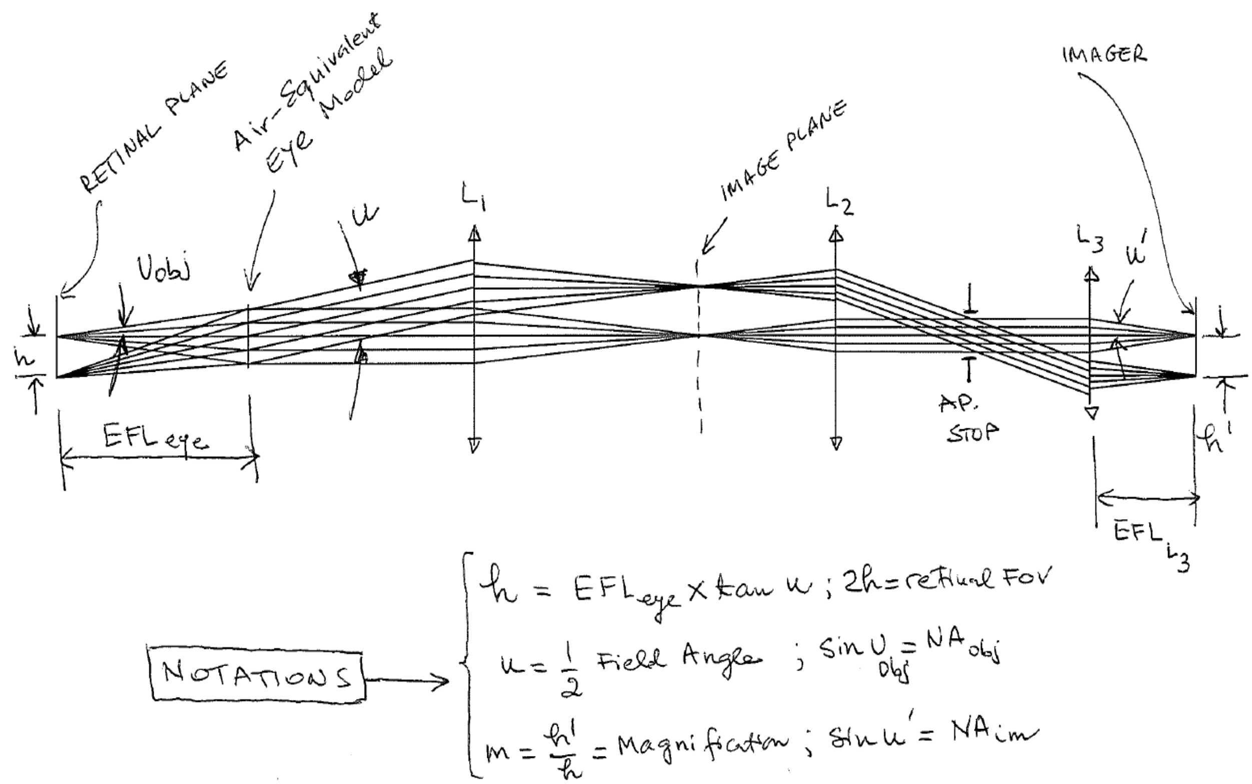
First-Order Analysis of Eye Imaging



The diagram above shows a generic lens train for digital imaging of the human retina. I am using this diagram to carry out a first-order analysis from the elementary diffraction theory of image formation. The system comprises three lenses, an aperture stop conjugate to the eye pupil and an imager. For simplicity, I assume aberration free imaging with an air-equivalent model eye whose focal length measures 17 mm (which corresponds to a normal unaccommodated eye having 1000/17 = 59 Diopters). I also assume that these are the main inputs to the optics design:

1) The required angular resolution at the imager plane is *R (# of pixels/degree of the field of view*).
2) The angular Field Of View (FOV) is FOV = θ degrees.

3) The central wavelength is placed at λ_c.

