Critique of the Formula Derivation for Light Deflection by a Gravitating Body in the General Relativity Theory

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Abstract

In this article it is shown that the hallmark of the General Relativity Theory (GRT), the calculation of the light deflection by a gravitating body, is incorrect. The reason is that the derivation does not follow the well know Fermat principle from optics. Instead an ad hoc principle is used that violates the basic tenets of relativity in order to force an agreement with observations. The Fermat principle has to be satisfied because it follows from the wave equation that guides the photon propagation. This is a fatal problem for the GRT with the root cause traced to the Schwarzschild metric, which does not describe the reality correctly. When the author's new metric is used the standard Fermat principle can be generalized and used leading to the results agreeing with observations and experiments quite well.

Key words: General Relativity Theory, Light deflection by a gravitating body, Schwarzschild metric, Fermat principle, Lagrange formalism, Natural coordinates, Harmonic coordinates, Isotropic coordinates.

1. Introduction

The confirmation of light deflection from a straight line by a gravitating body during the Solar eclipse in 1919 by Sir Arthur Eddington was one of the first major confirmations of the GRT and a milestone that initiated the general acceptance of the theory. Since then several new measurements and evaluations have been performed, for example with the Cassini space probe [1, 2], with a much greater precision. It would thus seem that there is no problem and the theory was, therefore, proclaimed proven. This paper will go over the basic calculations that are behind this phenomenon, investigate the assumptions used in the derivation of the formulas, and show by elementary means that the standard derivation presented in most major text books is unfortunately erroneous. The light deflection by a gravitating body is a hallmark of the GRT and should, therefore, be thoroughly scrutinized and analyzed. It is thus shocking and perplexing to find that the error described in this article has been hidden in the derivation and not discovered for so many years. The correct understanding of the light deflection by a gravitational field and the associated gravitational lensing become increasingly important today in studying the Universe and its galaxies.

2. Derivation of the GRT light deflection formula

The formula for the light deflection angle is derived from the Schwarzschild metric, which is in turn derived as a unique solution of the famous Einstein field equations for the centrally gravitating body of mass M, when the mass energy tensor T_{jk} is set to zero:

$$G_{jk} = R_{jk} - \frac{1}{2} R_{(c)} g_{jk} = -\frac{8\pi\kappa}{c^4} T_{jk}, \quad T^{k}_{j\cdot|k} = 0, \quad T_{jk} = 0 . \tag{1}$$

The Schwarzschild metric differential line element is then as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{tt}^{-1} dr^2 - r^2 d\Omega^2 , \tag{2}$$

where the angular coordinates are: $d\Omega^2 = \left(d\vartheta^2 + \sin^2 \vartheta d\varphi^2\right)$, and the coefficient g_{tt} is equal to:

$$g_{tt} = 1 - \frac{R_s}{r} , \tag{3}$$

with the Schwarzschild radius R_s defined as:

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$$R_s = \frac{2\kappa M}{c^2}\,. \tag{4}$$

The parameter κ is the gravitational constant and *c* the vacuum speed of light in our local intergalactic neighborhood. The path of the photons is found by setting the metric line element ds in Eq.2 to zero. For simplicity and without any loss of generality it is also considered that the photon motion in the spacetime of the centrally gravitating body can be limited to the equatorial plane: $\vartheta = \pi/2$. Finally, to find the particular photon trajectory on the remaining 2D surface it is necessary to use another condition derived from a suitable fundamental principle of physics such as, for example, from the well known Lagrangian formalism, that would provide the trajectory equation. There are many ways published in the literature how the light beam deflection can be calculated in the GRT, but most of them except Stephani [3] and perhaps some others are omitting to clearly mention the simple fact that they are all mathematically equivalent to the following [4, 5]:

$$\delta\int_r c\,dt = 0\,. \tag{5}$$

By substituting for the time variable from Eq.2 when $ds = 0$, and $\vartheta = \pi/2$, the variational integral becomes:

$$\delta\int_r \frac{1}{\sqrt{1-R_s/r}}\sqrt{\frac{1}{1-R_s/r} + r^2\left(\frac{d\varphi}{dr}\right)^2}\,dr = 0\,. \tag{6}$$

