The Schrödinger Equation and the Scale Principle

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Earlier this year (May) I wrote a paper entitled Scale Factors and the Scale Principle. In this paper I formulated a new law which describes a number of fundamental quantum mechanical laws. Since then I found that other quantum mechanical laws including the Bohr postulate and the De Broglie wavelength formula also obey the scale principle. Later I proved that this new law also describes the formula for the Schwarzschild radius, the equation for Einstein's relativistic energy and Newton's law of universal gravitation. Now I discovered that the Schrödinger's equation can also be explained in terms of the present formulation.

Keywords: *Schrödinger equation, De Broglie wavelength, wave number, wave function, differentiation, first order derivative, second order derivative, Laplacian operator.*

1. Introduction

In 2012 I formulated the *Scale Principle* or *Scale Law*. I published the first version of this paper in May this year (2014). In the first version this law was called the *Quantum Scale Principle*. However after finding that Einstein's relativistic energy also obeys this law, I changed its name to the *scale principle* or *scale law*. Since the first version the principle has evolved to the present form given by the following relationship:

(1)

Meta Law: *Scale Principle* or *Scale Law*	
(1a) Meta Form (Meta Quantities)	(1b) Explicit Form (ratio, exponents and scale factor)
$M_1 \Re \; S M_2$ M_1 = dimensionless Meta Quantity 1 M_2 = dimensionless Meta Quantity 2 S = dimensionless Meta scale factor $\Re$ = Meta Relationship Type	$\left(\frac{Q_1}{Q_2}\right)^n \left[< \middle\| \le \middle\| = \middle\| \ge \middle\| > \right] S \left(\frac{Q_3}{Q_4}\right)^m$ (See details below)

The above symbols stand for

a) ***Quantities:***
(i) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Temperature, etc), or
ii) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length). The physical quantities can be variables (including differentials, derivatives, Laplacians, divergence, integrals, etc.), constants, dimensionless constants, any mathematical operation between the previous quantities, etc.

b) ***Relationship type:*** The relationship is one of five possibilities: **less than or equal to** inequation ($\leq$), or **less than** inequation (<), or **equal to** - equation (=), or a **greater than or equal to** inequation ($\geq$), or a **greater than** inequation (>).

c) ***Scale factor**: S* is a dimensionless *scale factor*. This factor could be a real number, a complex number, a real function or a complex function (strictly speaking real numbers are a particular case of complex numbers). The scale factor could have more than one value for the same relationship. In other words a scale factor can be a quantum number. There must be one and only one scale factor per equation.

d) ***Exponents:*** *n* and *m* are integer exponents: 0, 1, 2, 3, …
Some examples are:
example 1: $n = 0$ and $m = 1$;
example 2: $n = 0$ and $m = 2$;
example 3: $n = 1$ and $m = 0$;
example 4: $n = 1$ and $m = 1$; (canonical form)
example 5: $n = 1$ and $m = 2$;
example 6: $n = 2$ and $m = 0$;
example 7: $n = 2$ and $m = 1$;
It is worthy to remark that:
i) The exponents, *n* and *m*, cannot be both zero in the same relationship.
ii) The number *n* is the exponent of both Q_1 and Q_2 while the number *m* is the exponent of both Q_3 and Q_4 regardless on how we express the equation or inequation (1). This means that the exponents will not change when we express the relationship in a mathematically equivalent form such as

$$\left(\frac{Q_4}{Q_3}\right)^m \left[< \middle| \leq \middle| = \middle| \geq \middle| > \right] S \left(\frac{Q_2}{Q_1}\right)^n$$

iii) So far these integers are less than 3. However we leave the options open as we don't know whether we shall find higher exponents in the future.
iv) When both exponents, n and m, are equal to *one,* then we say that the equation is in its *canonical form*. Whenever we express a particular law of physics in the form of the Scale Law, we should use its canonical form, if possible, provided we don't mix up the variables.

The scale law (1) can also be written as $Q_1^n Q_4^m \left[< \,|\, \leq \,|\, = \,|\, \geq \,|\, > \right] S\, Q_2^n\, Q_3^m$

2. Derivation of the Schrödinger Equation

How did Schrödinger derive his famous equation [2]? It must have come from at least the following knowledge: (a) the De Broglie's relationship between the wavelength and the momentum of a particle, (b) the classical mechanical energy of a particle, (c) the classical wave equation (which uses the second order derivative with respect to time as opposed to the first order derivative in Schrödinger's equation), and (c) differentiation. But the question still remains: where did the idea of using a complex wave function come from?

