The cosmic theory without the dark matter

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ABSTRACT

In the general relativity theory, using Einstein's revised gravity field equation (add the cosmological constant), discovers the solution of the cosmological problem. In this time, by the cosmological constant, understand the universe energy density without the dark matter.

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I.Introduction

This theory is that it treats the cosmology without the dark matter using the revised gravity field equation(add the cosmological constant).

Therefore, the general relativity theory's revised field equation (add the cosmological constant) is written completely.

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon} \quad (1)$$

Invariant time $d\tau$ of the cosmology is

$$d\tau^2 = dt^2 - \Omega^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (2)$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} \quad (3)$$

$$T_{\mu\upsilon} = p g_{\mu\upsilon} + (p/c^2 + \rho) U_\mu U_\upsilon, \quad U_\mu = (c,0,0,0)$$

$$T_{00} = \rho(t)c^2, \; T_{0i} = 0, \; T_{ij} = p(t) g_{ij}, \quad T^{\lambda}{}_{\lambda} = -\rho(t)c^2 + 3p(t) \quad (4)$$

$$R_{00} = 3\frac{\ddot{\Omega}}{c^2\Omega} = -\frac{4\pi G}{c^4}(\rho c^2 + 3p) + \Lambda \quad (5)$$

$$R_{ij} = -(\Omega\ddot{\Omega}\frac{1}{c^2} + 2\dot{\Omega}^2\frac{1}{c^2} + 2k)\frac{g_{ij}}{\Omega^2} = -\frac{4\pi G}{c^4}(\rho c^2 - p) g_{ij} - \Lambda g_{ij} \quad (6)$$

$$(\Omega\ddot{\Omega}\frac{1}{c^2} + 2\dot{\Omega}^2\frac{1}{c^2} + 2k)\frac{1}{\Omega^2} = \frac{4\pi G}{c^4}(\rho c^2 - p) + \Lambda \quad (7)$$

In this time, $\dot{\Omega} = \frac{\partial\Omega}{\partial t}$

Therefore, Eq(5) - $3\times$ Eq(7) is

$$-(6\dot{\Omega}^2\frac{1}{c^2} + 6k)\frac{1}{\Omega^2} = -\frac{16\pi G}{c^2}\rho - 2\Lambda$$

$$(\dot{\Omega}^2\frac{1}{c^2} + k)\frac{1}{\Omega^2} = \frac{8\pi G}{3c^2}\rho + \Lambda/3 \quad (8)$$

In this time,

$$\Omega(t) < \dot{\Omega}(t_0)(t - t_0) + \Omega(t_0) \quad (9)$$

If $t = 0$, $\Omega(0) = 0$

$$0 < -\dot{\Omega}(t_0)t_0 + \Omega(t_0) \quad , \quad t_0 < \frac{\Omega(t_0)}{\dot{\Omega}(t_0)} = \frac{\Omega_0}{\dot{\Omega}_0} = \frac{1}{H_0} \quad (10)$$

By Eq(8),Eq(10), the universe energy density ρ_0 is

$$\rho_0 = \frac{3c^2}{8\pi G}\{\frac{1}{c^2}(\frac{\dot{\Omega}_0}{\Omega_0})^2 + \frac{k}{{\Omega_0}^2} - \frac{\Lambda}{3})\} = \frac{3c^2}{8\pi G}\{{H_0}^2/c^2 + \frac{k}{{\Omega_0}^2} - \frac{\Lambda}{3}\} \quad (11)$$

In this time,

$k = 1$ or 0 or $-1 \rightarrow |k| \leq 1$

$\rho_0 \approx 2 \times 10^{-31} gm/cm^3$ (The observation result of the universe),

$$\rho_c = \frac{3{H_0}^2}{8\pi G} \approx 5 \times 10^{-30} gm/cm^3 \quad (12)$$

Therefore, in this time, the cosmological constant Λ is by Eq(11),Eq(12)

$$\frac{3c^2}{8\pi G}\frac{k}{{\Omega_0}^2} = \frac{3c^2}{8\pi G{\Omega_0}^2} \approx 0$$

$$\Lambda \approx \frac{8\pi G}{c^2}(\rho_c - \rho_0) \approx \frac{8\pi G}{c^2} \times (4.8 \times 10^{-30} gm/cm^3) \quad (13)$$

In this time, we understand the universe energy density by the cosmological constant Λ and it need not dark matter.