The first integral of Euler-Lagrange (EL) equation corresponding to this variational problem is as follows:

$$\frac{r^2}{\sqrt{1-R_s/r}}\left(\frac{d\varphi}{dr}\right) = \alpha\sqrt{\frac{1}{1-R_s/r} + r^2\left(\frac{d\varphi}{dr}\right)^2}\,, \tag{7}$$

where α is an arbitrary constant of integration. After some rearrangements and after determining the value of the integration constant from the condition that at the perihelion r_p the derivative of radial coordinate *r* with respect to the angle is zero, Eq.7 becomes:

$$\frac{d\varphi}{dr} = \frac{r_p}{r\sqrt{r^2(1-R_s/r_p) - r_p^2(1-R_s/r)}}\,. \tag{8}$$

This equation is further rearranged using the following common substitutions: $x = r_p/r$ and $a = R_s/r_p$ resulting in:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{1-x^2}}\frac{1}{\sqrt{1-a\dfrac{1-x^3}{1-x^2}}}\,, \tag{9}$$

where the variable x ranges from zero to unity: $0 \le x \le 1$. This expression is further simplified by assuming that the parameter a is very small compared to unity. This leads to the result:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{a}{2}\frac{1}{\sqrt{1-x^2}}\frac{1-x^3}{1-x^2}\,. \tag{10}$$

The trajectory deviation from a straight line is then obtained by integrating Eq.10 (perihelion to infinity), multiplying the result by two, and subtracting π. This leads to the final result:

$$\Delta\varphi = a\int_0^1 \left(1+\frac{x^2}{1+x}\right)\frac{dx}{\sqrt{1-x^2}} = 2a = 2\frac{R_s}{r_p} \ . \tag{11}$$

This result agrees with observations and the Cassini space probe measurements reasonably well, so this is a good reason for celebration and for a proclamation that the GRT is correct. But wait a minute, this is all nice, but where did Eq.5 come from? Shouldn't this equation be derived from some basic well-known general principle of optics to agree with the rest of the physics instead of just postulating a new principle? It is not reasonable to state a principle and claim that this is it when the result of using it seems to agree with observations. We have the rest of the physics to make the principle to agree with without contradictions in particular with the Fermat principle that follows from the wave equation that guides the photon propagation.

3. Derivation and application of the relativistic Fermat principle.

The optics engineering is well developed into a whole industry with a sophisticated science behind it and many optical instruments are fabricated every day to satisfy the various demands of customers and scientists. The lenses, prisms, and many optical communication devices are designed and calculated based on a simple concept that the photons propagate along the lines satisfying the Fermat principle, which states that the photon traveling time along the photon travelling path must be at minimum or at least stationary. It is well known that the gravitational field affects the speed of light, so one would expect that the Fermat principle should also work there and could be used to calculate the light trajectory by simply considering that the space around the gravitating body has an index of refraction: $n = c/v$ different from unity similarly as any other optical medium. It should thus hold that:

$$\delta\int_L dt_{ph} = 0 \, . \tag{12}$$

This is seemingly the same equation as Eq.5; however, in curved spacetimes the photon travel time needs to be evaluated at the same location where the photon arrival is observed, the location of a distant observer. Therefore, for the same differential of the photon traveling time dt at any location along the photon travelling path it is also necessary to include the time dilation factor. This leads to the following:

$$\delta\int_L \frac{dt}{\sqrt{g_{tt}}} = 0 \, . \tag{13}$$

This is the correct relativistic equivalent of the Fermat principle from optics that the photon path should follow in a gravitational field. This integral is also a requirement resulting from the fact that the photon is a wave guided by the wave equation and the condition of stationarity is necessary to make sure that the constructive interference occurs at the place of the photon arrival. More details of the derivation of Eq.13 are given in the Appendix.

In support of the correctness of Eq.13, it is necessary to understand where Eq.5 actually came from. The justification for this equation is usually obtained from the analogy of the trajectory calculation of massive particles. It is well known that such trajectories are found from the variational principle:

$$\delta\int_L ds = 0 \, . \tag{14}$$

For the photons it is just simplistically assumed that their trajectories can be calculated the same way. The differential line element *ds* is erroneously replaced by *cdt* when *ds* is set to zero in Eq.2. It is accepted that the same principle applies regardless of the fact that there is now one less dimension for variations. Also, the fact that photons do not have a gravitational mass is disregarded (photons are considered as typical massive particles). This is the essence of the fatal problem of the GRT light deflection calculations.