To derive the Schrödinger's equation we shall postulate the existence of a quantum field or complex wave function $\Psi = \Psi(x, y, z, t)$ that represent all there is to know about a given particle. Then we shall use differentiation. The derivation is a simple and straightforward mathematical process.

Before starting the derivation, it is worthy to remark that the units of the amplitude of the wave function are $1/m^{3/2} = m^{-3/2}$ (m stands for meters).

Now we shall derive both the time *independent* and *time dependent Schrödinger's equations*. The time dependent equation is:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + U\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{2}$$

In general the wave function Ψ will depend on the three spatial variables x, y, z and on the temporal variable t, mathematically

$$\Psi = f(x, y, z, t) \tag{3}$$

Let us postulate that there is complex quantum field or a complex wave function such as

$$\Psi(x, y, z, t) \equiv A e^{i(k_x x + k_y y + k_z z - wt)} \tag{4}$$

And that this function has a probabilistic interpretation as the one given by Max Born [3].

This is just an example of the wave function to illustrate the derivation process. We could have used other appropriate wave functions such as:

$\Psi(x,y,z,t) \equiv A\sin(k_x x)\sin(k_y y)\sin(k_z z)e^{-iwt}$, or

$\Psi(x,y,z,t) \equiv A\cos(k_x x)\cos(k_y y)\cos(k_z z)e^{-iwt}$, etc. and we would have got the same conceptual results.

The wave numbers of equation (4) are defined as

$$k_x \equiv \frac{2\pi}{\lambda_x} \tag{5}$$

$$k_y \equiv \frac{2\pi}{\lambda_y} \tag{6}$$

$$k_z \equiv \frac{2\pi}{\lambda_z} \tag{7}$$

Considering the *De Broglie relationship* $\lambda = h/p$ we can write

$$\lambda_x = \frac{h}{p_x} \tag{8}$$

$$\lambda_y = \frac{h}{p_y} \tag{9}$$

$$\lambda_z = \frac{h}{p_z} \tag{10}$$

where

p_x, p_y, p_z = components of the momentum of the particle along the *x* axis, *y* axis and *z* axis respectively

$\lambda_x, \lambda_y, \lambda_z$ = De Broglie wavelengths associated with a particle of mass *m.* (Although the particle has only one wavelength (λ) and since the momentum *p* of the particle has three components such as the ones that arise from using a rectangular coordinate system, we can associate a different wavelength ($\lambda_x, \lambda_y, \lambda_z$) with each component ($p_x, p_y, p_z$) of the momentum.)

h = Planck's constant

Then we can express the wave numbers in terms of the components of the momentum. This gives

$$k_x = \frac{p_x}{\hbar} \tag{11}$$

$$k_y = \frac{p_y}{\hbar} \tag{12}$$

$$k_z = \frac{p_z}{\hbar} \tag{13}$$

Now we express the angular frequency ω in terms of the classical mechanical energy E of the particle. (For massless particle such as photons the total relativistic energy equals its kinetic energy. However here we are dealing with the general case of massive particles and therefore we use the classical mechanical energy). This gives

$$\omega = \frac{E}{\hbar} \tag{14}$$

Substituting k_x, k_y, k_z and ω in equation (4) with equations (11), (12), (13) and (14) respectively yields

$$\Psi(x, y, z, t) = A e^{i\left(\frac{p_x}{\hbar}x + \frac{p_y}{\hbar}y + \frac{p_z}{\hbar}z - \frac{E}{\hbar}t\right)} \tag{15}$$

$$\Psi(x, y, z, t) = A e^{i\left(p_x x + p_y y + p_z z - Et\right)/\hbar} \tag{16}$$

Now we define the variable $\theta = \theta(x, y, z, t)$ to simplify the differentiation process

$$\theta = \frac{p_x x + p_y y + p_z z - Et}{\hbar} \tag{17}$$

Introducing θ in equation (16) we get

$$\Psi(x, y, z, t) = A e^{i\theta} \tag{18}$$

According to *Euler* equation (18) can be rewritten as

$$\Psi(x, y, z, t) = A\left(\cos\theta + i\sin\theta\right) \tag{19}$$

We now calculate the first order partial derivative of Ψ with respect to the spatial variable x

$$\frac{\partial\Psi}{\partial x} = A\left(-\sin\theta\,\frac{\partial\theta}{\partial x} + i\cos\theta\,\frac{\partial\theta}{\partial x}\right) \tag{20}$$

$$\frac{\partial\theta}{\partial x} = \frac{p_x}{\hbar} \tag{21}$$

$$\frac{\partial\Psi}{\partial x} = A\frac{p_x}{\hbar}\left(-\sin\theta + i\cos\theta\right) \tag{22}$$

