Bell's theorem refuted and 't Hooft's superdeterminism rejected as we factor quantum entanglements in full accord with commonsense local realism

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Abstract: Commonsense local realism (CLR) is the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively). Advancing our case for a local realistic quantum mechanics based solely on CLR, we use undergraduate maths and a single unifying thought-experiment (experiment Q) to jointly factor the quantum-entanglements in EPRB and Aspect (2002). Such CLR base-factors (one factor based solely on beables in Alice's domain, the other factor based solely on beables in Bob's domain), refute Bell's theorem and eliminate the need for 't Hooft's superdeterminism.

Key words: CLR base-factors, commonsense local realism, superdeterminism

On one supposition we absolutely hold fast; that of local/Einstein causality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).

"It is a matter of indifference ... whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, [Bell writes] as if λ were a single continuous parameter," Bell (1964:195). λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

1 Introduction

#**0.** NB: To facilitate discussion, all paragraphs and equations are numbered. Further, taking maths to be the best logic, we like the maths to do most of the talking.

#**1.** Bound by commonsense local realism (CLR), the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively), we refute Bell's theorem and eliminate the need for 't Hooft's superdeterminism. Our approach thus differs diametrically from 't Hooft's, who "did not refute Bell's theorem but by-passed it by accepting superdeterminism," after G 't Hooft (2014, pers. comm., 1 July).

#**2.** In the context of Bell (1964) (available online, with other essays; see References), let Alice and Bob be two independent experimenters, adequately separated. And let us assume that, in the absence of communication between them, Bob's free-will cannot possibly be correlated with Alice's free-will, nor with whatever the source of the particles does. And vice-versa.

#**3**. 't Hooft (2014:139, etc.) claims that "this assumption, natural as it seems, must be false.

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> Only 'superdeterminism' can explain the correlations you need to reproduce the quantum result [associated with Bell (1964)]. I accept superdeterminism, and the apparent 'conspiracy' that ensues, by arguing that the conspiracy is not at all as strange as it seems. Neither Alice nor Bob can change the settings they decided about earlier, without modifying the behaviour of the source of the particles. That's because they can't change the wave function of the universe from an ontic state into a superposition of ontic states," after G 't Hooft (2014, pers. comm., 27 June). "My deterministic particles know long in advance in what direction an experimenter will hold the polarisation filter," after G 't Hooft (2014, pers. comm., 1 July).

#**4.** With CLR directly opposed to 't Hooft's views, and seeking to eliminate the need for superdeterminism in physics, we proceed as follows:

2 Analysis

#**5.** Re Bell (1964), let the unnumbered equations between Bell's (14)-(15) be (14a)-(14c). Let unit-vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ replace his $\vec{a}, \vec{b}, \vec{c}$. Let Z be shorthand for EPRB, the experiment that Bell (1964) considers. Let expectation $\langle A(\mathbf{a})B(\mathbf{b})\rangle_Z$ replace Bell's equivalent term $P(\vec{a}, \vec{b})$; etc. Let $P(. \mid Z)$ denote a probability conditioned on Z; etc.

#**6.** Further, when clarity requires, let primes ($'$) identify elements in Bob's domain. Then, under the conservation of total angular momentum in EPRB and (for later) in Aspect (2002), let $\lambda_i + \lambda_i' = 0$; ie, $\lambda_i' = -\lambda_i$ in the ith test. We therefore use the shorthand $(\lambda, \lambda') = (\lambda, -\lambda)$ for all λ-pairs in our analysis, allowing that no two pairs need be the same. That is, we allow that no two particle-pairs need be the same in our analysis.

> "In a complete physical theory of the type envisioned by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can be thought of as initial values of these variables at some suitable instant," Bell (1964:196).

#**7.** Finally, let's combine Bell's (1)-(3) and (12)-(13) in our terms:

$$A(\mathbf{a}, \lambda) = \pm 1 \equiv A^{\pm};\ B(\mathbf{b}, \lambda') = B(-\lambda, \mathbf{b}) = \pm 1 \equiv B^{\pm};\ \int d\lambda \rho(\lambda) = 1; \tag{1}$$

$$\langle A(\mathbf{a})B(\mathbf{b})\rangle_Z = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(-\lambda, \mathbf{b}) \neq -\mathbf{a}\cdot\mathbf{b}. \tag{2}$$

#**8.** (1) captures the vital CLR assumption, an assumption in full accord with Bell (1964:196) and Einstein (1949:85): the result B does not depend on the setting $\mathbf{a}$; nor A on $\mathbf{b}$.