From the above considerations and from the derivations given in the Appendix it is thus obvious that the photon traveling time calculated only as an integral over the coordinate time with the constant speed of light c as is shown in Eq.5 cannot be correct. This formula certainly needs additional justification, which the author was not able to find anywhere in the published literature [3, 4, 5]. The coordinates r and t in the curved spacetime do not have a direct physical meaning as they do in the flat spacetime and the speed of light changes in the gravitational field from place to place depending on r. It is thus clear that Eq.5 can be definitively considered wrong and this serious error has been hidden in the GRT for many years perhaps for the reason that it yields an approximately correct result. The end thus justifies the means here regardless of the physics behind the phenomenon and this is not a good science. It is therefore clear that the variational integral in Eq.13 should be used instead of the integral in Eq.5 to calculate the photon trajectory. The calculations are simple to repeat starting from the integral:

$$\delta\int_r \frac{1}{1-R_s/r}\sqrt{\frac{1}{1-R_s/r}+r^2\left(\frac{d\varphi}{dr}\right)^2}\,dr=0\,, \tag{15}$$

with the result:

$$\frac{d\varphi}{dr}=\frac{r_p\sqrt{1-R_s/r}}{r\sqrt{r^2(1-R_s/r_p)^2-r_p^2(1-R_s/r)^2}}\,, \tag{16}$$

finally leading to the expression for the light beam deflection angle:

$$\Delta\varphi=2a\int_0^1\left(1-\frac{x}{2}+\frac{x^2}{1+x}\right)\frac{dx}{\sqrt{1-x^2}}=3a=3\frac{R_s}{r_p}\,. \tag{17}$$

Unfortunately, this result does not agree with observations and experiments. How is this possible? The hallmark of the GRT is not calculated correctly? This is a shocking discovery, what is wrong? Fortunately, the answer is not too difficult to find. The problem is the Schwarzschild metric and Einstein field equations from which the Schwarzschild metric is derived. It is absolutely necessary for the Fermat principle to be satisfied, otherwise the photon propagation in the GRT would not follow the wave equation and this is a fatal problem for the theory.

To clearly prove this point let's use another metric derived by the author elsewhere [6, 7], which does not predict the existence of Black Holes with their event horizons and which is not derived from Einstein field equations. The metric is:

$$ds^2=e^{-R_s/\rho}(cdt)^2-e^{R_s/\rho}dr^2-\rho^2e^{-R_s/\rho}d\Omega^2\,, \tag{18}$$

where the parameter $\rho(r)$ is the physical radius found from the differential equation: $d\rho=e^{R_s/2\rho}dr$. The variational integral according to the Fermat principle in Eq.13 is then as follows:

$$\delta\int_\rho e^{R_s/2\rho}\sqrt{e^{R_s/\rho}+\rho^2\left(\frac{d\varphi}{d\rho}\right)^2}\,d\rho=0\,. \tag{19}$$

Repeating the same steps as in the derivation from the Schwarzschild metric, EL equation and from that the differential equation for the trajectory becomes:

$$\frac{d\varphi}{d\rho}=\frac{\rho_p}{\rho\sqrt{\rho^2e^{-R_s/\rho_p}-\rho_p^2e^{-R_s/\rho}}}\,, \tag{20}$$

where ρ_p is the physical radial distance at the perihelion. Again, after the customary substitutions: $x = \rho_p / \rho$ and $a = R_s / \rho_p$ the result is:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{e^{-a} - x^2 e^{-a\,x}}} . \tag{21}$$

This formula is simplified assuming once more that the parameter a is small compared to unity:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{1-x^2}} \frac{1}{\sqrt{1 - a\dfrac{1-x^3}{1-x^2}}} . \tag{22}$$

After some simple rearrangements, integration from zero to unity, multiplying the result by two, and subtracting π the light beam deflection angle from a straight line is:

$$\Delta\varphi = a\int_0^1 \left(1 + \frac{x^2}{1+x}\right) \frac{dx}{\sqrt{1-x^2}} = 2a = 2\frac{R_s}{\rho_p} . \tag{23}$$

