PLOTS OF CYCLE GRAPHS OF THE FINITE GROUPS UP TO ORDER 36

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Abstract. The cycle graphs for all finite abstracts groups up to order 37 are plotted, except the trivial ring graphs of the cyclic groups themselves.

1. Introduction

1.1. **Element Order.** The small finite groups are fundamentally defined by their multiplication table (Cayley table). An illustration of the structure is extracted from there in forms of the cycle graph of the group, which we define as usual: For each element g of the group G define its order o as the smallest integer for which the power g^o equals e, the unit element. The elements e, g, g^2,...,g^{o-1} build a cyclic subgroup of G.

Remark 1. *Since subgroup orders divide the group orders according to Lagrange's theorem, all o are divisors of the group order* [6, p. 11].

A natural representation of such a cyclic subgroup is a graph with a single cycle with nodes defined by e, g, g^2 up to g^{o-1} and edges between each pair of nodes where the power of g differs by one. The element g^{o-1} is joined by an edge with e to complete the cycle. The same cyclic subgroup could as well be defined by using not g but its inverse $g^{o-1} = g^{-1}$ as the generating element; this means one could walk along the cycle graph also in the opposite direction, and consequently the edges are left *undirected*.

1.2. **Cycle Graph.** The union of all these cyclic graphs defines in essence the cycle graph. The number of vertices equals the group order. The unit element e is a node shared by all the cycles (and there may be more nodes common to one or more cycles), so the cycle graph is *connected*. A cleanup by removing some edges completes the definition of the cycle graph:

- For elements g of order two the edge from e to g and the edge that returns from g to e is replaced by a single edge. These elements are represented by leave nodes in the graph rather than cycles; in consequence the graph has no multiple edges and is *simple*. For some groups all elements but the unit element have order two (Figures 1, 4, 11, 31, ...); their cycle graph is a star graph.
- Elements of some order o which is a composite number generate cyclic subgroups that have further subgroups of orders which are divisors of o.

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Remark 2. *The orders of these elements g^i for $1 \le i \le o$ can be read off row o of triangle A054531 in the Online Encyclopedia of Integer Sequences (OEIS)* [8]. *There are $\varphi(o)$ different choices of generators for these cyclic subgroups, where φ is Euler's totient function, and this is illustrated by the diagonal elements in the triangle A054522 in the OEIS* [7, Theor. 2.10].

By convention the powers of the elements of these proper cyclic subgroups of cyclic subgroups are not connected by edges; these diagonals in the cycle graph of the generating function of the super-group are not drawn.

- Where group elements are shared by different cycles because the generators g_i and g_j of these cycles create some common element $g_i^k = g_j^l$ for some $k, l > 1$, the edges of cycles may be colored to indicate which edges attached to that common element are members of the same cyclic path.

We further note that the cycles have been extracted with GAP [3] from the small groups library, so the enumeration of the elements is the same as in the small groups library [1]. We shall denote the groups as G_o^i where o is the group order and the index i the same as in the small groups library.

Remark 3. *The maximum upper index i is the number of groups of order o, 1,1,2,1,2,1,5,2,... for $o \ge 1$* [8, A000001].

Almost all of the cycle graphs in the sequel are demonstrated to be planar by moving around the vertices with the aid of `P.I.G.A.L.E` [2]. The two exceptions for which this did not succeed are G_{32}^{17} in Figure 83 and G_{36}^{14} in Figure 131.

1.3. **Cycle Type.** For each graph we also provide the cycle type $(2^{m_2}, 3^{m_3}, \ldots, o^{m_o}, \ldots)$ indicating the multiplicity m_o of cycles of length o in the cycle graph. The degree of the node representing the unit element e, and labeled by index 1 in the graphs, is $2\sum_c m_c - m_2$, because general cycles start at and return to the unit element, but the elements of order 2 are connected only by single edges to the unit element.

Pólya's cycle index computed by GAP is printed as a polynomial in the free variables x_i [4, 5].

1.4. **Cayley Subtables.** The construction of a group is obviously complete if the generators of the cyclic subgroups and their defining relations—as represented by the cycle graph—are known. If one is interested in reducing that generating set of elements defined by the union of the cycle generators further, some of the generators may in generally be disposed of if they can be written as products of other cycle generators. To foster this type of (weak form of) finding small generator sets, we also generate subtables of each group multiplication table in which rows and columns are only printed if their element is a generator of a cyclic subgroup.

