Why is the General Relativity Theory Incorrect?

Jaroslav Hynecek[1] ©

ABSTRACT

In this article it is shown that the General Relativity Theory is an incorrect theory of gravity. It leads to erroneous predictions when it is extrapolated beyond its experimentally verified validity such as the prediction of the existence of Black Holes with their event horizons. The article describes the two fatal problems found in the theory in particular in the Schwarzschild metric, which is the "vacuum" solution of Einstein field equations for a mass point. The problems are the insufficient contravariance and the violation of conservation of angular momentum. The article then derives the correct metric for the centrally gravitating body, which does not have the above mentioned problems, presents the Christoffel coefficients, and derives the Riemann, and the Ricci tensors. From this metric it is also shown that the Einstein's Weak Equivalence Principle does not correspond to reality and that the inertial mass and the gravitational mass both depend on the gravitational potential as well as on the velocity in such a way that their product remains constant.

Keywords: General Relativity Theory, Schwarzschild metric, metric for the new Metric Theory of Gravity, Mass equivalence, violation of contravariance in Schwarzschild metric, violation of conservation of angular momentum in Schwarzschild metric, Christoffel coefficients, Riemann tensor, Ricci tensors, Einstein tensor, Planck length, minimum radius of maximally compacted mass, maximum gravitational red shift.

INTRODUCTION

The General Relativity Theory (GRT) is based on the famous Einstein field equations that were originally derived by applying the conservation rule to the energy-momentum tensor.

$$G_{jk} = R_{jk} - \frac{1}{2} R_{(c)} g_{jk} = -\frac{8\pi\kappa}{c^4} T_{jk}, \quad G^{j}_{k|j} = 0, \tag{1}$$

The equations were originally derived using the analogy adapted from the Maxwell Theory of Electromagnetic fields (EMT) where the Electromagnetic momentum stress tensor is constructed with its divergence equal to zero. This property is associated with the conservations of energy and momentum, which are considered to be among the few most important and fundamental laws of physics. By assuming that the metric coefficients are dependent on the gravitational potential the search for the second rank tensor, depending on the second derivatives of metric coefficients with its divergence equal to zero, led to finding the Einstein tensor G_{jk}. It was then assumed that in an empty space, "the vacuum without mass", the right hand side of Eq.1 should be zero. This in turn led to finding the Schwarzschild metric with its differential metric line element shown below as a solution of Eq.1 for the centrally gravitating mass M :

[1] E-mail: jhynecek@netscape.net

$$ds^2 = g_{tt}(cdt)^2 - g_{tt}^{-1}dr^2 - r^2 d\Omega^2 . \tag{2}$$

In this metric line element the coefficient g_{tt} equals to:

$$g_{tt} = 1 - \frac{R_s}{r} , \tag{3}$$

the angular variables are: $d\Omega^2 = \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$, and the Schwarzschild radius R_s is defined as:

$$R_s = \frac{2\ \kappa\ M}{c^2} \ , \tag{4}$$

with κ and c having their usual meanings. However, this is not the procedure that one finds in the Maxwell EM field theory. In that theory the momentum stress tensor is calculated from the fields not the other way around. The fields are not calculated from the divergence theorem, they are found from the Maxwell's field equations. It is therefore more reasonable to first find the metric and then from the metric calculate the mass energy tensor for the gravitational field. When this procedure is adopted it is, of course, found that the divergence theorem is still satisfied, but the mass energy tensor is not necessarily zero in the "empty" space around the centrally gravitating mass. This is certainly a more reasonable approach than an arbitrary apriori assumption that in a space where there is gravitation field the mass energy tensor must be zero.

CONTRAVARIANCE PROBLEM

The first problem that can be identified in the Schwarzschild metric is the problem of insufficient contravariance. This can be shown by assuming that a small test body is falling in a radial direction in the field of a centrally gravitating mass and derive equation for its motion. The Lagrangian describing such a motion is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{tt}^{-1}\left(\frac{dr}{d\tau}\right)^2 . \tag{5}$$

From the well known variational principle:

$$\delta\int_\tau L d\tau = 0 , \tag{6}$$

follow the corresponding Euler Lagrange (EL) equations of motion:

$$\frac{d}{d\tau}\left(g_{tt}\frac{dt}{d\tau}\right) = 0, \qquad -\frac{d}{d\tau}\left(2g_{tt}^{-1}\frac{dr}{d\tau}\right) = \frac{\partial L}{\partial r} . \tag{7}$$