This is the same result as obtained from the erroneously derived standard GRT formula, but now correctly derived from the new metric using the well known and many times verified Fermat principle. The only difference is that the standard coordinate radius is replaced by the physical radius $\rho(r)$. This result thus suggests that the Schwarzschild metric and the GRT are not describing the reality correctly. However, an objection could be raised that this is not due to the metric itself, which is not correct, but simply due to the choice of coordinates. It might be difficult, however, to overcome the large multiplication factor of $3/2$ existing between the correct GRT derivation result and observations, not just some small second order correction, but still such a claim could be made that if a correct coordinate system were found and the metrics transformed accordingly, the problem would go away. The Schwarzschild metric and Einstein field equations could then be proven correct again and the new metric proven wrong assuming that the new coordinate system could be shown to correspond to reality. The role of the coordinates in the light deflection tests thus needs to be discussed in more detail and this is addressed in the next section.

4. The role of coordinates in the light deflection tests.

By inspecting Eq.23 it can be observed that the formula has a certain asymmetry in it. The left hand side of the equation is the standard coordinate angle while the right hand side is an invariant independent of a particular choice of the radial coordinate system. It thus seems that by simply using different coordinates the equation could be changed. By examining both metrics, the Schwarzschild metric and the new metric, it is clear from the metric line element differentials that both metrics are using the same coordinates (t, r, ϑ, φ) called in this paper the standard or natural coordinates. The literature calls this coordinate system the Schwarzschild gauge, but from the above it is clear that other metrics are also using it, so the names "natural coordinate system" or "standard coordinate system" seem more appropriate. Still, even if the coordinates that both metrics are using are the same and it is now much more difficult to claim that the coordinates are causing the factor of 3 instead of factor of 2 in the correctly derived GRT deflection angle formula, it would be nice to derive a coordinate independent equation for the deflection angle. This is easily accomplished by introducing the physical angle differential: $d\varphi_{ph} = e^{-R_s/2\rho} d\varphi$ into Eq.19 and write for the variational integral the following relation.

$$\delta\int_\rho e^{R_s/\rho} \sqrt{1 + \rho^2 \left(\frac{d\varphi_{ph}}{d\rho}\right)^2}\, d\rho = \delta\int_\rho \frac{c}{c_r} \sqrt{1 + \rho^2 \left(\frac{d\varphi_{ph}}{d\rho}\right)^2}\, d\rho = 0 . \tag{24}$$

The derivation is simplified by substituting directly into Eq.21 for the physical angle the equivalent expression $d\varphi_{ph} = e^{-a \cdot x/2} d\varphi$ and carrying out the calculations. The result is:

$$\Delta\varphi_{ph} = \frac{R_s}{\rho_p} . \tag{25}$$

This equation now possesses the required symmetry with the physical quantities present on both sides, which are, of course, independent of a particular choice of the coordinate system, since there is only one physical spacetime. However, the result is surprisingly only one half of the expected value. This clearly leads to the conclusion that one half of the deflection angle is due to the index of refraction effect caused by the gravitational field in the Minkowski flat physical spacetime, as is apparent from Eq.24, and that the other half of the deflection angle is due to the gravity induced curvature of the coordinate spacetime itself, which depends on the particular choice of the coordinates. This finding is interesting since it is now possible to experimentally determine which coordinate system actually corresponds to reality and which coordinates are the observable quantities by comparing the theoretical predictions with observations. If the observations agree well with the predictions obtained using, for example, the natural coordinate system, the conclusion must be that other coordinate systems such as the isotropic, harmonic, or any other that has not been invented yet, must be ruled out and considered to be only mathematical speculations and manipulations with no basis in reality. The comparison of theoretical predictions with observations is discussed next.