#**9.** In that CHSH (1969) first coined the term "Bell's theorem" in the context of Bell 1964:(15), (2) is (in CLR terms), "Bell's second theorem" and "Bell's impossibility theorem" – see the line below Bell 1964:(3). Fortunately, as we'll show: CLR delivers Bell's hope (2004:167) for a simple constructive model of reality based on Einstein's local causality; echoing Bell's sentiment (2004:167) that 'what is proved by impossibility proofs is lack of imagination'.

#**10.** Now Watson (2014d) refutes Bell's 1964:(14b), the false equality which underpins Bell's 1964:(15) and the inequality in (2) above. And this result is consistent with Watson (2014a:7-8), which provides a functional refutation of (1)-(2) in the context of Mermin's (1990; 1990a) 3-particle GHZ-variant.

#**11.** However, to specifically address 't Hooft's reference to ontic states, we now take a different approach. Under CLR, we now show that the probability of any EPRB-style result — ie, we include the experiment with photons in Aspect (2002) — is solely determined by independent local factors (CLR base-factors) alone: one factor based solely on beables in Alice's domain, the other factor based solely on beables in Bob's domain.

#12. To that end, let s denote relevant spins: ie, $s = 1/2$ for the spin-$1/2$ particles in experiment Z (EPRB); $s = 1$ for the photons in experiment X (shorthand for the experiment in Aspect (2002)); $s = \frac{1}{2}$ or 1 for the particles in our unifying thought-experiment Q. That is, experiment Q reduces EPRB and Aspect (2002) to a single experiment in which CLR base-factors (see Appendix) deliver the generalized quantum mechanical expectation and relevant probabilities for Q. The expectations and relevant probabilities for Z and X then follow, as consequences of applying the appropriate spin s.

#13. Let $|A^+\rangle$ and $|A^-\rangle$ denote the ontic (observable) basis states for which Alice's outcomes are A^+ (ie, $A = +1$) and A^- (ie, $A = -1$) respectively, per (1); etc. Let a trigonometric argument like $(\mathbf{u},\mathbf{v})$ denote the angle between vectors $\mathbf{u}$ and $\mathbf{v}$.

#14. With the classical origin and nature of our base-factors addressed in the Appendix, we now show that CLR base-factors (34)-(35) represent the relevant physical reality in real terms. That is, the expectation of any experiment or the probability of any test outcome may be derived via a single rule: integrate over the product of the relevant conjugates.

#15. So, repeating (34)-(35) from the Appendix, here are the base-factors for experiment Q:

$$|\omega(\mathbf{a},\lambda)\rangle_Q \equiv (\sqrt{2}\cos^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})\,|A^+\rangle + (\sqrt{2}\sin^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})\,|A^-\rangle; \tag{3}$$

$$|\omega(\mathbf{b},-\lambda)\rangle_Q \equiv (\sqrt{2}\cos^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|B^+\rangle + (\sqrt{2}\sin^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|B^-\rangle. \tag{4}$$

#16. Further to (36)-(38), the coefficients of the composite state are also normalized to one:

$$|\Omega(A(\mathbf{a})B(\mathbf{b}))_Q \equiv \int_Q d\lambda\rho(\lambda)\,|\omega(\mathbf{a},\lambda)\rangle_Q\,|\omega(\mathbf{b},-\lambda)\rangle_Q \tag{5}$$

$$\begin{aligned}
= \int_0^{4\pi} \frac{d\lambda}{4\pi}[&(\sqrt{2}\cos^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})(\sqrt{2}\cos^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|A^+B^+\rangle \\
&+(\sqrt{2}\cos^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})(\sqrt{2}\sin^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|A^+B^-\rangle \\
&+(\sqrt{2}\sin^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})(\sqrt{2}\cos^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|A^-B^+\rangle \\
&+(\sqrt{2}\sin^2(s(\lambda,\mathbf{a})) \pm \tfrac{1}{2})(\sqrt{2}\sin^2(s(-\lambda,\mathbf{b})) \mp \tfrac{1}{2})\,|A^-B^-\rangle]
\end{aligned} \tag{6}$$

