations. The second beam splitter, BS_{output}, is actually made up by a series of devices: to one of these, an electro-optical modulator, an external voltage can be applied, which takes only two values: accordingly, either BS_{output} mixes the photon polarizations, thus getting the interference from the two paths, or it leaves the polarizations separated, thus recovering the which-path information.

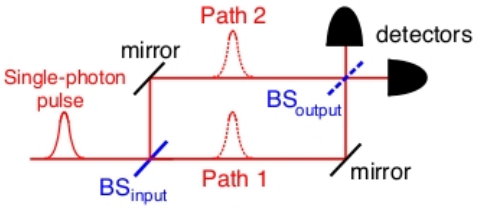

Figure 2: Wheeler's delayed choice Gedankenexperiment, as realized by V. Jacques et. al. in 2006 [25]; the picture is reprint from their article. A linearly polarized single-photon pulse enters the interferometer and arrives at the first polarization beam splitter, BS_{input}, after which the photon takes two different paths, Path 1 and Path 2. Simultaneously to the entering of the photon into the interferometer it is decided through a random number generator giving either 0 or 1 whether the second beam splitter, BS_{output}, should mix the photon polarizations coming from the two paths or not. BS_{output} turns to the desired configuration while the photon is already travelling along both paths.

When the photon enters the interferometer, a random number generator gives as output either 0 or 1, thus selecting the setting for BS_{output}: in this way the choice of which experiment to perform and the entering of the photon into the interferometer are two space-like separated events, because in the reference frame of the laboratory they happen simultaneously; therefore, the physical implementation of the selected configuration of BS_{output} takes place when the photon is already travelling down both Paths 1 and 2.

Whatever the setting, we apply eq. (13): the incoming photon α must entangle with the two apparatuses $D1$ and $D2$:

$$|\Psi_{pre}\rangle = |off\rangle_{D_1}|off\rangle_{D_2}\Big(|\alpha_1\rangle + |\alpha_2\rangle\Big), \tag{15}$$

where $|\alpha_1\rangle$ means that the photon is at detector 1 and $|\alpha_2\rangle$ that it is at detector 2. The global state must collapse either into $|on\rangle_{D_1}|off\rangle_{D_2}$ or $|off\rangle_{D_1}|on\rangle_{D_2}$, depending on where the photon is absorbed. In the experiment considered, the two detectors are silicon avalanche photodiodes operating in the photon counting regime. This means that, in our scheme, the photon will enter a reversely biased diode, will be absorbed by a superposition of indistinguishable electrons that will ionize and, subsequently, ignite an avalanche of secondly ionized electrons which will eventually form the electric signal at the readout.

First, if we maintain that the photon is at both detectors, then it entangles with both, but none of them has absorbed it yet. In order to write the instructions corresponding to eq. (15) let's first consider only two electrons, e_1 and e_2 belonging, respectively, to D1 and D2. The photon must entangle with the electrons present in the two diodes; furthermore, this must be a position entanglement such that the photon and the electrons must be at the same place: only then can the photon kick out the first electron. To describe the process we need the corresponding instructions. To further simplify, let's consider a discretized version of two detectors. Our photodiodes "occupy" only four points of space.

Then, $|\alpha_1\rangle_{ph}|off\rangle_{D1}|off\rangle_{D2}$ will result from instructions of the form

$ph \cdot e_1 \cdot e_2$	1 1 0	1 1 0	1 1 0	1 1 0	...	0 0 1	0 0 1	0 0 1	0 0 1

,

and, conversely, $|\alpha_2\rangle_{ph}|off\rangle_{D1}|off\rangle_{D2}$ will correspond to:

$ph \cdot e_1 \cdot e_2$	0 1 0	0 1 0	0 1 0	0 1 0	...	1 0 1	1 0 1	1 0 1	1 0 1

.

Within the present framework, $|\Psi_{pre}\rangle$ will then "dress" these instructions with the due complex coefficients which are the probability amplitudes for each state. In the end, we will have a superposition of instructions, like:

$$
\begin{array}{l|l}
ph \cdot e_1 \cdot e_2 & \Bigg(\begin{array}{|c|c|c|c|c|}\hline 1\,1\,0 & 0\,0\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_t + \\
& \begin{array}{|c|c|c|c|c|}\hline 1\,0\,0 & 0\,1\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_t + \\
& \begin{array}{|c|c|c|c|c|}\hline 0\,0\,0 & 1\,1\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_t + \ldots \\
& \begin{array}{|c|c|c|c|c|}\hline 0\,0\,0 & 0\,1\,0 & ... & 1\,0\,1 & 0\,0\,0 \\ \hline \end{array}_t + \ldots \Bigg)
\end{array}
\qquad (16)
$$