5. Comparison with observations

In order to find out what differences can be expected from the derived formulas it is convenient to evaluate the following ratio:

$$\varepsilon_{new} = \frac{\Delta\varphi(new)}{\Delta\varphi(Sch)} = \frac{r_p}{\rho_p(r_p)} . \tag{26}$$

First, numerically computing the physical distance of the trajectory perihelion for the Sun, the ratio becomes $\varepsilon_{new} = 1 - 2.8(66) \cdot 10^{-5}$. It is also possible to make more precise numerical calculations using Eq.21 without neglecting the higher order terms when assuming that the parameter a is small compared to unity. The result for this case is $\varepsilon_{new} = 1 - 2.7(02) \cdot 10^{-5}$. The classical result $\Delta\varphi = 1.7534481$ arc sec, which is the theoretical maximum of all possible deflections, thus slightly overestimates the real deflection that should be observed. The results of recent observations from the Cassini space probe found in the literature are presented in terms of the so called PPN parameter γ, which is related to the light deflection angle and the parameter ε as follows:

$$\varepsilon_{PPN} = \frac{\Delta\varphi(PPN)}{\Delta\varphi(Sch)} = \frac{1+\gamma}{2} . \tag{27}$$

The parameter γ in the Cassini space probe observations was actually obtained from the measurements of the Shapiro delay, but this is not important for the metric evaluation here, since the Shapiro delay is actually calculated from the same photon trajectory as the light deflection and γ is the same parameter for both cases. For the Schwarzschild metric $\gamma = 1$, and for the flat spacetime $\gamma = -1$, as claimed in the PPN theory. The results obtained from the new metric and from observations are for the convenience summarized in Table 1:

Unfortunately no definite conclusion can be made from this data at this time to separate the incorrectly derived deflection angle in the GRT from the new metric derived deflection angle, but the measurement accuracy is not too far from the range that is needed for the confirmation of the difference.

This is encouraging. Anderson's mean value of γ deviation from unity is in a negative direction in agreement with the new metric theory, but the accuracy is insufficient. Anderson also suggests ruling out the positive values of γ deviations from unity as erroneous measurements. For this case the mean value agrees well with the new metric. Bertotti's evaluation on the other hand if correct would disprove the new metric, but the positive deviation of γ from unity is theoretically not likely. Finally, it is important to note that Anderson is using the natural coordinates in the measurements and evaluations, which is important for the consistency and for the proof that the natural coordinates are the ones that correspond to reality. The erroneously derived GRT light deflection angle, of course, cannot be used to prove anything.

Table 1. Theoretical and experimental results of light deflection by the Sun

Parameter	**Value**
ε_{Sch}	1
ε_{new}	$1-2.7\cdot 10^{-5}$
ε_{PPN} Anderson	$1+\frac{(-1.3\pm 5.2)}{2}\cdot 10^{-5}$
ε_{PPN} Anderson (no $\gamma > 1$)	$1+\frac{(-5.5\pm 5.2)}{2}\cdot 10^{-5}$
ε_{PPN} Bertotti	$1+\frac{(2.1\pm 2.3)}{2}\cdot 10^{-5}$

Therefore, to summarize the experimental results and the theoretical predictions that follow from the new metric it is clear that the GRT computation based on the variational principle in Eq.5 is not correct. The Schwarzschild metric and consequently Einstein field equations are not physical, do not correspond to reality, and should be abandoned.

The coincidental agreement of the ad hoc formula in Eq.5 and the Schwarzschild metric with the measurement is not the confirmation of the GRT.

The variational formula in Eq.5 can thus be considered only a lucky guess to force an agreement with observations.

6. Conclusions

In this paper it was clearly shown that the traditional way the light deflection caused by the gravitation of a non-rotating centrally gravitating body as calculated in the GRT is not correct. The GRT calculation is a scientific fraud and a cover-up to save the “beautiful theory” by the main stream science. The problem was traced to the variational principle used in the calculation of the photon trajectory that was found not compatible with the generally accepted Fermat principle used in optics. This also questions the validity of the Schwarzschild metric and Einstein field equations, since the Schwarzschild metric is the unique solution of these equations when the Ricci tensor is zero. The Fermat principle must be satisfied, because it follows directly from the wave equation that describes the photon propagation. The failure to satisfy the Fermat principle is the fatal problem for the GRT. When the new metric, derived previously by the author, and the properly derived relativistic Fermat principle are used, the correct formula for the light deflection is obtained yielding results agreeing with observations quite well.