A second application of these reduced multiplication tables is to clarify whether two topologically equivalent cycle graphs have different group structures.

Example 1. *With the aid of Table 18 and Figure 20 we might take g_2 and g_6 as generators of two of the cycles of G_{16}^3, so g_5, g_8, g_{11} and g_{16} are defined by powers of these. Then $g_2 g_6 = g_{10}$, and $g_6 g_2 = g_{15}$ define two of the generators of two involutions. $g_2 g_{10} = g_{13} = g_{12}^{-1}$ and $g_6 g_{15} = g_{14} = g_7^{-1}$ define two generators for the remaining 4-cycles. The remaining three involutions are generated by (i) $g_9 = g_9^{-1}$ generated by $g_2 g_9 = g_{12}$, (ii) $g_4 = g_4^{-1}$ by $g_2 g_4 = g_7$, and (iii) $g_3 = g_3^{-1}$ by $g_2 g_3 = g_6$. This shows that the two generators g_2 and g_6 suffice to generate G_{16}^3.*

	2	1
2	3	2
1	2	1

TABLE 1. Cayley subtable for G_4^1.

	2	3	4	1
2	1	4	3	2
3	4	1	2	3
4	3	2	1	4
1	2	3	4	1

TABLE 2. Cayley subtable for G_4^2.

2. Orders up to 5

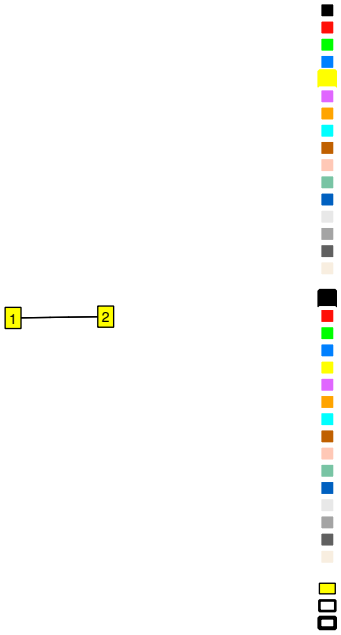

FIGURE 1. $G_2^1 \cong C_2 \cong S_2$, subgroup of S_2. (2^1). $(x_1^2 + x_2)/2$.

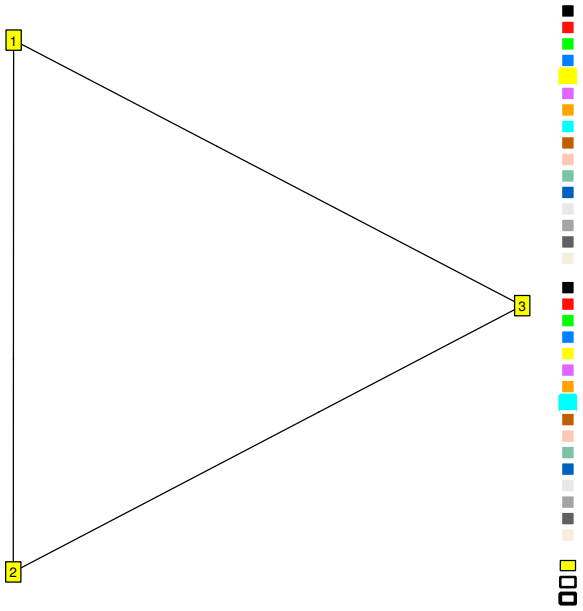

FIGURE 2. $G_3^1 \cong C_3$, subgroup of S_3. (3^1). $(x_1^3 + 2x_3)/3$.