If the photon gets absorbed by detector D1, then, starting from (16), the "chemical" reaction will result in:

$$
\begin{array}{l|l}
ph \cdot e_1 \cdot e_2 & \Bigg(\begin{array}{|c|c|c|c|c|}\hline 1\,1\,0 & 0\,0\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_{t+\Delta t} + \\
& \begin{array}{|c|c|c|c|c|}\hline 0\,0\,0 & 1\,1\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_{t+\Delta t} + \ldots \Bigg) + \\
& \begin{array}{|c|c|c|c|c|}\hline & 0\,0\,0 & 0\,1\,0 & ... & 1\,0\,1 & 0\,0\,0 \\ \hline \end{array}_{t+\Delta t} + \\
& \begin{array}{|c|c|c|c|c|}\hline & 1\,0\,0 & 0\,1\,0 & ... & 0\,0\,1 & 0\,0\,0 \\ \hline \end{array}_{t+\Delta t} + \ldots
\end{array}
$$

That is, those states dictating that the photon is in detector D2, or in detector D1 but not at the same point of space as e_1, will be separated from the rest of information making up a photon, and this process corresponds, in the present framework, to the collapse. However, the information regarding the states not survived after the measurement will not disappear, but simply detach from the rest of the information. The rest of the information encompasses the photon polarization, without which no interaction of light with matter is possible. It is straightforward to extend the reasoning to more realistic scenarios involving large numbers of electrons in the continuum.

In case the second beam splitter is not in the interferometer, we can idealize the set $\{\tau\}_t$ of all paths from the first beam splitter to the detectors and subdivide it into:

$$\{\tau\}_t = \{\tau_1\}_t + \{\tau_2\}_t,$$

where the two subsets represent the whole information regarding the position and the past history of the photon: if $\{\tau_1\}_t$ is selected, then the photon followed Path 1 and at t was detected by Detector 1; if, on the contrary, $\{\tau_2\}_t$ is chosen, then the photon went down Path 2 and was detected by Detector 2.

Let's consider now the second possible setting of the interferometer, when the second beam splitter, BS_{output}, is positioned at the crossroad of the two paths. In this way the wave-like interaction of the photon with itself is manifest. Ideally, we can imagine that the action of the beam splitter is to further subdivide $\{\tau\}_t$ into new subclasses, that is:

$$\{\tau\}_t = \{\tau_{1,D1}\}_t + \{\tau_{1,D2}\}_t + \{\tau_{2,D1}\}_t + \{\tau_{2,D2}\}_t,$$

where the first subscript, 1 or 2, refers to the path taken from the first to the second beam splitter, and the last subscript, D1 or D2, refers to the detector that found the particle. Now it is no longer clear where the particle comes from, and we get the interference.

4 Conclusions

A model was presented with the aim of trying to understand how, among entangled particles, the one on which a measurement is performed instantly communicates the outcome of the experiment to the others, irrespective of their relative distances. Therefore, the existence was imagined of global instructions τ's, corresponding to superposed information "underneath" the three-dimensional space. Space can be compared to a screen allowing for the information to be displayed: this information is global, i.e., the same everywhere, but the single points of space read just one part of it, the one that competes to them. When changes take place due to some measurement they are global, because the outcome of the experiment, in other words the collapse of the wave function into some state, is the consequence of a modification of the global information and, as such, it is simultaneously "there" all over the world.

This implies that a distinction is introduced between the wave function ψ and deeper states of information represented by the set $\{\tau\}$, each element of which is an infinite bit string. In this way the superposition principle is not at the level of ψ but, instead, it becomes an interplay between the wave function and an underlying set of different instructions: ψ lives upon the $\{\tau\}$, as if the whole of these instructions were a kind of "information ether", to every element of which its own probability amplitude is assigned, ψ.

The idea is thus introduced that a particle is a complex entity constituted by different types of information regarding position, spin, mass, charge, polarization, etc. This allows for the description of the collapse of the wave function in terms of the splitting up of parts of the information characterizing the initial state before the measurement. The particle loses information, but this lost information is an incomplete bit string which, as such, is not enough to constitute an interacting particle. For instance, there can be information about position at some point of space, but not information as to what is there. Position becomes then an *a priori*, an ontological entity. As a consequence of the reasoning, in this framework it is possible that these incomplete strings get filled again with other bits, until they gather enough information as to constitute another particle, giving rise to a new wave.

From this perspective, then, the wave function corresponds to a real physical state: in the double slit experiment, for instance, the wave really is at two different places at the same time, and really ends up at one single spot when measured.

The limits of this model are evident: first of all, these strings are not fully specified, as no attempt is made in order to account for momenta, masses, and all other relevant physical quantities. Secondly, there is no explanation as to how waves form out of these instructions. Thirdly, the formalism is completely *ad hoc*, with no other justification but that in this way it *should* work in a simple, basically trivial way. Finally, no indication regarding the Dynamics of the collapse is offered, and, at the present stage at least, the present model cannot even be called a theory, as it is untestable.