II. The revised Schwarzschild solution

In this theory, the general relativity theory's revised field equation (add the cosmological constant) is written completely.

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4}T_{\mu\upsilon} \quad (14)$$

Eq (14) multiply $g^{\mu\upsilon}$ and does contraction,

$$g^{\mu\upsilon}R_{\mu\upsilon} - \frac{1}{2}g^{\mu\upsilon}g_{\mu\upsilon}R - \Lambda g^{\mu\upsilon}g_{\mu\upsilon}$$

$$= -R - 4\Lambda = -\frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda} \quad (15)$$

Therefore, Eq (14) is

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}(-4\Lambda + \frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4}T_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} \quad (16)$$

In this time, the spherical coordinate system's vacuum solution is by $T_{\mu\upsilon} = 0$

$$R_{\mu\upsilon} = -\Lambda g_{\mu\upsilon} \quad (17)$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (18)$$

Using Eq(18)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = \Lambda A \quad (19)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = -\Lambda B \quad (20)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -\Lambda r^2 \quad (21)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \quad (22)$$

$$R_{tr} = -\frac{\dot{B}}{Br} \quad (23)$$

$$R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \quad (24)$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

By Eq(23),

$$\dot{B} = 0 \quad (25)$$

By Eq(19) and Eq(20),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}(\frac{A'}{A} + \frac{B'}{B}) = -\frac{(AB)'}{rAB^2} = 0 \quad (26)$$

Therefore,

$$A = \frac{1}{B} \quad (27)$$

If Eq(21) is inserted by Eq(27),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + (\frac{r}{B})' = -\Lambda r^2 \quad (28)$$

If it solves Eq(28)

$$\frac{r}{B} = r + C - \frac{1}{3}\Lambda r^3 \rightarrow \frac{1}{B} = 1 + \frac{C}{r} - \frac{1}{3}\Lambda r^2 \quad (29)$$

In this time,

$$\Lambda \approx \frac{8\pi G}{c^2}(\rho_c - \rho_0) \approx \frac{8\pi G}{c^2} \times (4.8 \times 10^{-30} gm/cm^3) \quad (30)$$

$$A=\frac{1}{B}=1+\frac{C}{r}-\frac{\Lambda}{3}r^{2} \tag{31}$$

To know Eq(31)'s third term, does Newton's limitation

$$\frac{d^{2}r}{dt^{2}}\approx\frac{1}{2}c^{2}\frac{\partial(-A)}{\partial r}=\frac{1}{2}c^{2}\frac{\partial}{\partial r}(-1-\frac{C}{r}+\frac{\Lambda}{3}r^{2})=-\frac{GM}{r^{2}}+\frac{\Lambda}{3}rc^{2}$$

$$\approx-\frac{GM}{r^{2}}+\frac{8\pi G}{3}(\rho_{c}-\rho_{0})r\,,\ C=-\frac{2GM}{c^{2}}$$

$$\approx-\frac{GM}{r^{2}}+\frac{8\pi G}{3}\times(4.8\times10^{-30}gm/cm^{3})r \tag{32}$$

$$\frac{d^{2}r}{dt^{2}}\approx\approx-\frac{GM}{r^{2}}+\frac{8\pi G}{3}\times(4.8\times10^{-30}gm/cm^{3})r \tag{33}$$

If it observes Eq(33)'s second term, the scale of r has to be the cosmological scale.

III. The revised Reissner-Nodstrom solution

Of course, this way success in the Reissner-Nodstrom solution.

$$R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R-\Lambda g_{\mu\upsilon}=-\frac{8\pi G}{c^{4}}T_{\mu\upsilon} \tag{34}$$

The spherical coordinate system's invariant time is

$$d\tau^{2}=A(t,r)dt^{2}-\frac{1}{c^{2}}[B(t,r)dr^{2}+r^{2}d\theta^{2}+r^{2}\sin^{2}\theta d\phi^{2}]$$

$$R_{\mu\upsilon}=-\frac{8\pi G}{c^{4}}(T_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda})-\Lambda g_{\mu\upsilon}$$

$$T_{\mu\upsilon}=\frac{E^{2}+H^{2}}{8\pi}\begin{pmatrix}A&0&0&0\\0&-B&0&0\\0&0&r^{2}&0\\0&0&0&r^{2}\sin^{2}\theta\end{pmatrix}$$

$E=\frac{k}{r^{2}}$, k is constant, $H=\frac{h}{r^{2}}$, h is constant (35)