Unfortunately the current level of observational precision is not yet good enough to unquestionably experimentally verify the accuracy of the new metric and the resulting difference from the incorrectly calculated light deflection angle as derived in the GRT. However, the measurement accuracy is good enough to conclude with certainty that the natural coordinates are the true observable quantities, mapping correctly the spacetime we are living in, and other coordinate systems obtained by various coordinate transformations from the natural coordinates are only useless mathematical manipulations without any physical meaning behind them and without any basis in reality. It will probably take much more time and much more hard work before the accuracy of experiments is improved to a level that is necessary for the unquestionable confirmation of correctness of the new metric. The author is hopeful, however, that with the help of the free internet access and many open minded readers the main stream science will eventually give up and finally abandon the GRT with its preposterous Black Holes, event horizons, and the nonphysical Schwarzschild metric.

References

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Appendix

The Fermat principle can also be defined as:

$$\delta\int_L ndl = 0, \tag{A1}$$

where dl is the physical length differential, n the index of refraction defined as: $n = c/v$, and v the speed of light in the medium that supports the light propagation. Let's consider for simplicity that the light propagates close to the radial direction for most of the travelled distance with the velocity $v = c_r$, which is the radial light speed obtained from Eq.2 when $ds = 0$:

$$c_r = cg_{tt}. \tag{A2}$$

For the physical length increment dl in the radial direction: $dl = \sqrt{g_{rr}}dr = \sqrt{g_{tt}^{-1}}dr$ follows also from Eq.2 when $ds = 0$ that:

$$\sqrt{g_{tt}}cdt = dl. \tag{A3}$$

By substituting these formulas from Eq.A2 and Eq.A3 into the Fermat principle in Eq.A1 the following result is obtained:

$$\delta \int_L \frac{cdt}{\sqrt{g_{tt}}} = 0 . \tag{A4}$$

It is important to use the physical radius in Eq.A1, instead of the standard coordinate radius since it is necessary to integrate over the same length increments. The standard coordinate radius differential varies its length depending on the location and becomes equal to the physical radius differential only at the location of a distant observer where $c_r = c$. The physical radius differential is therefore an invariant length reference, which provides the photon traveling time differential observed by a distant observer at the photon arrival place when divided by the speed of light along the photon travelling path.

Another way to derive this relation is to consider positioning stationary local observers along the photon travelling path and realizing that each will register his own photon travelling time:

$$dt_{phlo} = \frac{c}{c_{lo}} d\tau , \tag{A5}$$

where $cd\tau = dr_{ph}$ is the invariant length reference common to every observer. This invariant can be also related to the conservation of the photon linear momentum. The local observer radial speed of light is equal to: $c_{lo} = dr_{ph} / dt_{lo} = c\sqrt{g_{tt}}$ and this is derived again from Eq.2 with $ds = 0$ or directly from Eq.A2 as follows:

$$cg_{tt} = \frac{dr_{ph}}{d\tau} g_{tt} = \frac{dr_{ph}}{dt_{lo}} \frac{dt_{lo}}{d\tau} g_{tt} = c_{lo} \sqrt{g_{tt}} , \tag{A6}$$

where dt_{lo} is found from the relation: $d\tau = dt_{lo}\sqrt{g_{tt}}$. The local observer photon travelling time is then equal to:

$$dt_{phlo} = dt_{lo} = dt , \tag{A7}$$

where for $d\tau$ was substituted: $d\tau = dt\sqrt{g_{tt}}$. The distant observer will then observe the local observer's photon travelling times dilated as follows:

$$dt_{ph} = \frac{c}{c_{lo}} \frac{d\tau}{\sqrt{g_{tt}}} = \frac{c}{c_r} d\tau = \frac{dt}{\sqrt{g_{tt}}} . \tag{A8}$$

The principal error that occurs when using Eq.5 is considering that the photon travelling time at the location of a distant observer consists of just a simple addition of all the photon travelling times observed by the stationary local observers. This certainly cannot be correct, since each observer is located at a place with a different gravitational potential and each clock, which measures the particular photon travelling time, therefore, runs with a different rate. Using the integral of Eq.A8 instead of the integral in Eq.5 corrects this problem.

In the above derivations it was also considered that the light propagates in these light deflection tests close to the radial direction for most of the time. This assumption seems reasonable and it was also silently used in the GRT derivation. This is important when the light speed is not isotropic as is the case for the centrally gravitating body. The light speed anisotropy effect results in higher order corrections to formula in Eq.A4 that will cause some further reduction in the value of $\gamma < 1$. However, the detail study of this effect is outside of the scope of this paper and is therefore deferred to future work.