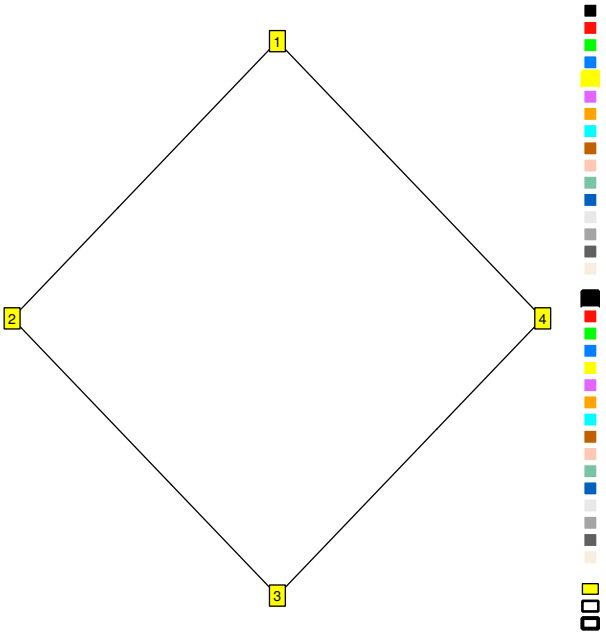

FIGURE 3. $G_4^1 \cong C_4$, subgroup of S_4. (4^1). $(x_1^4 + x_2^2 + 2x_4)/4$.

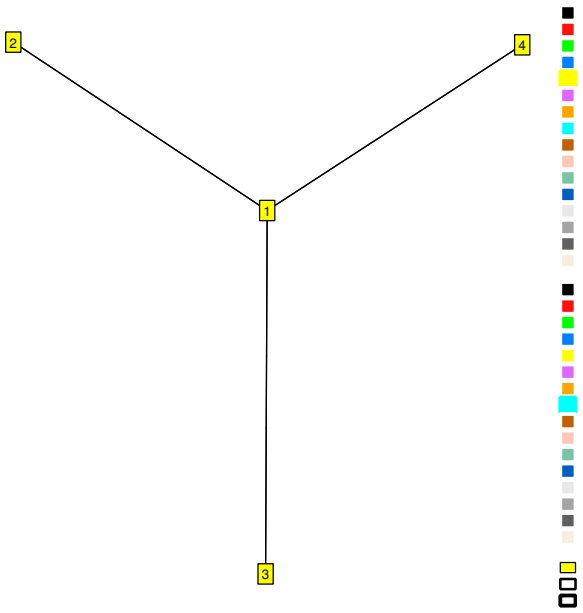

FIGURE 4. $G_4^2 \cong C_2 \times C_2$, subgroup of S_4. (2^3). $(x_1^4 + 2x_1^2x_2 + x_2^2)/4$.

	2	1
2	3	2
1	2	1

TABLE 3. Cayley subtable for G_5^1.

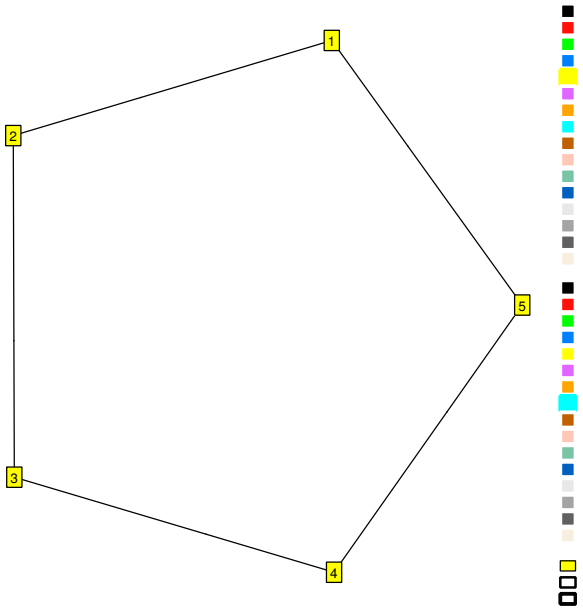

FIGURE 5. $G_5^1 \cong C_5$, subgroup of S_5. (5^1). $(x_1^5 + 4x_5)/5$.

	3	2	4	6	1
3	5	6	2	4	3
2	4	1	3	5	2
4	6	5	1	3	4
6	2	3	5	1	6
1	3	2	4	6	1

TABLE 4. Cayley subtable for G_6^1.