Likely Eq(19)~Eq(23),

$$R_{tt}=-\frac{A''}{2B}+\frac{A'B'}{4B^{2}}-\frac{A'}{Br}+\frac{A'^{2}}{4AB}+\frac{\ddot{B}}{2B}-\frac{\dot{B}^{2}}{4B^{2}}-\frac{\dot{A}\dot{B}}{4AB}=\Lambda A-G(k^{2}+h^{2})\frac{A}{r^{4}} \tag{36}$$

$$R_{rr}=\frac{A''}{2A}-\frac{A'^{2}}{4A^{2}}-\frac{A'B'}{4AB}-\frac{B'}{Br}-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^{2}}+\frac{\dot{B}^{2}}{4AB}=-\Lambda B+G(k^{2}+h^{2})\frac{B}{r^{4}} \tag{37}$$

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^{2}}+\frac{rA'}{2AB}=-\Lambda r^{2}-\frac{G(k^{2}+h^{2})}{r^{2}} \tag{38}$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0 \quad (39)$$

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}(\frac{A'}{A} + \frac{B'}{B}) = -\frac{(AB)'}{rAB^2} = 0 \quad (40)$$

$$-1 + (\frac{r}{B})' = -\Lambda r^2 - \frac{G(k^2 + h^2)}{r^2} \quad (41)$$

$$A = \frac{1}{B} = 1 + \frac{C}{r} - \frac{1}{3}\Lambda r^2 + \frac{G(k^2 + h^2)}{r^2}$$

$$C = -\frac{2GM}{c^2}, \quad \Lambda \approx \frac{8\pi G}{c^2}(\rho_c - \rho_0) \approx \frac{8\pi G}{c^2} \times (4.8 \times 10^{-30} gm / cm^3) \quad (42)$$

IV. Conclusion

Therefore, this theory describes the cosmology by the present energy density without the dark matter.

*Appendix.

The general relativity theory's revised field equation (add the cosmological term) is written completely.

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} \quad (43)$$

The co-moving system is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[U(t,r)dr^2 + V(t,r)(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$T_{00} = \rho c^2 \text{ ,otherwise } T^{ti} = T^{ij} = 0 \quad (44)$$

$$\frac{1}{U}R_{rr} = \frac{1}{U}(\frac{V''}{V} - \frac{V'^2}{2V^2} - \frac{U'V'}{2UV}) - \frac{\ddot{U}}{2U} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{V}}{2UV} = -\frac{4\pi G}{c^2}\rho - \Lambda \quad (45)$$

$$\frac{1}{V}R_{\theta\theta} = -\frac{1}{V} + \frac{1}{U}(\frac{V''}{2V} - \frac{U'V'}{4UV}) - \frac{\ddot{V}}{2V} - \frac{\dot{V}\dot{U}}{4VU} = -\frac{4\pi G}{c^2}\rho - \Lambda \quad (46)$$

$$U = R^2(t)f(r) \quad , \quad V = R^2(t)r^2$$

Eq(45),Eq(46) is

$$\frac{f'(r)}{rf^2(r)} + [\ddot{R}(t)R(t) + 2\dot{R}^2(t)] = \frac{4\pi G}{c^2}R^2(t)\rho(t) + \Lambda R^2(t) \quad (47)$$

$$[\frac{1}{r^2} - \frac{1}{r^2 f(r)} + \frac{f'(r)}{2rf^2(r)}] + [\ddot{R}(t)R(t) + 2\dot{R}^2(t)] = \frac{4\pi G}{c^2}R^2(t)\rho(t) + \Lambda R^2(t) \quad (48)$$

$$\frac{f'(r)}{rf^2(r)} = \frac{1}{r^2} - \frac{1}{r^2 f(r)} + \frac{f'(r)}{2rf^2(r)} = -2k \quad (49)$$

$$f(r) = \frac{1}{1 - kr^2} \quad (50)$$

Therefore, in the revised gravity field equation (add the cosmological constant), co-moving system's invariant time is

$$d\tau^2 = dt^2 - R^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \tag{51}$$

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[3]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[4]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[5]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[6]A.Raychaudnury, Theoretical Cosmology (Oxford University Press, 1979)

[7]P.Peebles, Physical Cosmology (Prenceton University,1971)

[8]E.Hubble, Proc, Nat, Acad. Sci. U.S. 15, 169(1929)

[9]A.Sandage, Astrophys. J. 178, 1(1972)