$$\begin{aligned}
&= \tfrac{1}{2}\cos^2(s(\pi + (\mathbf{a},\mathbf{b})))\,|A^+B^+\rangle + \tfrac{1}{2}\sin^2(s(\pi + (\mathbf{a},\mathbf{b})))\,|A^+B^-\rangle \\
&+ \tfrac{1}{2}\sin^2(s(\pi + (\mathbf{a},\mathbf{b})))\,|A^-B^+\rangle + \tfrac{1}{2}\cos^2(s(\pi + (\mathbf{a},\mathbf{b})))\,|A^-B^-\rangle.
\end{aligned} \tag{7}$$

$$\therefore \langle A(\mathbf{a})B(\mathbf{b})\rangle_Q \equiv P(A^+B^+ \mid Q) - P(A^+B^- \mid Q) - P(A^-B^+ \mid Q) + P(A^-B^- \mid Q) \tag{8}$$

$$= 2(\tfrac{1}{2})\cos^2(s(\pi + (\mathbf{a},\mathbf{b}))) - 2(\tfrac{1}{2})\sin^2(s(\pi + (\mathbf{a},\mathbf{b}))) = \cos(2s(\pi + (\mathbf{a},\mathbf{b}))). \tag{9}$$

$$\therefore \langle A(\mathbf{a})B(\mathbf{b})\rangle_Z = \langle A(\mathbf{a})B(\mathbf{b})\rangle_{Q,s=\frac{1}{2}} = \langle A(\mathbf{a})B(\mathbf{b})\rangle_{EPRB} = -\cos(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}; \tag{10}$$

$$\langle A(\mathbf{a})B(\mathbf{b})\rangle_X = \langle A(\mathbf{a})B(\mathbf{b})\rangle_{Q,s=1} = \langle A(\mathbf{a})B(\mathbf{b})\rangle_{Aspect\,(2002)} = \cos 2(\mathbf{a},\mathbf{b}).\ \blacksquare \tag{11}$$

#17. QED. In (10), Bell's second theorem, (2) above, is refuted via CLR's derivation of the correct EPRB expectation. Bell's first theorem, his 1964:(15), is consequently and independently refuted by substitutions based on (9). That is, for $s = \frac{1}{2}$ or $s = 1$:

$$\text{Bell 1964:(15)}\ : 1 + \langle A(\mathbf{b})B(\mathbf{c})\rangle_Q = 1 + \cos(2s(\pi + (\mathbf{b},\mathbf{c}))) \tag{12}$$

$$\geq |\cos(2s(\pi + (\mathbf{a}, \mathbf{b}))) - \cos(2s(\pi + (\mathbf{a}, \mathbf{c})))| = |\langle A(\mathbf{a})B(\mathbf{b})\rangle_Q - \langle A(\mathbf{a})B(\mathbf{c})\rangle_Q| \tag{13}$$

is absurd over 50% of the domain $0 \leq \theta \leq 2\pi$ if $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = \theta$ and $(\mathbf{a}, \mathbf{c}) = 2\theta$.

#18. Further: In (11), Aspect's (2002) expectation is correctly delivered. And in (3)-(4) and (6)-(7), 't Hooft's (2014) need for a superposition of ontic states is met. Then, comparing (8) with (7) and using (36):

$$P(A^+B^+ \mid Q) = \tfrac{1}{2}\cos^2(s(\pi + (\mathbf{a}, \mathbf{b}))) = P(A^+ \mid Q)P(B^+ \mid QA^+) = \tfrac{1}{2}P(B^+ \mid QA^+): \tag{14}$$

$$\therefore P(B^+ \mid QA^+) = \cos^2(s(\pi + (\mathbf{a}, \mathbf{b}))); \; P(B^- \mid QA^+) = \sin^2(s(\pi + (\mathbf{a}, \mathbf{b}))); \text{ etc.} \tag{15}$$

#19. So Alice's results are statistically dependent on Bob's results; and vice-versa. That is: (15) refutes Bell's many attempts to conflate locality, per (1) and (3)-(4), with statistical independence.

3 Objections answered

> **Objection 1:** "Without superdeterminism, the CHSH inequality is totally obvious," G 't Hooft (2014, pers. comm., 11 July).

