The Age of the Universe and other Cosmological Issues

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Abstract

Based on the Scale Law this paper explores several cosmological issues such as the age of the Universe, the Universe mass density, the radius of the present particle horizon and the Friedmann's equation. The Scale Law is the first Model Meta law we, humans, have discovered. The significance of this investigation is the formula for the age of the Universe: $T = \dfrac{h^2}{2\pi^2 c\, G\, m_e\, m_p^2}$. *This formula is in excellent agreement with the experimental data from the Planck spacecraft- 2013 and suggests that one of more of the constants it contains is a function of time.*

Keywords: *Planck scale, Planck's constant, Planck mass, Planck length, Planck spacecraft, rate of expansion, Friedmann equation.*

1. Introduction

In 2012 I formulated the Scale Principle or Scale Law. I published the first version of this paper in May this year (2014). In the first version this law was called the *Quantum Scale Principle*. However after finding that Einstein's relativistic energy also obeys this law, I changed its name to the *Scale Principle* or *Scale Law*. Since the first version the principle has evolved to the present form given by the following relationship:

(1)

Scale Principle or *Scale Law*
$M_1 \, \Re \, s \, M_2$

M_1 = dimensionless Meta-quantity 1
M_2 = dimensionless Meta-quantity 2
s = dimensionless Meta-scale-factor
$\Re$ = Meta-relationship-type

I invite the reader to read the following two papers: *Scale Factors and the Scale Principle (The Sacle Law)* [1] and *Where Do the Laws of Physics Come From?* [2] for a full description of the Scale Law.

2. The Age of the Universe

To find the expression for the age of the Universe we calculate the constant H defined as

$$H = T\, m_e c^2 \qquad (2)$$

Where

$H =$ Cosmic energy time-constant.
$T =$ age of the Universe measured by NASA (also known as universal time)
$m_e =$ electron rest mass
$c =$ speed of light in vacuum

Since we use the Cosmic energy-time constant, H, as a bridge to finding a relationship, we don't need to worry about the physical meaning of this constant.

Let's calculate H using the age of the Universe based on the combined Planck spacecraft's data with the previous missions' data [4, 5]

$$T_{COMB}(2013) = (13.798 \pm 0.037) \times 10^9 \; years$$

Taking into account the errors we calculate the minimum and the maximum combined values as follows

$$T_{COMB-MIN}(2013) = (13.798 - 0.037) \times 10^9 \; years = 13.761 \times 10^9 \; years = 13{,}761 \; million \; years$$

$$T_{COMB-MAX}(2013) = (13.798 + 0.037) \times 10^9 \; years = 13.835 \times 10^9 \; years = 13{,}835 \; million \; years$$

Now we calculate the Cosmic energy-time constant

$$H = (13798 \times 10^6 \times 365.25 \times 24 \times 60 \times 60)(9.10938291 \times 10^{-31} \times (299792458)^2) \; JS$$

$$H \cong 35649.256\,07 \; JS$$

Now we calculate the following products

$$m_p^2 H = (1.672\,621\,777 \times 10^{-27})^2 \times 35649.25607 \; Kg^2 JS = 9.973462639 \times 10^{-50}$$

$$M_P^2 h = (2.176\,509\,252 \times 10^{-8})^2 \times 6.626\,069\,57 \times 10^{-34} \, Kg^2 JS = 3.138\,896\,723 \times 10^{-49}$$

Then we draw a table as shown below

Constants *(Cosmic scale)*	**Generation 1** *(Proton scale)*	**"Generation 4"** *(Planck scale)*	**Constants** *(Planck scale)*
H	m_p^2	M_P^2	h

***SCALE TABLE 1**: This table is used to find the age of the Universe*

Based on this table we establish the following fundamental relationship

$$H\, m_p^2 = S\, M_P^2\, h \tag{3}$$

Let's express this relationship in the form of the Scale Law with *n=2 and m=1* (this is not the canonical form)

$$\frac{m_p^2}{M_P^2} = S\,\frac{h}{H} \tag{4}$$

We shall determine the scale factor *S* as follows

$$\frac{1}{S} = \frac{M_P^2\, h}{m_p^2\, H} = \frac{3.138\,896\,723 \times 10^{-49}}{9.973462639 \times 10^{-50}} = 3.147\,248\,69 \tag{5}$$

