A Conjecture on Near-square Prime Number Sequence of Mersenne Primes

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Abstract

In this paper we present a conjecture that there is a near-square prime number sequence of Mersenne primes to arise from the near-square number sequence $W_p = 2M_p^2 - 1$ generated from all Mersenne primes M_p, in which every term is larger prime number than corresponding perfect number $P_p = (M_p^2 + M_p)/2$. The conjecture has been verified for the first few prime terms in the near-square prime number sequence and we may expect appearing of near-square prime numbers of some known Mersenne primes with large p-values will become larger primes to be searched than the largest known Mersenne prime $M_{57885161}$.

Keywords: Mersenne prime; near-square prime number sequence of Mersenne primes; the largest known Mersenne prime.

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It is well known that every Mersenne prime $M_p=2^p-1$ will lead to appearing of a corresponding perfect number $P_p=(M_p^2+M_p)/2$ and the largest known prime has almost always been a Mersenne prime in modern times[1]. In order to find a new way for searching larger prime numbers than the largest known Mersenne prime $M_{57885161}$ from known Mersenne primes themselves, we give the following discussion.

The traditional relation formula between perfect number P_p and Mersenne prime M_p can be expressed as

$$P_p = (M_p^2 + M_p)/2. \qquad (1)$$

From (1) we have

$$W_p = 2(2P_p-M_p) -1, \qquad (2)$$

where

$$W_p = 2M_p^2 -1 \qquad (3)$$

is a near-square number of a Mersenne prime M_p, so that there is a near-square number sequence $W_p = 2M_p^2 -1$ generated from all Mersenne prime M_p. If Mersenne primes M_p are infinite then $W_p = 2M_p^2 -1$ is an infinite sequence. From $M_p=2^p-1$ we get structure of near-square number $W_p = 2M_p^2 -1$ as follows

$$W_p = 2^{2p+1}-2^{p+2}+1, \qquad (4)$$

where p is exponent of Mersenne prime $M_p=2^p-1$. Hence we have the following conjecture.

Conjecture. There is a near-square prime number sequence of Mersenne primes to arise from the near-square number sequence $W_p = 2M_p^2 -1$ generated from all Mersenne primes M_p, in which every term is larger prime number than corresponding perfect number $P_p = (M_p^2 + M_p)/2$.

Obviously, the conjecture does not mean every near-square number W_p of Mersenne prime M_p is a prime number. In this near-square number sequence, we have verified the first few prime terms as follows[2]

$W_2=2^5–2^4+1=17$,

$W_3=2^7–2^5+1=97$,

$W_7=2^{15}–2^9+1=32257$,

$W_{17}=2^{35}–2^{19}+1=34359214081$

$W_{19}=2^{39}–2^{21}+1=549753716737$

…

From (2) we see every prime W_p is larger than corresponding perfect number $P_p = (M_p^2 + M_p)/2$. Therefore, we obtain

$W_2=17 > P_2 =6$

$W_3=97 > P_3 =28$

$W_7=32257 > P_7 =8128$

$W_{17}=34359214081> P_{17} =8589869056$

$W_{19}=549753716737> P_{19} =137438691328$

…

Above discussion makes us fell larger prime numbers than the largest known Mersenne prime $M_{57885161}$ will arise from near-square numbers of some known Mersenne primes i.e. $W_p = 2M_p^2 –1$ for M_p with large p-values. In other words, such large primes possibly appear among the following near-square numbers of known

Mersenne primes:

$$W_{30402457}=2^{2\cdot 30402457+1}-2^{30402457+2}+1=2^{60804915}-2^{30402459}+1,$$

$$W_{32582657}=2^{2\cdot 32582657+1}-2^{32582657+2}+1=2^{65165315}-2^{32582659}+1,$$

$$W_{37156667}=2^{2\cdot 37156667+1}-2^{37156667+2}+1=2^{74313335}-2^{37156669}+1,$$

$$W_{42643801}=2^{2\cdot 42643801+1}-2^{42643801+2}+1=2^{85287603}-2^{42643803}+1,$$

$$W_{43112609}=2^{2\cdot 43112609+1}-2^{43112609+2}+1=2^{86225219}-2^{43112611}+1,$$

$$W_{57885161}=2^{2\cdot 57885161+1}-2^{57885161+2}+1=2^{115770323}-2^{57885163}+1.$$

References

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