However, the hope is that splitting space into two entities, an instruction and an implementation space, could offer advantages for a simple explanation of puzzling phenomena of the quantum world such as non-locality and the principle of complementarity.

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Appendix

A Space and Time

A.1 Distances

In our framework points of space are functions implementing truth-values given by the τ's; distances, on the contrary, are not points, but, rather, they are represented by vectors connecting pairs of points, regardless of the truth-values these points implement: evaluating and connecting are then two different concepts, so that we should treat points and vectors in two different ways. To be more precise, we can think of $\mathbb{R}$ as the set of all slots constituting an infinite τ. In this way, we can write the "slot" $x_2 = x_1 + dx$ as the sum of the slot x_1, plus a distance, dx. In other words, $dx = x_2 - x_1$ is not a slot but, if added to some slot x, gives another slot, namely $x_1 + dx = x_2$.

We can define another kind of difference, among successive τ's, that is, between τ_1 and τ_2: $d\tau = \tau_2 - \tau_1$. For example, in the discrete case:

$$\Delta\vec{\tau}_{4,3} \quad = \quad \boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{1}\boxed{0}\boxed{0}\ldots \qquad ,$$

where the leftmost slot is the zeroth one. In this way, by adding each slot of $\Delta\vec{\tau}_{4,3}$ to the corresponding slot of

$$\vec{\tau}_3 \quad = \quad \boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\ldots$$

one obtains

$$\vec{\tau}_4 \quad = \quad \boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\ldots$$

Subtraction between two different points evaluating different τ's holds:

$$\begin{aligned} \phi(x_2)[\tau_2] - \phi(x_1)[\tau_1] \quad &= \quad x_2 * \tau_2(x_2) - x_1 * \tau_1(x_1) \\ &:= \quad (x_2 - x_1) * (\tau_2(x_2) - \tau_1(x_1)). \end{aligned}$$

However, we can depict the difference between different τ's in much the same way as the τ's themselves, thus recovering in the end a shift operator and an abstract definition of distances. Coming back again to the discrete case for a good visualization, we can write, for example:

$$\Delta\vec{\tau}_4 := \boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\ldots \tag{A.1}$$

If the first slot on the left is slot 0, then array (A.1) will express a jump $\Delta x = 4$, and it corresponds to

$$\Delta\vec{\tau}_{i+4,i} = \ldots\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\ldots \tag{A.2}$$

where i is the slot of $\vec{\tau}$ where the true-value of the position of the particle is located, $\tau_i = 1$, so that the first 1 on the left of instruction (A.2) is exactly at the same slot i. In other words, the displaced particle has to pass through 4 slots, from where it was to where it is now. If the 1 of array (A.1) had been in the first slot, this would have meant that the particle hadn't moved. In general we can define $\Delta\tau$ as the "vector" $\Delta\tau$ whose only non-empty slot is the Δx-th

one: $\Delta\tau(\Delta x) = 1$, whereas $\Delta\tau(\Delta x') = 0, \forall\, \Delta x' \neq \Delta x$. With this representation we need to redefine the operation of addition accordingly. For example:

$$\vec{\tau}_3 + \Delta\vec{\tau}_1 \quad = \quad \boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{0} + \boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{0} = \boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{1}\,.$$

Although representation (A.1) will not be used in the following, it is important as it shows that, in this framework, "point"-spaces and vector spaces are, in fact, equivalent.

Using this abstract representation, we are able to define a shift operator ϕ_S:

$$\phi_S(dx)[d\tau] = dx * d\tau(dx)$$

such that, for $\tau_2 = \tau_1 + d\tau$ and $x_2 = x_1 + dx$, we get:

$$\begin{aligned} \phi(x_1)[\tau_1] + \phi_S(dx)[d\tau] &= x_1 * \tau_1(x) + dx * d\tau(dx) \\ &:= (x_1 + dx) * [\tau_1 + d\tau](x_1 + dx) = x_2 * \tau_2(x_2). \\ &= \phi(x_2)[\tau_2]. \end{aligned}$$

If $d\tau = 0$, then:

$$\phi(x_1)[\tau_1] + d\phi_S(dx)[0] = \phi(x_1 + dx)[\tau_1] = \phi(x_2)[\tau_1].$$

In case more particles are involved, then $\tau = \sum_a \tau_a$, hence

$$\phi(x_2)[\tau_2] - \phi(x_1)[\tau_1] = (x_2 - x_1) * \sum_a \left[\tau_{2,a}(x_2) - \tau_{1,a}(x_1)\right].$$