Because the above ratio (equation 5) is based on experimental data, it seems logical to assume that the value of this ratio is the number π. Therefore we write

$$S = \frac{1}{\pi} \cong \frac{1}{3.141\,592\,654} \tag{6}$$

Then equation (4) transforms into

$$m_p^2\, H = \frac{h}{\pi}\, M_P^2 \tag{7}$$

We shall substitute *H* with the value given in equation (2), this gives

$$m_p^2\,(T\, m_e c^2) = \frac{h}{\pi}\, M_P^2 \tag{8}$$

Now we solve this equation for *T*

$$T = \frac{\pi\, h}{c^2}\,\frac{M_P^2}{m_p^2\, m_e} \tag{9}$$

Taking into consideration that the Planck mass is defined as

$$M_P = \sqrt{\frac{hc}{2\pi\, G}} \tag{10}$$

And that

$$M_P^2 = \frac{hc}{2\pi\, G} \tag{11}$$

We substitute M_P^2 in equation (9) with the second side of equation (11) to obtain the formula for the age of the Universe

$$T = \frac{h^2}{2\pi^2 c\, G\, m_e\, m_p^2} \qquad \text{(Formula for the age of the Universe)} \tag{12}$$

Now T is the theoretical age of the Universe (not the measured one).

It is worthy to emphasize that even if any of the parameters h, c, G, m_e , and m_p where found to be a function of time such as $G = f(T)$, we wouldn't be able to use it in equation (12) because the first side of this equation is precisely the universal time (T). In other words you cannot express the universal time as a function of itself.

To illustrate this point let us assume that G is given by

$$G = \frac{K}{T} \qquad \text{(Not a law of physics)}$$

then substituting G in equation (12) with K/T we get

$$T = \frac{h^2}{2\pi^2 c \left(\frac{K}{T}\right) m_e m_p^2} \qquad \text{(Not a law of physics)}$$

$$T = \frac{h^2}{2\pi^2 c\, K\, m_e m_p^2}\, T \qquad \text{(Not a law of physics)}$$

$$1 = \frac{h^2}{2\pi^2 c\, K\, m_e m_p^2} \qquad \text{(Not a law of physics)}$$

Thus we have "lost" the variable (in this case T) we are interested in. We have to devise another method to deal with this unique situation. But this will be the subject for another article.

I discovered formula (12) in 2012 through a different method not shown in this paper. The value this formula yields is

$$T \cong 4.362\,157\,043 \times 10^{17}\,S \cong 4.362 \times 10^{17}\,S$$

Converting to Julian years

$$T \cong \frac{4.362\,157\,043 \times 10^{17}\,S}{365.25 \times 24 \times 60 \times 60\,S\,/\,year} = 1.382\,284\,154 \times 10^{10}\,years$$

The difference with the universal age I calculated in version 1-6 of my previous paper [1] was because of the length of the year. In that paper I used 365 days/year while here I used 365.25 days/year (Julian year) to increase the accuracy.

$$T \cong 13{,}822.84\ million\ years$$

This value fits perfectly within the Planck 2013 data when we consider the errors (See Table 1)

Description	Age of the Universe
$T_{COMB-MAX}$ (2013)	13,835 *million years*
$T_{THEORETICAL}$ (2014) (*equation* 12)	13,822.84 *million years*
$T_{COMB-MIN}$ (2013)	13,761 *million years*

TABLE 1*: This table compares the theoretical value with the combined best estimate range.*

Thus the error of the *best combined estimate* (Planck Data and previous missions) is

Error (Best combined estimate) =
$(13{,}798 - 13{,}822.84)\ million\ years = -24.84\ million\ years$

The minus sign means that the *best combined estimate* underestimates the age of the Universe by 24.84 million years.

Based on the Planck spacecraft the European Space Agency (ESA) estimated the age of the Universe to be 13,820 million years. Assuming that formula (12) is the correct formula for the age of the Universe, the error of ESA's estimation is

$$\text{Error (ESA)} = (13{,}820 - 13{,}822.84)\ \textit{million years} = -2.84\ \textit{million years}$$

The minus sign means that ESA could have underestimated the age of the Universe by 2.84 million years which is a very small relative error indeed. Table 2 summarizes the errors.