Here it was also considered that for the static and spherically symmetric case the metric coefficient is a function of only the coordinate radius r. The first integrals of these equations are easily found to be:

$$g_{tt}\frac{dt}{d\tau} = k , \tag{8}$$

$$\left(\frac{dr}{d\tau}\right)^2 = c^2k^2 - c^2 g_{tt}, \qquad (9)$$

where k is an arbitrary constant of integration ($L = c^2$). To find the forces that govern the motion of a test body, Eq.9 is differentiated with respect to τ with the following result:

$$\frac{d^2r}{d\tau^2} = -\frac{c^2}{2}\frac{\partial g_{tt}}{\partial \varphi_n}\frac{\partial \varphi_n}{\partial r}. \qquad (10)$$

In this formula it was assumed that the metric coefficient is a function of the Newton gravitational potential φ_n in order to satisfy the experimental fact that the Newton potential becomes the limiting function as $r \to \infty$. From this equation then follows that since the left hand side is a component of a contravariant geometrical object, the right hand side must also be a component of a contravariant geometrical object. It is therefore necessary, considering that in the orthogonal spherical coordinate system it holds: $g^{rr} = g_{tt}$ and that the gradient of a potential is a covariant vector [1], that the following condition must be satisfied:

$$\frac{c^2}{2}\frac{\partial g_{tt}}{\partial \varphi_n} = g^{rr} = g_{tt}. \qquad (11)$$

The solution of this equation, assuming a flat space at infinity, is simple to find and is equal to:

$$g_{tt} = e^{\frac{2\varphi_n}{c^2}}. \qquad (12)$$

From this result it is then clear that the Schwarzschild metric satisfies this requirement only approximately to the first order. Therefore, the Schwarzschild solution of the Einstein field equations is a nonphysical solution. This also includes the Schwarzschild solution as originally published [2]. Even if the Schwarzschild metric leads to the correct predictions for the Mercury perihelion advance, the light bending by the Sun, etc., it is not correct to extrapolate it and use it for the astronomical objects where there is a strong gravitational field. The examples are the neutron stars and similar compact objects, the whole universe etc.. The Black Holes are, therefore, a mathematical artifact of an incorrect extrapolation of the Schwarzschild metric and cannot exist in reality.

CONSERVATION OF THE ANGULAR MOMENTUM PROBLEM

The second problem of the Schwarzschild metric is the problem of conservation of angular momentum. This is a well recognized law of physics that has to be satisfied as is well known, for example, from the quantum physics where the spin is one of the well conserved quantum numbers. In order to illustrate this problem the metric given in Eq.2 is generalized as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{tt}^{-1}dr^2 - g_{\varphi\varphi}d\Omega^2, \qquad (13)$$

where the metric coefficient $g_{\varphi\varphi}$ standing by the angular coordinates was introduced. Again, the motion of a small test body described by the Lagrangian corresponding to this metric will be investigated. For simplicity only the motion in an equatorial plane, $\vartheta = \pi/2$, will be considered. The first integrals of EL equations are thus as follows:

$$g_{tt}\frac{dt}{d\tau} = 1, \tag{14}$$

$$g_{\varphi\varphi}\frac{d\varphi}{d\tau} = \alpha, \tag{15}$$

where α is the integration constant corresponding to the angular momentum. By eliminating the proper time differential $d\tau$ from these equations the result becomes:

$$\frac{g_{\varphi\varphi}}{g_{tt}}\frac{d\varphi}{dt} = \alpha. \tag{16}$$

This is the correct equation for the conservation of the angular momentum. Many GRT textbooks, however, claim that Eq.15 is the correct equation for the conservation of the angular momentum [3]. This is a very strange claim, which is absolutely wrong. The linear velocity is defined and measured as the coordinate distance increment divided by the coordinate time increment, but for some unexplained reason the angular velocity should be the angular increment divided by the invariant $d\tau$? This does not make sense. The angular velocity has always been determined by measuring the coordinate angle increment divided by the coordinate time increment. From Eq.16 it is then clear that the ratio of the angular metric coefficient and the coordinate time metric coefficient for the spherically symmetric coordinate system has to be some generalized, most likely the proper distance squared, function of the coordinate radius. It must therefore hold that:

$$g_{\varphi\varphi} = \rho(r)^2 g_{tt}, \tag{17}$$

where $\rho(r)$ will be called the physical distance defined as:

$$d\rho = \sqrt{g_{rr}}dr = \frac{1}{\sqrt{g_{tt}}}dr. \tag{18}$$

Clearly, this is not satisfied by the Schwarzschild metric. There is no suitable function, which would satisfy Eq.18 and at the same time the condition: $g_{\varphi\varphi} = r^2$ found in the Schwarzschild metric. The correct metric for the centrally gravitating body in the metric theory of gravity (MTG) is thus as follows [4]:

$$ds^2 = e^{\frac{2\varphi_n}{c^2}}(cdt)^2 - e^{\frac{-2\varphi_n}{c^2}}dr^2 - \rho^2 e^{\frac{2\varphi_n}{c^2}}d\Omega^2, \tag{19}$$

where again the angular variables are: $d\Omega^2 = \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$ and where for the physical distance and for the potential hold the following formulas:

$$d\rho = e^{\frac{-\varphi_n}{c^2}}dr, \tag{20}$$

$$\varphi_n = -\frac{\kappa M}{\rho(r)}. \tag{21}$$

In this metric both the contravariance and the conservation of angular momentum are satisfied. It can also be shown that the well known four tests of gravitational theory: the Mercury perihelion advance, the bending of the light by the Sun, the Shapiro delay, and the gravitational red shift are all leading to the identical results obtained from the Schwarzschild metric for the case of the weak gravitational field. These tests have been already verified by observations and experiments to a reasonable degree of accuracy, so any new metric must also provide equations that lead to the same results [4].

GRAVITATIONAL RED SHIFT AND THE MASS EQUIVALNCE PRINCIPLE

The metric derived in Eq.19 can now be used to answer questions about the Einstein Weak Equivalence Principle (WEP), which in one formulation states that the inertial and the gravitational masses are identical independent of any inertial motion. In order to derive the relation between these two masses, it is necessary to return back to Eq.10. It is also necessary to find the general expression for the coordinate time.

When an observer is stationary in a gravitational field his coordinate time is given by the following relation:

$$d\tau = dt\sqrt{g_{tt}} \ . \tag{22}$$

From this equation then directly follows the gravitational red shift formula, since it is known that the frequency of emitted light from the source located in the gravitational field is determined by the time rate at that place. For the moving test body, however, it is also necessary that the Lorentz coordinate transformation holds. Therefore, there are clearly two factors that must enter into the formula for the coordinate time, the metric coefficient and the Lorentz velocity factor. To proceed further in the derivation it is necessary to find the dependence of the light speed in the radial direction on the gravitational field. This is found from the metric in Eq.13 by setting the ds to zero. The result is:

$$c_r = c \ g_{tt} \, . \tag{23}$$

By eliminating the proper time differential $d\tau$ from Eq.9 and using Eq.23 it is possible to write for the metric coefficient g_{tt} the following:

$$g_{tt} = 1 - v^2 / c_r^2 \, . \tag{24}$$

Finally, by considering that from Eq.8 it holds that: $d\tau = g_{tt}dt$, and from Eq.22 for $v = 0$ that it holds: $d\tau = dt\sqrt{g_{tt}}$, it is clear that for the coordinate time in general it must hold:

$$d\tau = dt\sqrt{g_{tt}}\sqrt{1 - v^2 / c_r^2} \ . \tag{25}$$

This formula reverts to the standard red shift formula for time when $v = 0$ and to the standard Lorentz time dilation formula for the flat spacetime when $g_{tt} = 1$. It is now simple to substitute this result into Eq.10, multiply it by the rest mass m_0 of the test body, and write:

$$\frac{d}{dt}\left(\frac{m_0 v}{\sqrt{g_{tt}}\sqrt{1-v^2/c_r^2}}\right) = -g^{rr}\frac{\partial\varphi_n}{\partial r}m_0\sqrt{g_{tt}}\sqrt{1-v^2/c_r^2}\ , \qquad (26)$$

where $v = dr/dt$. The left hand side of this equation is the familiar Newton inertial force law and the right hand side is the Newton gravitational force law. It is therefore clear that for the inertial and the gravitational masses it must hold the following:

$$m_i = \frac{m_0}{\sqrt{g_{tt}}\sqrt{1-v^2/c_r^2}}\ , \qquad (27)$$

$$m_g = m_0\sqrt{g_{tt}}\sqrt{1-v^2/c_r^2}\ . \qquad (28)$$

The Einstein's WEP ($m_i = m_g$) is therefore not consistent with this result. In view of the above findings that the Schwarzschild metric does not correspond to reality, the WEP does not correspond to reality also. The derived dependencies of the inertial and the gravitational masses on the gravitational potential have significant consequences in cosmology, where they can be used to calculate the duration of long GRB explosions [5]. This mass dependence on the gravitational potential is also in line with the philosophy of Mach where he believed that the inertial and the gravitational masses of bodies on Earth must be influenced by the masses present in the surrounding distant universe.

CHRISTOFFEL SYMBOLS, RIEMANN, RICCI, AND EINSTEIN TENSORS

Since the metric is now known, it is straight forward to calculate the Christoffel coefficients, Riemann and Ricci tensors, and the Einstein tensor. For the sake of simplicity the coordinates defined as: $(ct, r, \vartheta, \varphi)$ will be changed to a numeric convention: (0,1,2,3). By introducing an abbreviation: $\varphi_c = \varphi_n / c^2$ the Christoffel symbols are calculated to be:

$$\Gamma^0_{0\,1} = \frac{\phi_c}{\rho}e^{\phi_c}$$

$$\Gamma^1_{0\,0} = \frac{\phi_c}{\rho}e^{-3\phi_c} \qquad \Gamma^1_{1\,1} = -\frac{\phi_c}{\rho}e^{\phi_c} \qquad \Gamma^1_{2\,2} = -\rho(1+\phi_c)e^{-3\phi_c} \qquad \Gamma^1_{3\,3} = \Gamma^1_{2\,2}\sin^2\vartheta$$

$$\Gamma^2_{1\,2} = \frac{(1+\phi_c)}{\rho}e^{\phi_c} \quad \Gamma^2_{3\,3} = -\sin\vartheta\cos\vartheta$$

$$\Gamma^3_{1\,3} = \frac{(1+\phi_c)}{\rho}e^{\phi_c} \quad \Gamma^3_{2\,3} = \frac{\cos\vartheta}{\sin\vartheta}. \qquad (29)$$

The Riemann tensor is calculated according to the relation:

$$R^\lambda_{\mu\,\nu\,\sigma} = \partial_\nu\Gamma^\lambda_{\mu\,\sigma} - \partial_\sigma\Gamma^\lambda_{\mu\,\nu} + \Gamma^\eta_{\mu\,\sigma}\Gamma^\lambda_{\eta\,\nu} - \Gamma^\eta_{\mu\,\nu}\Gamma^\lambda_{\eta\,\sigma}\,, \qquad (30)$$

with the following results:

$$R^0_{1\,0\,1} = \frac{\phi_c(2-\phi_c)}{\rho^2} e^{2\phi_c} \qquad R^2_{0\,2\,0} = \frac{\phi_c(1+\phi_c)}{\rho^2} e^{-2\phi_c}$$

$$R^0_{2\,0\,2} = -\phi_c(1+\phi_c)e^{-2\phi_c} \qquad R^2_{1\,2\,1} = -\frac{\phi_c^2}{\rho^2} e^{2\phi_c}$$

$$R^0_{3\,0\,3} = R^0_{2\,0\,2}\sin^2\vartheta \qquad R^2_{3\,2\,3} = R^3_{2\,3\,2}\sin^2\vartheta$$

$$R^1_{0\,1\,0} = -\frac{\phi_c(2-\phi_c)}{\rho^2} e^{-2\phi_c} \qquad R^3_{0\,3\,0} = \frac{\phi_c(1+\phi_c)}{\rho^2} e^{-2\phi_c}$$

$$R^1_{2\,1\,2} = -\phi_c^2 e^{-2\phi_c} \qquad R^3_{1\,3\,1} = -\frac{\phi_c^2}{\rho^2} e^{2\phi_c}$$

$$R^1_{3\,1\,3} = R^1_{2\,1\,2}\sin^2\vartheta \qquad R^3_{2\,3\,2} = \; 1-(1+\phi_c)^2 e^{-2\phi_c}\,, \tag{31}$$