Now we turn to the description of distances that have been covered by some moving particle p. A trajectory is the set of points that, in our simile, illuminate; therefore we define, for the discrete case:

$$Q_p[\vec{\tau}_p] := i_p * 1_p$$

and for the continuum one:

$$\Phi_p[\tau_p] := x_p * 1_p$$

to be the functions giving as output the point of space, either i_p or x_p, where p is located. We will then have:

$$\Phi_p[\tau + d\tau] - \Phi_p[\tau] = d\Phi_p[d\tau]. \tag{A.3}$$

The careful steps leading to the integration of (A.3) are discussed in the appendix, which yields:

$$\Delta L_p = \int_{\tau_{p,in}}^{\tau_{p,fin}} d\Phi_p[d\tau_p] \equiv \Phi_p[\tau_{p,fin}] - \Phi_p[\tau_{p,in}] = (x_{p,fin} - x_{p,in}) * 0_p.$$

A.2 Time

In Analytical Mechanics the formalism allows to write time as the $(n+1)$-th coordinate, eq. (2), which, like the other generalized coordinates, depends on

some other variable τ. To specify the dependence of t from the global instructions τ in the present context we will be following Ernst Mach's line of thought[1], thus considering time as the abstraction of the output of single physical systems called clocks. To begin with, let's consider the "vector" in one dimension

$$\tau_{in} = \sum_{l=1}^{n} \tau_{in,l} + \sum_{p=1}^{m} \tau_{in,p},$$

containing information about $n + m$ particles. Let's imagine now that, in the reference system we are in, at least one particle among the m ones is moving, say the m-th one, and let's call it c. If we pick up some successive configuration of τ, we can write:

$$\tau = \sum_{l=1}^{n} \Big(\tau_{in,l} + d\tau_l\Big) + \sum_{p=1}^{m-1} \Big(\tau_{in,p} + d\tau_p\Big) + (\tau_{in,c} + d\tau_c).$$

Assumed that c is moving, looking at its position alone allows to identify the τ it belongs to. In other words, in this framework an idealized clock C is a machine made up of $m - 1$ particles capable of keeping track of the position of the m-th particle c. To simplify, we further assume that the clock bears no influence on the system of n particles one wants to describe.

We will then have:

$$t_c[\tau_c; \tau_{c,in}] \quad := \quad C\left(\int_{\tau_{c,in}}^{\tau_c} \Big|d\Phi_c[d\tau_c]\Big|\right) \tag{A.4}$$

Function C of eq. (A.4) is physically implemented by the particles composing the clock, which convert the the length of the path of c into another quantity, time. A realization of C could consist in two reflecting walls where light bounces forth and back; we could imagine that every time the light hits the walls, part of it is detected and converted into an electric signal.

For example, let's consider the system of two particles in a discrete case, described by $\vec{\tau} = \vec{\tau}_a + \vec{\tau}_c$. For clarity, let's rewrite instructions like

1_c	$0_a + 0_c$	$1_a + 0_c$	$0_a + 0_c$	$0_a + 0_c$	$0_a + 0_c$	$0_a + 0_c$

as

c	0	a	0	0	0	0

and let's imagine that the system be given by:

c	0	a	0	0	0	0
0	c	0	a	0	0	0
c	0	0	0	a	0	0
0	c	0	0	0	a	0

$\vdots$

[1]He wrote, in 1883: "It is utterly beyond our power to measure the changes of things by time ... time is an abstraction at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected ", as quoted in [20]. For a recent experimental test dealing with the problem of time, see [21].

We can now describe C as a counter that adds 1 every time c moves, that is, every time $\vec{\tau}_c$ is updated. From (A.4), since $\Delta Q_c[\Delta\vec{\tau}_{c,l}] = 1 * 0_c$, $\forall\, l$, we get:

$$t_c[\tau_c;\tau_{c,in}] \;=\; C\left(\sum_{l=1}^{n}\left|\Delta Q_c[\Delta\vec{\tau}_{c,l}]\right|\right) = C\left(\sum_{l=1}^{n} 1 * 0_c\right) := n.$$

It is easy to specify a time dependence for the τ's. This can be best illustrated through an example. Let's consider a trajectory $x_a = x_a(t)$ for the material point a, for example

$$x_a(t) = x_{a,0}\sin\omega t. \tag{A.5}$$

In our framework $x_a(t)$ becomes $\Phi_a[\tau_a(t_c)] = x_a(t_c) * 1_a = x_{a,0}\sin\omega t_c * 1_a$. That is, to the trajectory (A.5) corresponds the series of $\tau_a(t_c)$ such that:

$$[\tau_a(t_c[\tau_c;\tau_{c,in}])](x) = \begin{cases} 1_a & \text{if } x = x_a(t_c) \\ 0_a & \text{otherwise} \end{cases} \tag{A.6}$$

It is useful now to work out the expression $[\tau_a(t_c)](x_a)$ in def. (A.6).