Method	Possible error of the age of the Universe in comparison with formula 12
Combining Planck spacecraft with previous missions	-24.84 *million years*
Planck spacecraft	-2.84 *million years*

TABLE 2*: This table shows the possible errors.*

3. The Mass Density of the Universe

Let us calculate the mass density of the Universe form the Scale Law. We start by drawing the corresponding scale table as shown below

Density *(Cosmic scale)*	**Time squared** *(Cosmic scale)*	**Time squared** *(Planck scale)*	**Density** *(Planck scale)*
ρ	T^2	T_P^2	ρ_P

SCALE TABLE 2*: This table is used to find the Universe mass density*

Where

ρ = mass density of the Universe

T = age of the Universe

T_P = Planck time

ρ_P = Planck density

$$T_P \equiv \sqrt{\frac{hG}{2\pi\ c^5}} \tag{13}$$

$$\rho_{\ P} \equiv \frac{2\pi\ c^5}{hG^2} \tag{14}$$

From the table we write the following equation

$$\rho\ T^2 = S\,T_P^2 \rho_{\ P} \tag{15}$$

$$\rho = S\frac{T_P^2}{T^2}\rho_{\ P} \tag{16}$$

$$\rho = S\left(\frac{h\,G}{2\pi\ c^5}\right)\left(\frac{2\pi^{\ 2} c\ G\,m_e\,m_p^2}{h^2}\right)^2\left(\frac{2\pi\ c^5}{h\,G^2}\right) \tag{17}$$

$$\rho = S\left(\frac{4\pi^{\ 4} c^2\ G\,m_e^2 m_p^4}{h^4}\right) \tag{18}$$

$$\rho = S\left(7.874\,475\,141 \times 10^{-26}\,\frac{Kg}{m^3}\right)$$

The WAP value is

$$\rho_{\ WAP} = 9.9 \times 10^{-27}\,\frac{Kg}{m^3}$$

Comparing with the experimental value

$$R_\rho = \frac{9.9 \times 10^{-27}}{7.874\,475\,141 \times 10^{-26}} = 0.125\,7226\,65$$

Then I adopt a scale factor of

$$S = \frac{3}{8\pi} \cong 0.119\,366\,207$$

Should we have used 1/8 instead of 3/8π we would have got an incorrect radius, R_0 (see next section equation 33). Then final formula for the mass density of the Universe is

$$\rho = \frac{3\pi^3}{2} \frac{G c^2 m_e^2 m_p^4}{h^4}$$ (Formula for the mass density of the Universe) (19)

$$\rho = 9.399\,462\,317 \times 10^{-27} \frac{Kg}{m^3}$$

Considering that the Compton wavelength for the electron is

$$\lambda_{Ce} = \frac{h}{m_e c}$$ (Compton wavelength for the electron) (20)

We can express the formula for the mass density of the Universe in terms of the above wavelength. This yields

$$\rho = \frac{\pi^4}{2} \frac{G m_p^4}{\lambda_{Ce}^2}$$ (Formula in terms λ_{Ce}) (21)

4. The Radius of the Present Particle Horizon

We define the present particle horizon, R_0, as the distance light could have travelled from the beginning of the Universe (*t = 0*) to the present time (*t = T*) (where *T* is the age of the Universe) if there were no expansion of space. (It is possible that astronomy textbooks use a different name for this definition. Should the reader find that this is the case please let me know and I shall change it for a more appropriate name).

To continue we draw a scale table as follows

Length *(Cosmic scale)*	**Mass** *(Planck scale)*	**Mass** *(Cosmic scale)*	**Length** *(Planck scale)*
R_0	M_P	M_0	L_P

SCALE TABLE 3*: This table is used to find the present particle horizon.*

From the table we write the following equation

$$R_0 M_P = S M_0 L_P$$ (22)

$$R_0 = S M_0 \frac{L_P}{M_P}$$ (23)

We calculate the ratio $\frac{L_P}{M_P}$ separately

$$\frac{L_P}{M_P} = \sqrt{\frac{hG}{2\pi c^3}\frac{2\pi G}{hc}} \tag{24}$$

$$\frac{L_P}{M_P} = \frac{G}{c^2} \tag{25}$$

$$R_0 = S\frac{GM_0}{c^2} \tag{26}$$

Now taking $S = 2$ yields

$$R_0 = \frac{2GM_0}{c^2} \tag{27}$$

From the definition of the Universe mass density

$$\rho = \frac{M_o}{\frac{4}{3}\pi R_0^3} \tag{28}$$

Where

M_0 = mass in the spherical volume of radius R_0

Solving for M_0

$$M_0 = \frac{4}{3}\pi R_0^3 \rho \tag{29}$$

Now substituting M_0 in equation (27) with equation (29) gives

$$R_0 = \sqrt{\frac{3c^2}{8\pi\ G\rho}} \qquad \text{(Formula for the present particle horizon)} \tag{30}$$