where only the nonzero symmetric terms were listed. After contracting the Riemann tensor the Ricci tensor becomes:

$$R_{0\,0} = \frac{3\phi_c^2}{\rho^2} e^{-2\phi_c} \qquad R^0_0 = \frac{3\phi_c^2}{\rho^2}$$

$$R_{1\,1} = \frac{\phi_c(2-3\phi_c)}{\rho^2} e^{2\phi_c} \qquad R^1_1 = -\frac{\phi_c(2-3\phi_c)}{\rho^2}$$

$$R_{2\,2} = 1-\left(1+3\phi_c+3\phi_c^2\right)e^{-2\phi_c} \qquad R^2_2 = -\frac{e^{2\phi_c}}{\rho^2} + \frac{1+3\phi_c+3\phi_c^2}{\rho^2}$$

$$R_{3\,3} = R_{2\,2}\sin^2\vartheta \qquad R^3_3 = R^2_2\,. \tag{32}$$

The Ricci scalar then becomes as follows:

$$R_c(r) = \frac{2}{\rho(r)^2}\left(1-e^{\frac{R_s}{\rho(r)}}\right) + \frac{2R_s}{\rho(r)^3} + \frac{3R_s^2}{\rho(r)^4}\,. \tag{33}$$

For small R_s/ρ this expression reduces to:

$$R_c(r) \approx \frac{2R_s^2}{\rho(r)^4}\cdots. \tag{34}$$

Since the terms in powers of ρ^{-2} and ρ^{-3} cancel after expanding Eq.33 into a power series, the Ricci curvature becomes for most of the space outside the Schwarzschild radius very small as can be clearly seen in Fig.1. It is thus easy to understand now why the Einstein field equations provide the Schwarzschild solution that describes the reality reasonably well, when the Ricci scalar is set to zero, $R_c = 0$. However, it is now also easy to understand that the Schwarzschild solution must fail with large errors and nonphysical Black Hole artifacts such as the singularity and the event horizon, where the Ricci scalar is not equal to zero, $R_c \neq 0$. From this result it is not evident that the Ricci tensor or the

Einstein tensor should be zero everywhere in the empty space around the gravitating body or the Einstein tensor simply proportional to the traditional stress-energy tensor. The Einstein tensor that should be used on the right hand side of Eq.1 instead of zero to obtain the correct solution for the metric of the centrally gravitating body is as follows:

$$G_0^0 = \frac{e^{2\phi_c}}{\rho^2} - \frac{1+2\phi_c+3\phi_c^2}{\rho^2} \qquad G_1^1 = G_0^0 - \frac{2\phi_c}{\rho^2} \qquad G_2^2 = \frac{\phi_c - 3\phi_c^2}{\rho^2} \qquad G_3^3 = G_2^2 . \tag{35}$$

The Einstein field equations thus become identities and cannot be used to search for the metric similarly as the momentum stress tensor in EMT is not used to find the electric and magnetic fields [6]. Furthermore, since there is no event horizon, it is now possible to calculate the energy stored in the gravitational field outside of the radius *r*. This is:

$$W(r) = -Mc^2 = -\frac{1}{2}\frac{\kappa \cdot M^2}{\rho(r)} . \tag{36}$$

At some point this energy becomes equal to the mass equivalent energy of the gravitating body. This occurs at the mass equivalent physical radius $\rho_e = \frac{1}{4} R_s$, or the coordinate mass equivalent radius $r_e = 0.009384 \cdot R_s$. This also places an upper limit on the observation of the gravitational red shift $Z_{\max} = 6.389$. In Fig.1, the normalized energy of the field $W_n(r) = W(r)/(Mc^2)$ is represented by a dashed curve and the mass equivalent coordinate radius is shown as a vertical dot-dashed line. It is an interesting fact that the largest *Z* shift observed to date is: *Z* = 6.29 from the Gamma Ray Buster GRB 050904. This is very close to what is predicted above for the maximum intrinsic Z shift without including any applicable cosmological red shift and thus clearly confirms the correctness of the theory.