3. Order 6 and 7

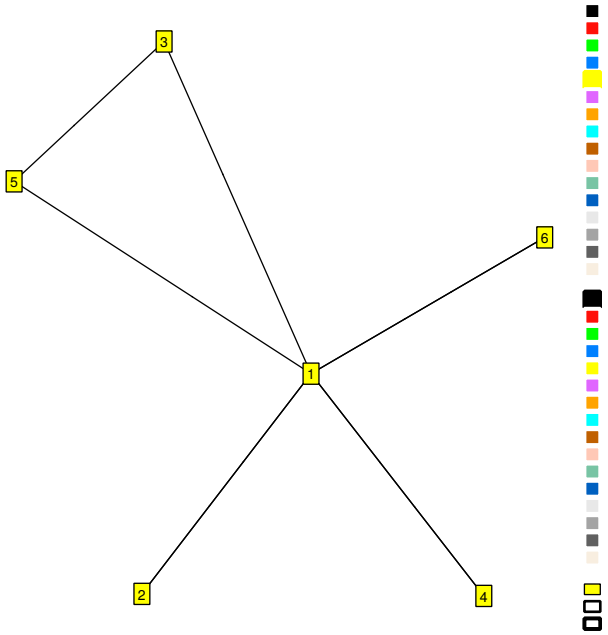

FIGURE 6. $G_6^1 \cong S_3$. $(2^3, 3^1)$. $(x_1^6 + 3x_2^3 + 2x_3^2)/6$.

	4	1
4	5	4
1	4	1

TABLE 5. Cayley subtable for G_6^2.

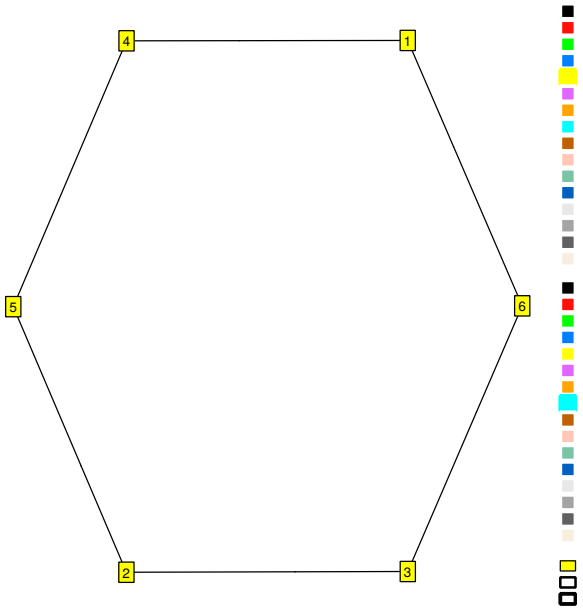

FIGURE 7. $G_6^2 \cong C_6$, subgroup of S_5. (6^1). $(x_1^5 + x_1^3x_2 + 2x_1^2x_3 + 2x_2x_3)/6$.

	2	5	3	7	1
2	4	7	5	8	2
5	7	4	2	6	5
3	5	2	1	4	3
7	8	6	4	1	7
1	2	5	3	7	1

TABLE 6. Cayley subtable for G_8^2.

The graph of the group $G_7^1 \cong C_7$ is the simple cyclic ring with 7 elements and not shown for that reason. The group is a subgroup of S_7 with cycle index $(x_1^7+6x_7)/7$.

4. Order 8 and 9

The graph of the group $G_8^1 \cong C_8$ is the simple cyclic ring with 8 elements and not shown for that reason. The group is a subgroup of S_8 with cycle index $(x_1^8 + x_2^4 + 2x_4^2 + 4x_8)/8$.

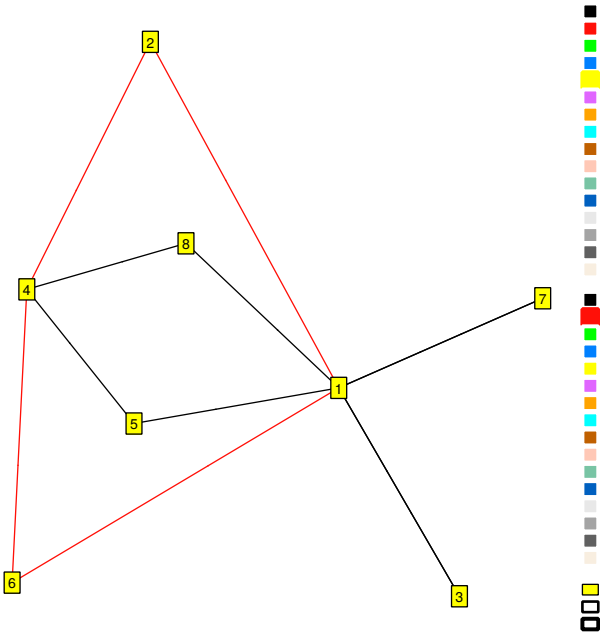

FIGURE 8. $G_8^2 \cong C_4 \times C_2$, subgroup of S_6. $(2^2, 4^2)$. $(x_1^6 + x_1^4x_2 + x_1^2x_2^2 + 2x_1^2x_4 + x_2^3 + 2x_2x_4)/8$.