Strictly speaking

$$R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\Omega_0 - 1}} \tag{31}$$

however for a flat Universe $k = 0$ equation (31) transforms into

$$R_0 = \frac{c}{H_0} = cT \tag{32}$$

The value of R_0 as per equation (30) is

$$R_0 = \left(\frac{2.99792458 \times 10^8}{2} \sqrt{\frac{3}{2\pi \ (6.67384 \times 10^{-11})(9.399\,462\,317 \times 10^{-27})}} \right) m$$

$$R_0 = 1.307\,741\,782 \times 10^{26}\,m$$

$$R_0(ly) = \frac{1.307\,741\,782 \times 10^{26}\,m}{9.460\,730\,473 \times 10^{15}\,m/ly} \cong 1.382\,284\,154 \times 10^{10}\,ly = 13{,}822.84154 \times 10^6\,ly$$

We can express R_0 as a function of the age of the Universe, *T*. Thus eliminating the Universe mass density from equation (30) by means of equation (19) gives

$$R_0 = \sqrt{\frac{3c^2}{8\pi \ G}\left(\frac{2h^4}{3\pi^{\,3}c^2 G\, m_e^2\, m_p^4}\right)} = c\,T \tag{33}$$

This indicates that the age of the Universe's scale factor and the mass density's scale factor are both correct. The reader who is interested in reading more about cosmology can consult an excellent article from T. Davis and C. Lineweaver on the misconceptions about the Universe [6].

5. Friedmann Equation

Friedmann assumed that, on average, the Universe mass density, ρ , is the same everywhere. To prove that Friedmann's equation [7] obeys the Scale Law we define the following variables

$$R_0 \equiv \sqrt{\frac{3c^2}{8\pi \ G\rho}} \tag{34}$$

$$K = K(R, k, \Lambda\,,) \equiv \sqrt{k + \frac{\Lambda}{3}\frac{R^2}{c^2}} \qquad \text{(dimensionless variable)} \tag{35}$$

Then we draw the following *cosmic scale table*

Expansion rate "Speed" *(Cosmic scale)*	**Expansion rate "Speed"** *(Cosmic scale)*	**Speed** *(Cosmic scale)*	**Speed** *(Cosmic scale)*
$\dot{R}$	$\dot{R}$	$\left(\frac{R}{R_0}+K\right)c$	$\left(\frac{R}{R_0}-K\right)c$

SCALE TABLE 4*: Cosmic scale table. This table is used to find the Friedmann's equation*

From the above table and according to the Scale Law we build the following equation

$$\dot{R}\,\dot{R} = S\left(\frac{R}{R_0}+K\right)c\left(\frac{R}{R_0}-K\right)c \tag{36}$$

where

G= Newton's gravitational constant
Λ = Cosmological constant
k = Curvature constant
c = Speed of light
ρ = mass density of the Universe
$\dot{R}$ = Rate of expansion (Because the units of this variable are *m/S* we have labeled the first and the second columns of Table 4 as Speed).

$v(R, R_0, K) \equiv \left[(R/R_0)+K\right]c$ = This variable represents the velocity of light multiplied by a variable factor.

Working mathematically and taking $S = 1$ we get

$$\frac{1}{c^2}\left(\dot{R}\right)^2 = \left(\frac{R}{R_0}\right)^2 - K^2 \tag{37}$$

Multiplying by c^2/R^2 both sides

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{c}{R_0}\right)^2 - \frac{c^2K^2}{R^2} \tag{38}$$

Substituting K with equation (35) yields

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{c}{R_0}\right)^2 - \frac{k\,c^2}{R^2} + \frac{\Lambda}{3} \tag{39}$$

Substituting R_0 with the second side of equation (34) yields the Friedmann's equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi\ G\rho\ - \frac{k\,c^2}{R^2} + \frac{\Lambda}{3} \tag{40}$$

Now we take equation (36) into the form of the Scale Law

$$\frac{\dot{R}}{\left(\frac{R}{R_0} + K\right)c} = S\,\frac{\left(\frac{R}{R_0} - K\right)c}{\dot{R}} \tag{41}$$

where
$n = 1$

$Q_1 = Q_2 = \dot{R}$

$m = 1$

$$Q_3 = \left(\frac{R}{R_0} + K\right)c$$

$$Q_4 = \left(\frac{R}{R_0} - K\right)c$$

S = the *scale factor* in this case is 1.