$$\begin{aligned} \tau_a(x_a(t_c)) \;&=\; \tau_a\left(x_{a,0}\sin\omega C\left(\int_{\Phi_c[\tau_{c,0}]}^{\Phi_c[\tau_c]} |d\Phi_c[d\tau_c]|\right)\right) \\ &=\; \tau_a\left(x_{a,0}\sin\omega C\left(\int_{\phi,x_{c,0}}^{x_c} |dx_c| * 0_c\right)\right) = 1_a. \end{aligned}$$

In this way we come back to our starting point, eq. (2), which now – for all the details see the appendix – can be rewritten, for a system of n interacting particles, as:

$$\delta\int_{\tau_{c,1}}^{\tau_{c,2}} L\left(\Phi_1[\tau_1(t_c[\tau_c])],\ldots,\Phi_n[\tau_n(t_c[\tau_c])];\frac{\Phi_1'[\tau_1]}{t_c'}[\tau_c],\ldots,\frac{\Phi_n'[\tau_n]}{t_c'}[\tau_c]\right)\cdot \cdot\frac{dt_c}{d\Phi_c[d\tau_c]}d\Phi_c[d\tau_c] = 0. \tag{A.7}$$

Solving the Euler-Lagrange equations implies virtual displacements of the functions Φ_i, but this, in turn, entails virtual variations of the τ_i themselves.

In this model every part of space could be used as a clock, provided it is sufficiently far apart from the subsystem one wants to describe. However, in the same reference system clocks must be synchronized, in order to define time consistently. The simplest condition would be:

$$\frac{dt_c}{d\Phi_c[d\tau_c]}d\Phi_c[d\tau_c] = \frac{dt_d}{d\Phi_d[d\tau_d]}d\Phi_d[d\tau_d] \equiv dt,$$

where d represents some other particle. This ensures that, if for any observer some time elapses according to some clock, the same amount of time elapses if he considers another portion of space as his time-defining clock. In this way clocks themselves can be seen as subsystems changing in time, where time is given by another subsystem somewhere else in space, that we once more name clock, and this can be repeated indefinitely. This entails that, in this framework, there can be no flowing of time if there is no movement, at least in a small subregion

of space. Furthermore, we can leave aside the subscript c and turn from t_c to t, being understood that the definition of the time unit must be consistent, irrespective of the choice of the clock used, or, to say it differently, of the kind of movement one uses as his own "time source".

As a second remark, we could note that, for a classical material point, specifying its coordinates at every instant implies that at that instant the particle is not somewhere else, as eq. (A.5) states. This is by no means trivial. In fact, non-locality means, at least from our perspective, that the same information can be simultaneously at different places, and that there it can be simultaneously updated. If we regard the existence of things as an information, then it shouldn't be taken for granted, even in classical physics, that the information concerning the existence of a particle is just at one spot. If this is not granted, then, in this framework, we should specify that this is actually the case. This is why we recur to so many 0's :

$$\vec{\tau}_a \quad = \quad ... \quad \boxed{0_a}\boxed{0_a}\boxed{0_a}\boxed{1_a}\boxed{0_a}\boxed{0_a}\boxed{0_a} \quad ...$$

or, more generally, to instructions like (A.6). What could be here nothing more than a quibble becomes relevant if we superpose different information: in fact, if we retain that the wave function collapses –that it reduces to a point when its position is being measured– this has implications for all space.

A.3 Reference Systems

The model up to now could work at best for a single reference system. In fact, if a material point remains fixed in one system, it may be moving in another one: this should mean that, depending on where we are looking at, we would have different vectors τ. Let's then imagine two material points, a and b, moving in opposite directions along a straight line. For convenience let's label the content of the slots where the points are with a and b respectively, instead of with 1_a and 1_b; we also turn to discrete space and time for a better visualization. For a in its rest frame Σ_a we would have the following series of instructions:

...	0	0	0	0	0	a, b	0	0	0	0	0	...
...	0	0	0	0	0	a	b	0	0	0	0	...
...	0	0	0	0	0	a	0	b	0	0	0	...
...	0	0	0	0	0	a	0	0	b	0	0	...
...	0	0	0	0	0	a	0	0	0	b	0	...

etc. For the material point b in its rest frame Σ_b we would have instead:

...	0	0	0	0	0	a, b	0	0	0	0	0	...
...	0	0	0	0	a	b	0	0	0	0	0	...
...	0	0	0	a	0	b	0	0	0	0	0	...
...	0	0	a	0	0	b	0	0	0	0	0	...
...	0	a	0	0	0	b	0	0	0	0	0	...

etc. The two sets of messages, for Σ_a and Σ_b respectively, must be equivalent.