We continue by calculating the second order partial derivative of Ψ with respect to the spatial variable x

$$\frac{\partial^2 \Psi}{\partial x^2} = A \frac{p_x}{\hbar} \left(- \cos\theta \, \frac{\partial \theta}{\partial x} - i \sin\theta \, \frac{\partial \theta}{\partial x} \right) \tag{23}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = - A \frac{p_x^2}{\hbar^2} \left(\cos\theta + i \sin\theta \right) \tag{24}$$

Comparing equations (24) with (19) we can write the following relationship

$$\frac{\partial^2 \Psi}{\partial x^2} = - \frac{p_x^2}{\hbar^2} \Psi \tag{25}$$

A similar process leads to

$$\frac{\partial^2 \Psi}{\partial y^2} = - \frac{p_y^2}{\hbar^2} \Psi \tag{26}$$

$$\frac{\partial^2 \Psi}{\partial z^2} = - \frac{p_z^2}{\hbar^2} \Psi \tag{27}$$

Adding the three previous equations yields

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = - \frac{p_x^2}{\hbar^2} \Psi - \frac{p_y^2}{\hbar^2} \Psi - \frac{p_z^2}{\hbar^2} \Psi \tag{28}$$

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = - \left(\frac{p_x^2 + p_y^2 + p_z^2}{\hbar^2} \right) \Psi \tag{29}$$

$$p^2 = p_x^2 + p_y^2 + p_z^2 \tag{30}$$

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{p^2}{\hbar^2} \Psi = 0 \tag{31}$$

Now we consider the classical mechanical energy E of the particle

$$E = K + U \tag{32}$$

Where

E = classical mechanical energy of the particle

K =classical kinetic energy of the particle
U = potential energy of the particle

In general the mechanical energy will be a function of time.
The classical kinetic energy is

$$K = \frac{1}{2} m v^2 \tag{33}$$

which can be written as

$$K = \frac{p^2}{2m} \tag{34}$$

where p is the momentum of the particle and is given by

$$p = mv \tag{35}$$

Where m is the mass of the particle and v is the speed

The mechanical energy can be expressed in terms of the kinetic energy as follows

$$E = \frac{p^2}{2m} + U \tag{36}$$

Solving for the square of the momentum yields

$$p^2 = 2m\left(E - U\right) \tag{37}$$

Substituting p^2 in equation (31) with the second side of equation (37) yields

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m\left(E - U\right)}{\hbar^2} \Psi = 0 \tag{38}$$

Considering that

$$\hbar \equiv \frac{h}{2\pi} \tag{39}$$

we finally find the *time independent (TI) Schrödinger's partial differential equation*

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{8\pi^2 m\left(E - U\right)}{h^2} \Psi = 0 \quad (\textit{TI Schrödinger's equation}) \tag{40}$$

If we introduce the *Laplacian operator* ∇^2 which is defined as

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \text{(\textit{Laplacian operator})}$$

we can express equation (40) in terms of the *Laplacian*

$$\nabla^2 \Psi + \frac{8\pi^2 m(E - U)}{h^2} \Psi = 0 \qquad \text{(\textit{TI Schrödinger's equation - Laplacian form})} \tag{41}$$

We now calculate the first order partial derivative of Ψ with respect to the temporal variable t

$$\frac{\partial \Psi}{\partial t} = A\left(- \sin\theta \, \frac{\partial \theta}{\partial t} + i \cos\theta \, \frac{\partial \theta}{\partial t} \right) \tag{42}$$

$$\frac{\partial \theta}{\partial t} = - \frac{E}{\hbar} \tag{43}$$

$$\frac{\partial \Psi}{\partial t} = - \frac{AE}{\hbar} \left(- \sin\theta + i \cos\theta \right) \tag{44}$$

Multiplying by $i\hbar$ yields

$$\frac{\partial \Psi}{\partial t} = - AE \left(- i \sin\theta + i^2 \cos\theta \right) \tag{45}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = AE \left(\cos\theta + i \sin\theta \right) \tag{46}$$