Thus the Friedmann equation can be expressed as

$$\frac{\dot{R}}{\left(\frac{R}{R_0} + K\right)c} = \frac{\left(\frac{R}{R_0} - K\right)c}{\dot{R}} \tag{42}$$

Equation (42) tells us that the Friedmann equation obeys the Scale Law. In fact we can state that

All laws of physics obey the Scale Law [2].

6. The Cosmic Scale Table: The Big Picture

This section presents the sequence of steps to find both the data used by the Friedmann's equation (except the cosmological constant) and the Friedmann's equation itself. For the first table an operation, $(\ )\times m_e c^2$, is applied to the age of the Universe T.
For the next table, an operation is carried out on the cosmological equation calculated with the previous table. The result of this operation is a new equation which is part of the data for the next table. The process stops after the calculation for the fourth table is complete. ***Table 3*** shows the input/s and the result (equation) for each *scale table* while the ***Cosmic Scale Table*** shows this process.

	Input/s (Data is shown within parenthesis)	Output/Result (Data for the next Table)	Equation number
Scale Table 1	$(T)\times m_e c^2$	$T = \dfrac{h^2}{2\pi^2 c\, G\, m_p^2\, m_e}$	(12)
Scale Table 2	$(T)^2$	$\rho = \dfrac{3\pi^3 G\, c^2 m_e^2 m_p^4}{2h^4}$	(19)
Scale Table 3	$(\rho)\times 4/3(\pi R_0^3)$	$R_0 = \sqrt{\dfrac{3c^2}{8\pi\, G\rho}}$	(30)
Scale Table 4	$[(R_0)^{-1}R + K]c$ $[(R_0)^{-1}R - K]c$	$\left(\dfrac{\dot{R}}{R}\right)^2 = \dfrac{8}{3}\pi\, G\rho - \dfrac{k\,c^2}{R^2} + \dfrac{\Lambda}{3}$	(40)

TABLE 3: *This table shows the input/s and the result (equation) for each scale table.*

The following color code is used in the ***Cosmic Scale Table***:

Unknown	Data	Unknown and Data

Cosmic Scale Table

$$T \xrightarrow{OPERATION} (T)\times m_e c^2$$

Constants *(Cosmic scale)*	**Generation 1** (Proton scale)	**"Generation 4"** *(Planck scale)*	**Constants** (Microscopic scale)
H $\left(T\, m_e c^2\right)$	m_p^2	M_P^2	h

$$T \xrightarrow{OPERATION} (T)^2$$

Density *(Cosmic scale)*	**Time squared** *(Cosmic scale)*	**Time squared** *(Planck scale)*	**Density** *(Planck scale)*
ρ	T^2	T_P^2	ρ_P

$$\rho \xrightarrow{OPERATION} (\rho)\times 4/3(\pi R_0^3)$$

Length *(Cosmic scale)*	**Mass** *(Planck scale)*	**Mass** *(Cosmic scale)*	**Length** *(Planck scale)*
R_0	M_P	M_0	L_P

$$R_0 \xrightarrow{OPERATIONS} [(R_0)^{-1}R + K]\,c \quad ; \quad [(R_0)^{-1}R - K]\,c$$

Rate of Expansion *(Cosmic scale)*	**Rate of Expansion** *(Cosmic scale)*	**Speed** *(Cosmic scale)*	**Speed** *(Cosmic scale)*
$\dot{R}$	$\dot{R}$	$\left(\frac{R}{R_0} + K\right)c$	$\left(\frac{R}{R_0} - K\right)c$

Cosmic Scale Table: This table shows the four previous *scale tables* and the corresponding links.

7. Conclusions

In summary, the present paper shows the age of the Universe, the Universe mass density, the radius of the present particle horizon and the Friedmann's equation are special cases of a more general law: the *Scale Law* [1, 2, 3, 9, 10, 11, 12].

But this law, unlike all other known laws, is not a normal law but a Model Meta-law: *a law that nature use as a model to create the known laws of physics*. Now we are closer to the truth because we have answered the question: Where do the laws of physics come from? The answer is they come from Meta laws (assuming there is more than one). But then another question arises: Where do Meta laws come from? And the answer could be: they came from a Meta-universe that "existed before" the Big Bang. But then again a new question arises: How? Then is when we run out of answers.

I would like to add that, this simple natural mechanism can now be applied not only to the microscopic quantum mechanical scale but also to the macroscopic realm of the Universe. I would like to conclude by saying that, according to the research I have carried out so far, it seems that the potential of this formulation is promising.

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