Let's imagine now two reference systems Σ and Σ', being the latter connected to the first through the coordinate transformation $x' = f(x, t)$. This means that

at time t the information content of point x in Σ corresponds to the information content of x' in Σ' : $[\tau(t)](x) = [\tau'(t)](x')$, where τ' represents the same message as τ, but as seen from Σ'. Let's define the operator $\zeta_f \tau$:

$$\begin{aligned} \zeta_f[\tau(t)](f(x,t)) &:= [(\tau \circ f^{-1})(t_c)](f(x,t)) \equiv [\tau'(t)](f(x,t)) \\ &= [\tau'(t)](x'). \end{aligned}$$

We can write:

$$\phi(x)[\tau] = x \otimes \tau(x) \longrightarrow \phi(x')[\tau'] = x' \otimes \tau'(x') = x' \otimes \tau(x).$$

This means:

$$\begin{aligned} \phi(x')[\tau'] &= \phi(f(x,t))[\zeta_f \tau] \\ &= f(x,t) \otimes \tau'(f(x,t)) \\ &:= \phi'_f(x,t)[\tau]. \end{aligned}$$

B Functions of space, derivation, integration and Lagrangians

Let's consider some function $f = f(\phi(x)[\tau])$, and let's us suppose that :

$$f(\phi(x)[\tau]) = f(x * [\tau](x)) = f_\phi(x) * f_B([\tau](x)), \tag{B.1}$$

where f_ϕ, with values in $\mathbb{R}$, denotes the spatial part and f_B, with values in $2 = \{0, 1\}$, the boolean part of f; that is, f_B is one of the four boolean functions from 2 to 2. To our purposes it is convenient to define the derivative of f with respect to the trajectory of c as:

$$\begin{aligned} \frac{df}{d\Phi_c[d\tau_c]}\Big|_{\tau_c} &= \lim_{d\tau_c \to 0} \frac{f(\tau_c + d\tau_c) - f(\tau_c)}{d\Phi_c[d\tau_c]} \\ &= \left(\frac{df_\phi}{dx_c}\Big|_{x_c}\right) * \left(\frac{df_B}{[d\tau](dx_c)}\Big|_{[\tau](x_c)}\right)_B \\ &= \lim_{[d\tau_c](dx_c) \to 0} \frac{f_\phi(x_c + dx_c) - f_\phi(x_c)}{dx_c} * \\ &\quad * \left(f_B\Big([\tau_c + d\tau_c](x_c + dx_c)\Big) - f_B\Big(\tau(x_c)\Big)\right), \end{aligned}$$

where x_c represents the point where particle c is. Since $\Phi_c[\tau_c + d\tau_c] = (x_c + dx_c) * 1_c$ and $\Phi_c[\tau_c] = (x_c) * 1_c$ we get that $f_B\Big([\tau_c + d\tau_c](x_c + dx_c)\Big) = f_B\Big([\tau_c](x_c)\Big) = f_B(1_c)$, so that

$$\frac{df}{d\Phi_c[d\tau_c]}\Big|_\tau := \frac{df_\phi}{dx_c}\Big|_{x_c} * 0_c.$$

Alternatively, it can be useful to define functions with just the spatial part. Time can be described as

$$t = t(\Phi_c[\tau_c]) = t(x_c * \tau_c(x_c)) = t(x_c).$$

Its derivative will be:

$$\begin{aligned}\frac{dt}{d\Phi_c[d\tau_c]}\Big|_{\tau_c} &= \lim_{d\tau_c\to 0}\frac{t(\Phi_c[\tau_c+d\tau_c])-t(\Phi_c[\tau_c])}{d\Phi_c[d\tau_c]}\\ &:= \lim_{dx_c\to 0}\frac{t(x_c+dx_c)-t(x_c)}{dx_c}\equiv\frac{1}{v_c}(\Phi_c[\tau_c]).\end{aligned}$$

Let's consider now the trajectory of particle a, given by $\Phi_a[\tau_a(t)]$; it is then easy to define

$$\begin{aligned}\dot{\Phi}_a[\tau_a(t)] &= \lim_{d\tau_c\to 0}\frac{\Phi_a[\tau(t(\tau_c+d\tau_c))]-\Phi_a(\tau[t(\tau_c)])}{dt[d\tau_c]}=\lim_{dt_c\to 0}\frac{x_a(t+dt)-x_a(t)}{dt}*0_a\\ &= \left(\frac{\frac{d\Phi_a[d\tau_a]}{d\Phi_c[d\tau_c]}}{\frac{dt}{d\Phi_c[d\tau_c]}}\right)\Bigg|_{\tau_c}=\frac{\Phi'_a}{t'},\end{aligned}$$

where

$$\begin{aligned}\frac{d\Phi_a[d\tau_a]}{d\Phi_c[d\tau_c]}\Big|_{\tau_c} &= \lim_{d\tau_c\to 0}\frac{x_a[\tau_c+d\tau_c]-x_a[\tau_c]}{dx_c}*\left(\Big(\tau_a[\tau_c+d\tau_c]\Big)(x_a[\tau_c+d\tau_c])-\Big(\tau_a[\tau_c]\Big)(x_a[\tau_c]\Big)\right)\\ &= \frac{dx_a}{dx_c}\Big|_{\tau_c}*0_a. \qquad \text{(B.2)}\end{aligned}$$