#20. We counter by showing that the CHSH inequality is totally absurd. To this end, and in our terms, let Γ_{CHSH} denote the CHSH (1969) inequality per Peres 1993:(6.30), as defended in Mermin (2005). Let additional subscripts identify relevant experiments. Then:

$$\Gamma_{CHSH\,\&\,Aspect\,(2002)} = \Gamma_{CHSH-EPRB} \equiv |\langle AB\rangle + \langle BC\rangle + \langle CD\rangle - \langle DA\rangle| \leq 2. \tag{16}$$

#21. Let Γ_{CLR-Q} denote the equivalent CLR inequality that applies to experiment Q:

$$\Gamma_{CLR-Q} \equiv |\langle A(\mathbf{a})B(\mathbf{b})\rangle_Q + \langle B(\mathbf{b})C(\mathbf{c})\rangle_Q + \langle C(\mathbf{c})D(\mathbf{d})\rangle_Q - \langle D(\mathbf{d})A(\mathbf{a})\rangle_Q| \tag{17}$$

$$\begin{aligned} = |\, & P(A^+B^+ \mid Q) - P(A^+B^- \mid Q) - P(A^-B^+ \mid Q) + P(A^-B^- \mid Q) \\ & + P(B^+C^+ \mid Q) - P(B^+C^- \mid Q) - P(B^-C^+ \mid Q) + P(B^-C^- \mid Q) \\ & + P(C^+D^+ \mid Q) - P(C^+D^- \mid Q) - P(C^-D^+ \mid Q) + P(C^-D^- \mid Q) \\ & - P(D^+A^+ \mid Q) + P(D^+A^- \mid Q) + P(D^-A^+ \mid Q) - P(D^-A^- \mid Q)\,|\,. \end{aligned} \tag{18}$$

#22. Then, given the common-cause pairing of λ and $-\lambda$ in each probability function:

$$P(A^+B^+ \mid Q) = P(A^+ \mid Q)P(B^+ \mid QA^+); P(A^+B^- \mid Q) = P(A^+ \mid Q)P(B^- \mid QA^+); \text{ etc.} \tag{19}$$

#23. And with λ a random variable:

$$P(A^+ \mid Q) = P(B^+ \mid Q) = P(C^+ \mid Q) = P(D^+ \mid Q) = \tfrac{1}{2}. \tag{20}$$

#24. And from symmetry, comparing (8) with (7):

$$P(A^+B^+ \mid Q) = P(A^-B^- \mid Q); \; P(A^+B^- \mid Q) = P(A^-B^+ \mid Q); \text{ etc.} \tag{21}$$

#25. So (17)-(18) reduces to:

$$\begin{aligned} \Gamma_{CLR-Q} = |\, & P(A^+ \mid QB^+) - P(A^- \mid QB^+) + P(B^+ \mid QC^+) - P(B^- \mid QC^+) \\ & + P(C^+ \mid QD^+) - P(C^- \mid QD^+) + P(D^+ \mid QA^-) - P(D^- \mid QA^-)\,|\,. \end{aligned} \tag{22}$$

#26. Thus, using $P(A^+ \mid QB^+) + P(A^- \mid QB^+) = 1$, etc., then (13), (22) reduces to:

$$\Gamma_{CLR-Q} = 2\,|\,P(A^+ \mid QB^+) + P(B^+ \mid QC^+) + P(C^+ \mid QD^+) + P(D^+ \mid QA^-) - 2\,| \tag{23}$$

$$= 2\,|\cos^2(s(\pi+(\mathbf{a},\mathbf{b})))+\cos^2(s(\pi+(\mathbf{b},\mathbf{c})))+\cos^2(s(\pi+(\mathbf{c},\mathbf{d})))+\sin^2(s(\pi+(\mathbf{d},a)))-2|\,. \tag{24}$$

#27. In (24), let $(\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{c}) = (\mathbf{c},\mathbf{d}) = \theta$ and $(\mathbf{d},\mathbf{a}) = 3\theta$ respectively. Then (24), in full agreement with quantum mechanics and experiment, exceeds the CHSH limit of two — in (16) — over more than 75% of the domain $0 \leq \theta \leq 2\pi$. That is: (16), the CHSH inequality, is absurd over 75% of θ's domain; and obviously wrong.

> **Objection 2:** "You seem to be using quantum states whereas Bell's hidden variable hypothesis is that one should use classical hidden variables. The inequalities say that these are impossible," G 't Hooft (2014, pers. comm., 11 July).

#28. In that our ontic states deliver observables, our states are both classical and quantum. Further, the pairs $(\lambda, -\lambda)$ are equally classical and quantum, being existents/beables correlated by the conservation of total angular momentum. Further: any appeal to Bell's inequalities falls to the arguments given in #17 and #27; while any Bellian arguments to impossibilities fail under the same CLR-style analysis that led to (10) versus (2) above.