Comparing equation (46) with (19) we get

$$i\hbar \frac{\partial \Psi}{\partial t} = E \Psi \tag{47}$$

Now we shall substitute $E \Psi$ in equation (41) with the first side of equation (47) to get the *time-dependent (TD) Schrödinger's partial differential equation*. A simple mathematical work leads to the result we are looking for: the *time dependent Schrödinger's equation*

$$- \frac{\hbar^2}{2m} \nabla^2 \Psi + U \Psi = i\hbar \frac{\partial \Psi}{\partial t} \qquad \text{(\textit{Time dependent Schrödinger's equation})} \tag{48}$$

3. The Schrödinger Equation as a special case of the Scale Principle

We shall show that the *time-independent Schrödinger equations* (given by equation (41)) obeys the *scale law*. The *scale factor* will be deduced from comparison with the previous section.

This particular case requires a more elaborated *scale table* than the ones we used before. While a *scale table* with a pair of balanced columns was enough for the previous cases [4, 5, 6, 7, 8] now it isn't. What we need is to create a balanced table with unbalanced columns. This can be achieved if we consider the De Broglie equation

$$\lambda = \frac{h}{p} \tag{49}$$

This can be re-written as

$$\lambda = \frac{h}{\sqrt{m^2 v^2}} \tag{50}$$

If inside the square root we multiply and dividing by 2 the above equation takes the following form

$$\lambda = \frac{h}{\sqrt{\frac{2}{2} m^2 v^2}} = \frac{h}{\sqrt{2m \left(\frac{1}{2} m v^2 \right)}} \tag{51}$$

We recognize $\frac{1}{2} m v^2$ as the classical kinetic energy *K* of the particle. Then we write

$$\lambda = \frac{h}{\sqrt{2m\,K}} \tag{52}$$

According to equation (32) we substitute *K* with $E - U$. This gives

$$\lambda = \frac{h}{\sqrt{2m\,(E - U)}} \tag{53}$$

Squaring both sides

$$\lambda^2 = \frac{h^2}{2m\,(E - U)} \tag{54}$$

The idea is to place λ in the first half of the scale table and 1/ λ in the other half so that when we multiply them we shall get λ^2.

To balance the table we perform a simple dimensional analysis: we place both $\nabla^2 \Psi$ $(LENGTH^{-7/2})$ and λ $(LENGTH)$ on one half of the table; and $\frac{1}{\lambda}$ $(LENGTH^{-1})$ and Ψ $(LENGTH^{-3/2})$ on the other one. With these considerations we can draw the following *scale table*

Length Laplacian of the Wave function	Length Wavelength	Length 1/Wavelength	Length Wave function
$m^{-7/2}$	m	m^{-1}	$m^{-3/2}$
$\nabla^2 \Psi$	λ	$\frac{1}{\lambda}$	Ψ

***TABLE 1**: This simple scale table is used to show that the Schrödinger equation obeys the scale law. The table must be balanced as a whole.*

Now we write the following relationship

$$\nabla^2 \Psi \, \lambda = S \frac{1}{\lambda} \Psi \tag{55}$$

$$\nabla^2 \Psi = S \frac{1}{\lambda^2} \Psi \tag{56}$$

From equations (54) and (56) we get

$$\nabla^2 \Psi = S \frac{2m(E - U)}{h^2} \Psi \tag{57}$$

If we adopt

$$S = -4\pi^2 \tag{58}$$

Equation (57) transforms into equation (41)

$$\nabla^2 \Psi + \frac{8\pi^2 m(E - U)}{h^2} \Psi = 0 \qquad \textit{(Time independent Schrödinger equation)} \tag{59}$$

which is the *time independent Schrödinger equation*.

Now we consider equation (55). This equation can be written in the form of the *scale law* as follows

$$\left(\frac{\lambda}{\sqrt{\frac{\Psi}{\nabla^{2}\Psi}}} \right)^{2} = S \tag{60}$$

where

$$n = 2$$
$$Q_1 = \lambda$$
$$Q_2 = \sqrt{\frac{\Psi}{\nabla^{2}\Psi}}$$
$$m = 0$$
$$Q_3 = 1 \ (doesn't\ matter\ \ because\ \ m = 0)$$
$$Q_4 = 1 \ (doesn't\ matter\ \ because\ m = 0)$$
$$S = -4\pi^{2}$$

We should express equation (60) in the canonical form ($n = m = 1$). Thus we write

$$\frac{\lambda}{\sqrt{\frac{\Psi}{\nabla^{2}\Psi}}} = -4\pi^{2} \frac{\sqrt{\frac{\Psi}{\nabla^{2}\Psi}}}{\lambda} \qquad \text{(\textit{Scale Law} for the \textit{Schrödinger's equation})} \tag{61}$$

where

$$n = 1$$
$$Q_1 = \lambda$$
$$Q_2 = \sqrt{\frac{\Psi}{\nabla^{2}\Psi}}$$
$$m = 1$$
$$Q_3 = \sqrt{\frac{\Psi}{\nabla^{2}\Psi}}$$
$$Q_4 = \lambda$$
$$S = -4\pi^{2}$$

Thus we have proved that the *time independent Schrödinger's equation* obeys the *scale law*.