The derivative with respect to time, $\dot{\Phi}_a[\tau(t)]$, then becomes

$$\dot{\Phi}_a[\tau_a(t)]=\left(v_c\frac{dx_a}{dx_c}\right)[\tau_c]*0_a.$$

There could be a more general case, where the function

$$\Phi[\tau_a(t)]=x(t)*[\tau_a(t)](x(t)),$$

evaluates at every t the point $x(t)$, which does not necessarily correspond to the position of particle a, that is, to $x_a(t)$. Its time derivative, then, will be written as:

$$\dot{\Phi}(t)[\tau_a(t)] = \lim_{dt\to 0}\frac{\Phi(x(t+dt))[\tau_a(t+dt)]-\Phi(x(t)(\tau_a[t])}{dt[d\tau_c]}=\frac{\Phi'}{t'}[\tau_c],$$

where now

$$\begin{aligned}\frac{d\Phi[d\tau_a]}{d\Phi_c[d\tau_c]}\Big|_{\tau_c} &= \lim_{d\tau_c\to 0}\frac{x[\tau_c+d\tau_c]-x[\tau_c]}{dx_c}*\left(\Big(\tau_a[\tau_c+d\tau_c]\Big)(x[\tau_c+d\tau_c])-\Big(\tau_a[\tau_c]\Big)(x[\tau_c]\Big)\right)\\ &= \frac{dx}{dx_c}\Big|_{\tau_c}*\tau'_a(x_c). \qquad \text{(B.3)}\end{aligned}$$

In this way, the trajectory $\Phi(t)$ is independent from the actual position of particle a. If $\tau'_a(x_c)=0_a$, then the trajectory, for the infinitesimal time interval $dt_c[d\tau_c]$, either doesn't intercept the position of a or it coincides with it; otherwise, if $\tau'_a(x_c)=1_a$, either the trajectory, during $dt_c[d\tau_c]$, stopped coinciding with that of particle a, or it just began coinciding with it. Let's consider now some function L – the Lagrangian – of the trajectory and velocity of particle a:

$$L:=L\Big(\Phi_a[\tau_a(t)],\dot{\Phi}_a[\tau_a(t)]\Big)=L_\Phi(x_a,\dot{x}_a)*L_B\Big([\tau_a(t)](x_a),\tau'_a(x_c)\Big), \qquad \text{(B.4)}$$

where the $\tau'_a(x_c) = 0_a$ in L_B comes from $\dot{\Phi}_a[\tau_a(t)]$. For our purposes it is most convenient to define L_B as a simple projection:

$$L_B\Big([\tau_a(t)](x_a), \tau'_a(x_c)\Big) := \tau'_a(x_c) \equiv 0_a. \tag{B.5}$$

After derivation, we have to define now integration, and eventually arrive, via eqs. (B.4) and (B.5), at eq. (2). Since the integrand will be of the form $f = f_\phi * f_B$, we will have a corresponding integral $\int = \int_\phi * \int_B$. The first one, $\int_\phi$, applies to the spatial part, and it is defined as usual; the second one, $\int_B$ can be defined as follows. Given some trajectory $\Phi = \Phi[\tau]$, such that $\Phi[\tau_{in}] = x_{in} * \tau_{in}(x_{in})$ and $\Phi[\tau_{fin}] = x_{fin} * \tau_{fin}(x_{fin})$, the integral of f_B is:

$$\int_{B,\Phi[\tau_{in}]}^{\Phi[\tau_{fin}]} f_B(\tau) d\tau[dx] \quad := \quad \lim_{N\to\infty} \sum_{i=0}^{N-1} \Big(f_B[\tau_{i+1}](x_{i+1}) - f_B[\tau_i](x_i) \Big), \tag{B.6}$$

where $\tau_0(x_0) = \tau_{in}(x_{in})$ and $\tau_N(x_N) = \tau_{fin}(x_{fin})$.

