Using Matrix Method to Find What is the Next Number in this Sequence

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Given a sequence of numbers , that is generated by matrix I described an easy way to write down the matrix by computing the next value of this sequence which was located on the next number on the diagonal of the matrix.

A sequence in mathematics is defined as an ordered list of elements. The order of the elements is very important and changing even one element would change the meaning of the entire sequence, so finding the next number in a sequence play important part in IQ test, discovering pattern numbers and puzzles.

Many efforts have been devoted to solve problems that contain missing numbers in a sequence , for example R.K Guy called this the ' strong law of small numbers' (reference 1), this approach confirmed by C.E.Lindeholm (reference 2). Recently P.C.Toh, and E.G.Tay solve given a sequence of numbers by easy way to write down the polynomial or the recurrence relation by comparing successive differences of the terms of the sequence (reference 3).

Our problem is to find the next number on the following sequence ,

$$S_n : a_1 , a_2 , a_3 , \ldots, a_{n,} \ldots \qquad (1)$$

This sequence must be finite , and increasing

So to find the next term of (1) , consider the following square matrix,

$$\begin{pmatrix} a11 \; a12 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ an1 \; an2 & \cdots & ann \end{pmatrix}, \qquad (2)$$

Know the problem is reduced as this , if we can find the next number on the diagonal of the matrix on (2) , we can find the next number on (1), before dive in it is necessary to mention the relationship between (1) and (2) , which is diagonal of the matrix is the same as (1).

Now we need to generate matrix (2) from the given sequence (1) as follows:

$a_2 + a_1$, $a_3 + a_2$, …, $\underline{a_n + a_{n-1}}$ ← upper vertex

$a_3 + a_2 + a_1$, $a_4 + a_3 + a_2$, …, $a_n + a_{n-1} + a_{n-2}$

and so on until we got one value

a_1 , a_2 , a_3 , …, a_n

$a_2 - a_1$, $a_3 - a_2$, …, $a_n - a_{n-1}$

lower vertex→ $\underline{a_3 - a_2 - a_1}$, $a_4 - a_3 - a_2$, …, $a_n - a_{n-1} - a_{n-2}$

and so on until we got one value

the matrix we have generated from the given sequence has the following properties :

1) all the values on the right side of the diagonal are bigger than the values on the left side of the diagonal , because the value we got on the right side came from addition of the terms of (1) , while the left side generated by subtraction the terms of (1).

2) The terms of the diagonal can be computed by the following algorithm:

$a_{11} = \frac{a12-a21}{2}$, $a_{22} = \frac{a12+a21}{2}$,…, $a_{nn} = \frac{a1n-an1}{2}$.

3) To find the next number on the sequence , we use the this procedure :

- First add all numbers on the latest row of matrix except a_{nn} . i.e. so $x=\sum_{i=an1}^{a(n-1)} ai = a_{n1}+a_{n2}+\ldots+a_{n(n-1)}$ if there is no common factor between any two rows, if there is common factor to two consecutive rows or more add lower vertex to x , so $x=\sum_{i=an1}^{a(n-1)} ai +$ lower vertex $= a_{n1}+a_{n2}+\ldots+a_{n(n-1)} +$ lower vertex.

- Second we need to find the number (say y) located exactly below a_{nn} , like this $y = 2\,a_{nn}+x$.

- Finally , the next number to a_{nn} say $z = \frac{y+x}{2}$.

Finally , the following examples illustrate the method.

Example 1 : Find the next number of the following sequence

23, 48, 84, 133, …

522← upper vertex

203 349

71 132 217

23 48 84 133

25 36 49

11 13

lower vertex→ 2

Now I convert the above calculation to the following matrix A:

$$A=\begin{pmatrix} \mathbf{23} & 71 & 203 & 522 \\ 25 & \mathbf{48} & 132 & 349 \\ 11 & 36 & \mathbf{84} & 217 \\ 2 & 13 & 49 & \mathbf{133} \end{pmatrix}$$

We can note :

$\frac{71-25}{2} = 23$, $\frac{71+25}{2} = 48$, $\frac{132-36}{2} = 48$, $\frac{132+36}{2} = 84$, $\frac{217-49}{2} = 84$,

$\frac{217+49}{2} = 133$.

So to find the next number of 133, you follow the above instruction such that, x=2+13+49 = 64 , because there is no any common factor between the numbers (2,13,49), or any two numbers on this row except a_{nn}, then the next number to 133 on its right hand side , but not on the diagonal is y = 2(133) +64 = 330.So the next number of 133 is , $z = \frac{330+64}{2} = 197$, while , $\frac{330-64}{2} = 133$. I can summarize this procedure simply like this:

The next number = 2+13+49 +133 = 197

Example 2 : what is the next number in the following sequence :

1, 3 , 8 , 19 , 42 , 89 , …

Solution : first generate the terms of matrix

585

179 406

53 126 280

15 38 88 192

4 11 27 61 131

1 3 8 19 42 89

2 5 11 23 47

3 6 12 24

3 6 12

3 6

3

Second, convert to square matrix B

$$B=\begin{pmatrix} \mathbf{1} & 4 & 15 & 53 & 179 & 585 \\ 2 & \mathbf{3} & 11 & 38 & 126 & 406 \\ 3 & 5 & \mathbf{8} & 27 & 88 & 280 \\ 3 & 6 & 11 & \mathbf{19} & 61 & 192 \\ 3 & 6 & 12 & 23 & \mathbf{42} & 131 \\ 3 & 6 & 12 & 24 & 47 & \mathbf{89} \end{pmatrix}$$

Third, x= 3+6+12+24+47 + lower vertex (3) = 95. We have common factor between (3,6,12,24) =3

Forth , y = 2(89) + 95 = 273.

Finally , $z = \frac{x+y}{2} = \frac{95+273}{2}$ = 184, which is the solution of our problem.

. I can summarize this procedure simply like this: The next number = 3+6+12+24+47 +89 + lower vertex (3)= 184.

Example 3 : what is the next number in the following sequence :

1,7,19,45,109 , …

Let C be the matrix generated by this sequence , so

$$C=\begin{pmatrix} 1 & & & & \\ 6 & 7 & & & \\ 6 & 12 & 19 & & \\ 8 & 14 & 26 & 45 & \\ 16 & 24 & 38 & 64 & 109 \end{pmatrix}$$

So the next number is 16+24+38+64+109+ lower vertex (16)= 267.

Example4 : what is the next number in the following sequence :

1,2,4,8,16 , …

Let D be the matrix generated by this sequence , so

$$D=\begin{pmatrix} 1 & & & & \\ 1 & 2 & & & \\ 1 & 2 & 4 & & \\ 1 & 2 & 4 & 8 & \\ 1 & 2 & 4 & 8 & 16 \end{pmatrix}$$

So the next number is 1+2+4+8+16+ lower vertex (1)= 32

Example5 : what is the next number in the following sequence :

7,27,58,102 , …

Let E be the matrix generated by this sequence , so

$$E=\begin{pmatrix} \mathbf{7} & & & \\ 20 & \mathbf{27} & & \\ 11 & 31 & \mathbf{58} & \\ 2 & 13 & 44 & 102 \end{pmatrix}$$

So the next number is 2+13+44+102 = 161

References

1 R.K Guy, The strong law of small numbers , Amer. Math. Monthly 95 (1988), pp. 697-712.

2 C. E. Lindeholm, Mathematics Made Difficult (world Publishing , New York, 1972) .

3 P.C.Toh, and E.G.Tay, What is the Next Number in this Sequence ?, Mathematical Spectrum 46 (2014), pp. 125 -131.