Let us define the function *Units*(Q) that will give us the units of the quantity Q and let us introduce a new concept to quantum mechanics: the *Psi radius* R_Ψ which we shall define as

$$R_\Psi \equiv \sqrt{\frac{\Psi}{\nabla^2 \Psi}} \qquad \text{(\textit{Psi Radius})} \tag{62}$$

Then let us verify the units of R_Ψ

$$Units\left(R_\Psi\right) = Units\left(\sqrt{\frac{\Psi}{\nabla^2 \Psi}}\right) = \sqrt{\frac{m^{-3/2}}{m^{-7/2}}} = \sqrt[4]{\frac{m^{-3}}{m^{-7}}} = \sqrt[4]{m^4} = m$$

Thus the units of R_Ψ are meters as it should be.

Now we express the *Schrödinger's equation* in terms of the *Psi radius*

$$\frac{\lambda}{R_\Psi} = -4\pi^2 \frac{R_\Psi}{\lambda} \tag{63}$$

$$\left(\frac{\lambda}{R_\Psi}\right)^2 = -4\pi^2 \tag{64}$$

or

$$\frac{\lambda}{R_\Psi} = i\,2\pi \tag{65}$$

Finally

$$\lambda = i\,2\pi\, R_\Psi \qquad \text{(\textit{Schrödinger's equation in terms of the Psi radius})} \tag{66}$$

Now we see that the wavelength of the so called "material waves" is the length of a circle of radius $R_\Psi \equiv \sqrt{\frac{\Psi}{\nabla^2 \Psi}}$ multiplied by the imaginary number: $i = \sqrt{-1}$. For this reason equation (62) was defined as a radius and not as a wavelength. Thus we see that λ is real if and only if R_Ψ is imaginary.

To obtain the *time dependent Schrödinger equation* we simply substitute E in equation (53) with the second side of the following equation

$$E = i\hbar \frac{1}{\Psi} \frac{\partial \Psi}{\partial t} \tag{67}$$

This gives

$$\lambda = \frac{h}{\sqrt{2m\left(i\hbar \frac{1}{\Psi} \frac{\partial \Psi}{\partial t} - U \right)}} \tag{68}$$

Now after simple algebraic steps we get the *time dependent Schrödinger's equation*

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + U\Psi = i\hbar\frac{\partial \Psi}{\partial t} \qquad (\textit{Time dependent Schrödinger's equation}) \tag{69}$$

3. Conclusions

In summary, we have derived both the *time dependent* and *the time independent Schrödinger equations* from the following assumptions:

(a) "There is" a complex quantum field or complex wave function which cannot be measured,
(b) The particle has an associated wavelength that obeys the *De Broglie relationship between the wavelength and the momentum*,
(c) The kinetic energy of the particle is the classical kinetic energy
(d) The particle obeys the principle of conservation of energy.

We have also defined a new quantum mechanical quantity we called the *Psi radius* whose full physical meaning is unknown.

Although in order to obtain the value of the *scale factor* we had to compare two equations, we have proved that the *Schrödinger equation* is a special case of the Scale Law. Thus the Scale Law describes quantum mechanics' most powerful formulation – *the Schrödinger equation*.

Taking into consideration that the Scale Law describes several normal laws as I have shown both on previous papers [4, 5, 6, 7, 8] and on this paper, we can consider the *scale law* as a *more fundamental law* than the specific laws it describes because the *scale law* "must have been conceived before" the Big Bang. This means that the Scale Law wouldn't be a normal law of physics but a Meta Law*: a law that would have spawned other laws of physics*. But why would there be only one Meta Law? Common sense indicates that there must be other Meta Laws which we haven't been able to discover yet.

Thus each normal or specific law of physics must obey one or more Meta Laws and one Meta Law governs a number of natural laws. The answer to the question "Where do the laws of physics come from?" is: they come from Meta Laws [9].

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