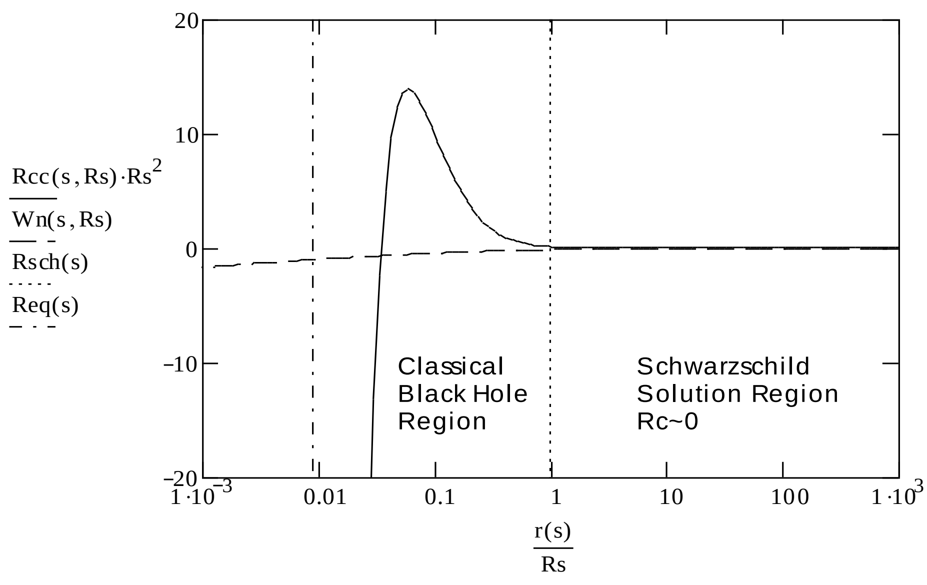

Fig.1. Graph of normalized Ricci curvature as a function of the normalized coordinate distance for a body with the mass equal to the mass of the Sun. The plot is based on the metric derived in this article. The dashed curve Wn is the normalized energy stored in the field outside of the radius r(s). The vertical dotted line indicates the location of the Schwarzschild radius; the vertical dot-dash line indicates the location of the mass equivalent radius.

The derived MTG metric can now also be applied to very small objects. If it is considered that the mass-energy that is compressed into the minimum volume is quantized and represented by oscillations in the azimuthal direction with the maximum wavelength equal to one half of the circumference the minimum mass is equal to:

$$M = \frac{h}{\pi \, \rho_e c}, \tag{37}$$

After substituting this value into Eq.36 the minimum possible physical radius is found to be equal to:

$$\rho_e = \sqrt{\frac{h\kappa}{2\pi \, c^3}} = 1.61625 \cdot 10^{-35} m. \tag{38}$$

This value is identical to the Planck length found only on the basis of dimensional analysis.

CONCLUSIONS

In this article it was shown, using the well established fundamental principles of physics, that the GRT is a wrong theory of gravity. The main reason for this problem is that the right hand side of the Einstein field equations is determined from some other apriori assumption rather than calculated directly from the gravitational field. This procedure when applied to the centrally gravitating body leads to the nonphysical solution, the Schwarzschild metric. The new metric presented in this article, which is replacing the Schwarzschild metric, leads to the equation of motion of a test body that satisfies the contravariance requirement as well as the conservation of angular momentum. The new metric is also used for the derivation of the formulas for the dependency of the inertial mass and the gravitational mass on the gravitational potential and velocity. From this result it is shown that the Einstein's WEP does not hold in reality. Finally the new metric theoretically disproves the existence of Black Holes with their event horizons and allows giving the direct physical meaning to the Planck length.

REFERENCES

1. J. L. Synge, A. Schild, "Tensor Calculus", Univ. of Toronto Press, Toronto, 1966.
2. K. Schwarzschild, "On the Gravitational Field of a Mass Point According to Einstein's Theory", translation and foreword of original paper: Sitzungsberichte der Koeniglich Preussischen Akademie der Wissenschaften zu Berlin, Phys-Math Klasse 1916, pp. 189-196, S. Antoci and A. Loinger, arXiv:physics/9905030 v1, 12 May 1999.
3. S. Carroll, "Spacetime and Geometry an Introduction to General Relativity", Addison Wesley, New York, 2004. p. 208.
4. J. Hynecek, "New Space-Time Metric, Four Tests of Gravitation Theory, and Newton's Law of Gravitation", Physics Essays, vol.20, No.2. p. 313.
5. **http://www.wbabin.net/physics/hynecek12.pdf**
6. J. Hynecek, "Geometry based critique of general relativity theory" Physics Essays, vol.24, No.2. p. 2011.