4 Conclusions

#29. As expected, our results continue to refute Bell's theorem to our total satisfaction: for all loopholes are closed under CLR. The physical significance of CLR's factor-analysis is evident in (9)-(11); for there we find the correct expectations for experiment Q, for EPRB, and for Aspect 2002:(6). Moreover, any probability $P(.\mid Q)$ can be derived from such factors.

#30. Based on (2) above, Bell's 'impossibility theorem' is doubly refuted, via (10)-(11), as CLR continues to deliver the correct quantum mechanical expectations for important experiments.

#31. That is: Based on the way Bell presents his case, the implicit assumption in Bell's 1964:(2) integral – see (2) above – is statistical independence, particle-pair by particle-pair. But the facts are otherwise: particle-pair by particle-pair, the results are causally independent per (1) and statistically dependent/correlated per (7). As shown, in full accord with our CLR mantra: Correlated tests (correlated by $(\mathbf{a},\mathbf{b})$) on correlated things (particle-pairs with $\lambda = -\lambda'$ in each pair) produce correlated results (A, B) and the appropriate expectation without mystery.

#32. We conclude that Bell's theorem is irrelevant to any serious physical theory. In particular, it should no longer be a constraint on 't Hooft's (2014) program, especially not at 't Hooft 2014:(8.22)-(8.23). Finally, reviewing paragraph #2 in the light of all our results, we conclude that Alice and Bob have sufficient free-will to complete any experiment to our CLR satisfaction. For, in refuting Bell's theorem, we eliminate the need for 't Hooft's superdeterminism in physics.

#33. So the story that began with Mermin (1988) continues. And thanks to viXra.org, there's http://vixra.org/abs/1405.0020: a preliminary draft that also meets Bell's (1990:10) expectation that relativity and quantum mechanics would be reconciled; ie, it too delivers Bell's hope (2004: 167) for a simple constructive locally-causal (CLR) model of reality like that above.

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6 Appendix: CLR base-factors

#**A.1.** To demonstrate the classical background to our CLR base-factors (identified by lower-case Greek letters), consider experiment C – C for Classical – the classical analog of experiment Q. That is, instead of Q's twinned-particles being correlated by the conservation of angular momentum via $(\lambda, \lambda') = (\lambda, -\lambda)$: C's twinned-particles are correlated by anti-parallel polarizations via $(\phi, \phi') \equiv (\phi, -\phi)$. Thus, from wholly classical considerations, including Malus' Law via its application to polarizations:

$$|\omega(\mathbf{a}, \phi)\rangle_C \equiv \cos^2(s(\phi, \mathbf{a}))\left|A^+\right\rangle + \sin^2(s(\phi, \mathbf{a}))\left|A^-\right\rangle; \tag{25}$$

$$|\omega(\mathbf{b}, -\phi)\rangle_C \equiv \cos^2(s(-\phi, \mathbf{b}))\left|B^+\right\rangle + (\sin(s(-\phi, \mathbf{b}))\left|B^-\right\rangle. \tag{26}$$

#**A.2.** Integrating over the product of relevant coefficients in (25)-(26), the normalized probabilities of C's composite states follow:

$$|\Omega(A(\mathbf{a})B(\mathbf{b})\rangle_C \equiv \int_C d\phi\rho(\phi)\,|\omega(\mathbf{a}, \phi)\rangle_C\,|\omega(\mathbf{b}, -\phi)\rangle_C \tag{27}$$

$$= \int_0^{2\pi} \frac{d\phi}{2\pi}[(\cos^2(s(\phi, \mathbf{a})))(\cos^2(s(-\phi, \mathbf{b})))\left|A^+B^+\right\rangle$$

$$+(\cos^2(s(\phi, \mathbf{a})))(\sin^2(s(-\phi, \mathbf{b})))\left|A^+B^-\right\rangle + (\sin^2(s(\phi, \mathbf{a})))(\cos^2(s(-\phi, \mathbf{b})))\left|A^-B^+\right\rangle$$

$$+ (\sin^2(s(\phi, \mathbf{a})))(\sin^2(s(-\phi, \mathbf{b})))\left|A^-B^-\right\rangle] \tag{28}$$

$$= \tfrac{1}{8}(1 + 2\cos^2(s(\pi + (\mathbf{a}, \mathbf{b}))))\left|A^+B^+\right\rangle + \tfrac{1}{8}(1 + 2\sin^2(s(\pi + (\mathbf{a}, \mathbf{b}))))\left|A^+B^-\right\rangle$$