4) The system must be able to resolve a minimum line width of $\delta\,(mm)$.

5) The pixel footprint is $\Delta\,(mm)$ per side.

6) The diameter of the eye pupil is $D\,(mm)$.

<u>Computation of the angular resolution *R*</u>

The numerical aperture in image space is given by

$$NA_{im} = \sin u' \tag{1}$$

and the diameter of the diffraction blur (Airy disk) is

$$\Phi = \frac{1.22\lambda_c}{NA_{im}} \tag{2}$$

The optical invariant (étendue) for this system can be written as

$$Et = h \times NA_{obj} = h' \times NA_{im} \tag{3}$$

Using (1) and (2) yields

$$h' = \frac{E_t \Phi}{1.22\lambda_c} \tag{4}$$

which automatically fixes the image height h', that is, the radius of the image circle. The lateral magnification is also fixed by

$$m = \frac{h'}{h} = \frac{NA_{obj}}{NA_{im}} \tag{5}$$

For a real optical system whose performance in diffraction limited, it is natural to assume that the diameter of the diffraction blur matches the linear size of the pixel, that is,

$$\Phi \approx \Delta \tag{6}$$

The angular resolution R is related to the diameter of the image circle via

$$2h'(mm) = FOV(\text{deg}) \times R(\text{pix/deg}) \times \Delta(mm) \tag{7}$$

From (4) and (7) we obtain

$$E_t = 0.61\lambda_c \times FOV \times R \tag{8}$$

On the other hand, the optical invariant can be expressed as

$$E_t = h \times NA_{obj} = EFL_{eye} \times \tan u \times \frac{D}{2 \times EFL_{eye}} \tag{9}$$

or

$$E_t = \frac{D \tan u}{2} \tag{10}$$

Combining (8) and (10) leads to

$$\boxed{R = \frac{D \tan u}{1.22\lambda_c \theta}} \tag{11}$$

Taking a 40 deg. FOV and a 4 mm diameter non-mydriatic pupil (*D* = 4), returns an angular resolution of $R = 50.8$ pixels per degree. If the pupil diameter measures 3 mm instead, the angular resolution becomes $R = 38$ pixels per degree.

Bottom line is that, when the FOV is 40 deg., the circle image needs to cover an array of 1520 x 1520 = 2.3 Mpix for a non-mydriatic 3 mm diameter pupil and an array of 2032 x 2032 = 4.1 Mpix for a 4 mm diameter pupil.

Theoretical modulation required to resolve δ

The Modulation Transfer Function (MTF) for an aberration-free monochromatic optical system is given by

$$MTF(\nu) = \frac{2}{\pi}(\Psi - \cos\Psi \sin\Psi) \tag{12}$$

where ν is the spatial frequency in line-pairs/mm and

$$\Psi(\nu) = \arccos(\frac{\lambda_c \nu}{2NA_{im}}) \text{ (radians)} \tag{13}$$

The limiting cases of (12) are

1) $MTF(0) = 1$

2) Cutoff frequency $\nu_c = \frac{2NA_{im}}{\lambda_c} \Rightarrow MTF(\nu_c) = 0$

Since the system must be able to resolve a retinal detail whose line-width measures δ (mm), the corresponding spatial frequency in line-pairs/mm may be computed from

$$\nu_0 = \frac{1}{2\delta} \tag{14}$$

in which 2δ represents the length of a black and a white bar forming the line-pair. From (2), (6) and (14) we get

$$\Psi(\nu_0) = \arccos\left(\frac{\Delta}{4.88 \times \delta}\right) \qquad (15)$$

Further assuming that the line-width δ is comparable in size with the pixel footprint ($\delta \approx \Delta$) leads to

$$\Psi(\nu_0) = 1.36(rad) = 0.8658 \times \frac{\pi}{2}(rad) \qquad (16)$$

and a modulation of

$$MTF(\nu_0) = 0.738 = 73.8\,\% \qquad (17)$$