We proceed with an example, by calculating the length of the trajectory of a. For two successive positions of particle a infinitesimally close to each other, $x_{a,i}$ and $x_{a,i+1}$, it holds:

$$d\Phi_a[d\tau_a] = (x_{a,i+1} - x_{a,i}) * (\tau_a(x_{a,i+1}) - \tau_a(x_{a,i})) = dx_a * 0_a.$$

Then, for $x_{a,i+1} - x_{a,i} = \frac{x_{a,fin} - x_{a,in}}{N}$,

$$\begin{aligned}
\int_{\Phi_a[\tau_{a,in}]}^{\Phi_a[\tau_{a,fin}]} d\Phi_a[d\tau_a] &= \lim_{N\to\infty} \sum_{i=0}^{N-1} d\Phi_a[d\tau_{a,i}] \\
&= \lim_{N\to\infty} \sum_{i=0}^{N-1} (x_{a,i+1} - x_{a,i}) * (\tau_{a,i+1}(x_{a,i+1}) - \tau_{a,i}(x_{a,i})) \\
&:= \int_{\Phi,x_{a,in}}^{x_{a,fin}} * \int_{B,[\tau_{a,in}](x_{a,in})}^{[\tau_{a,fin}](x_{a,fin})} dx_a[d\tau_a](dx) \\
&= (x_{a,2} - x_{a,1}) * \Big([\tau_{a,2}](x_2) - [\tau_{a,1}](x_1) \Big) \\
&= (x_{a,2} - x_{a,1}) * (1_a - 1_a) \equiv (x_{a,2} - x_{a,1}) * 0_a,
\end{aligned}$$

so that

$$\begin{aligned}
\Phi_a[\tau_{a,2}] &= \Phi_a[\tau_{a,1}] + \int_{\tau_{a,1}}^{\tau_{a,2}} d\Phi_a[d\tau_a] \\
&= x_{a,1} * 1_a + (x_{a,2} - x_{a,1}) * 0_a = x_{a,2} * 1_a.
\end{aligned}$$

We then have:

$$\begin{aligned}
L(x_a(t), \dot{x}_a(t))dt &\to L\left(\Phi_a(t)[\tau_c], \frac{\Phi'_a}{t'}[\tau_c]\right) \frac{dt}{d\Phi_c[d\tau_c]} d\Phi_c[d\tau_c]. \\
&= L_\Phi\left(x_a, \frac{x'_a}{t'}\right) \frac{dx_c}{v_c} * \tau'_a(x_c)
\end{aligned} \tag{B.7}$$

Integration yields then, with a somewhat incorrect notation:

$$\left(\int_{x_{a,in}}^{x_{a,fin}} L_\Phi\left(x_a, \frac{x'_a}{t'}\right)\frac{dx_c}{v_c}\right) * \Big(\Big(\tau_a[\tau_{c,fin}]\Big)(x_a(x_{c,fin})) - \Big(\tau_a[\tau_{c,in}]\Big)(x_a(x_{c,in}))\Big) \tag{B.8}$$

The spacial part of (B.8), once varied, corresponds to eq. (2); as far as particle a is concerned, it doesn't disappear at this stage, and one could equally write the boolean part of the integrand as 0_a.

In sec. 2.2 it is dealt with a wave function ψ containing an integration over all possible paths from one point to another. To each of these paths we are now going to assign its own "particle": when a position measurement is being performed, the wave function "collapses", and of all paths only a subset remains – as if many particles would disappear at once. In that case, then, the boolean part of (B.8) at the instant of the collapse will yield 1_a, where the 1_a means that a change took place for particle a.

To perform this labelling we assign to each path in the path integral, which at time t_i can be identified by the series of points $(x_i, x_{i-1}, ...)$, its own "particle", whose corresponding instruction is going to be dubbed $\tau_{(x_i,x_{i-1},...)}$. To be coherent we should turn from Φ_a to $\Phi_{(x_i,x_{i-1},...)}$, but we omit the subscripts for brevity. The action S, eq. (10), then becomes, accordingly:

$$\begin{aligned}
S(x_{i+1}, x_i) &= \min \int_{\tau_{c,i}}^{\tau_{c,i+1}} * \int_{B,\tau_{c,i}}^{\tau_{c,i+1}} L \\
&\quad \left(\Phi_a\Big[\tau_{a,(x_i,x_{i-1},...)}[\tau_c]\Big], \frac{\Phi'_a\Big[\tau_{a,(x_i,x_{i-1},...)}[\tau_c]\Big]}{t'[\tau_c]}\right)\frac{dt}{d\Phi_c[d\tau_c]}d\Phi_c[d\tau_c]. \\
&= \min\left(\int_{x_{c,i}}^{x_{c,i+1}} L_\Phi\left(x_a, \frac{x'_a}{t'}\right)t'dx_c\right) * \\
&\quad \Big(\tau_{a,x_{i+1},x_i,x_{i-1},...}(x_{a,i+1}(x_{c,i+1})) - \tau_{a,x_i,x_{i-1},...}(x_{a,i}(x_{c,i}))\Big) \\
&= \min\left(\int_{x_{c,i}}^{x_{c,i+1}} L_\Phi\left(x_a, \frac{x'_a}{t'}\right)t'dx_c\right) * 0_{a,x_{i+1},x_i,x_{i-1},...}
\end{aligned} \tag{B.9}$$