$$+ \tfrac{1}{8}(1 + 2\sin^2(s(\pi + (\mathbf{a}, \mathbf{b}))))\left|A^-B^+\right\rangle + \tfrac{1}{8}(1 + 2\cos^2(s(\pi + (\mathbf{a}, \mathbf{b}))))\left|A^-B^-\right\rangle. \tag{29}$$

$$\therefore \langle A(\mathbf{a})B(\mathbf{b})\rangle_C \equiv P(A^+B^+ \mid C) - P(A^+B^- \mid C) - P(A^-B^+ \mid C) + P(A^-B^- \mid C) \tag{30}$$

$$= 2(\tfrac{1}{8})(1 + 2\cos^2(s(\pi + (\mathbf{a}, \mathbf{b})))) - 2(\tfrac{1}{8})(1 + 2\sin^2(s(\pi + (\mathbf{a}, \mathbf{b})))) = \tfrac{1}{2}\cos(2s(\pi + (\mathbf{a}, \mathbf{b}))). \tag{31}$$

$$\therefore \langle A(\mathbf{a})B(\mathbf{b})\rangle_Y \equiv \left\langle A(\mathbf{a})B(\mathbf{b})\,|\,C, s = \tfrac{1}{2}\right\rangle = -\tfrac{1}{2}\cos(\mathbf{a}, \mathbf{b}) = -\tfrac{1}{2}\mathbf{a}\cdot\mathbf{b}; \tag{32}$$

$$\langle A(\mathbf{a})B(\mathbf{b})\rangle_W \equiv \langle A(\mathbf{a})B(\mathbf{b})\,|\,C, s = 1\rangle = \tfrac{1}{2}\cos 2(\mathbf{a}, \mathbf{b}). \blacksquare \tag{33}$$

#**A.3.** In that (32) and (33) are one-half their quantum counterparts, and comparing (29) with (25)-(26), we conclude that the CLR base-factors for experiment Q are:

$$|\omega(\mathbf{a}, \lambda)\rangle_Q \equiv (\sqrt{2}\cos^2(s(\lambda, \mathbf{a})) \pm \tfrac{1}{2})\left|A^+\right\rangle + (\sqrt{2}\sin^2(s(\lambda, \mathbf{a})) \pm \tfrac{1}{2})\left|A^-\right\rangle; \tag{34}$$

$$|\omega(\mathbf{b}, -\lambda)\rangle_Q \equiv (\sqrt{2}\cos^2(s(-\lambda, \mathbf{b})) \mp \tfrac{1}{2})\left|B^+\right\rangle + (\sqrt{2}\sin^2(s(-\lambda, \mathbf{b})) \mp \tfrac{1}{2})\left|B^-\right\rangle; \tag{35}$$

noting that each $\pm$ sign creates two real (not complex/imaginary) conjugates.

#**A.4.** To derive relevant probabilities, normalized to one, we employ this CLR rule: integrate over the product of the relevant conjugates. Noting that this rule is compatible with integration over the absolute squares of quantum amplitudes (the product of complex conjugates), we next demonstrate the rule in the context of experiment Q:

$$P(A^+ \mid Q) = \int_0^{4\pi} \frac{d\lambda}{4\pi}(\sqrt{2}\cos^2(s(\lambda, \mathbf{a})) + \tfrac{1}{2})(\sqrt{2}\cos^2(s(\lambda, \mathbf{a})) - \tfrac{1}{2}) = \tfrac{1}{2}; \tag{36}$$

$$P(A^- \mid Q) = \int_0^{4\pi} \frac{d\lambda}{4\pi}(\sqrt{2}\sin^2(s(\lambda, \mathbf{a})) + \tfrac{1}{2})(\sqrt{2}\sin^2(s(\lambda, \mathbf{a})) - \tfrac{1}{2}) = \tfrac{1}{2};\ \text{etc.} \tag{37}$$

$$P(A^+A^- \mid Q) = \int_0^{4\pi} \frac{d\lambda}{4\pi}(\sqrt{2}\cos^2(s(\lambda,\mathbf{a})) + \tfrac{1}{2})(\sqrt{2}\sin^2(s(\lambda,\mathbf{a})) - \tfrac{1}{2}) = 0. \qquad (38)$$

#**A.5.** In the same way — ie, under the same rule: integrate over the product of the relevant conjugates — the normalized probabilities of Q's composite states follow at (5)-(9) above, as we return to the main text at #**15** above.

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