	5	2	3	6	7	1
5	4	7	2	3	6	5
2	3	1	5	4	8	2
3	6	8	1	5	4	3
6	7	4	8	1	5	6
7	2	5	4	8	1	7
1	5	2	3	6	7	1

TABLE 7. Cayley subtable for G_8^3.

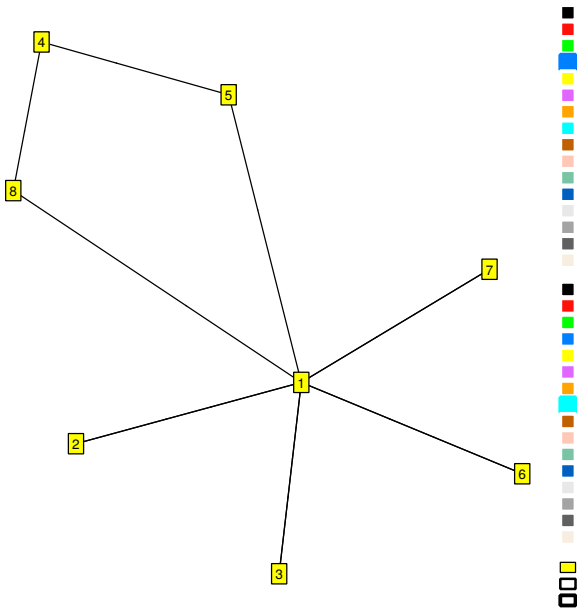

FIGURE 9. $G_8^3 \cong D_8$, subgroup of S_4. $(2^4, 4^1)$. $(x_1^8 + 5x_2^4 + 2x_4^2)/8$.

	2	3	5	1
2	4	5	7	2
3	8	4	2	3
5	3	6	4	5
1	2	3	5	1

TABLE 8. Cayley subtable for G_8^4.

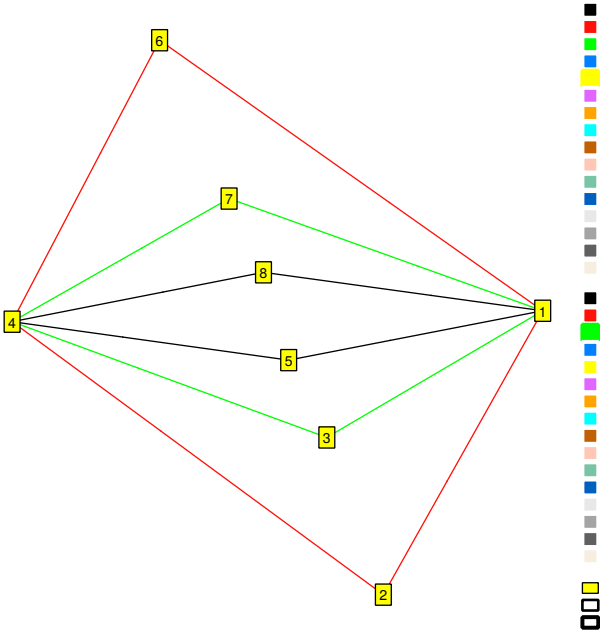

FIGURE 10. $G_8^4 \cong Q_8$, subgroup of S_8. (4^3). $(x_1^8 + x_2^4 + 6x_4^2)/8$.

	2	3	4	5	6	7	8	1
2	1	5	6	3	4	8	7	2
3	5	1	7	2	8	4	6	3
4	6	7	1	8	2	3	5	4
5	3	2	8	1	7	6	4	5
6	4	8	2	7	1	5	3	6
7	8	4	3	6	5	1	2	7
8	7	6	5	4	3	2	1	8
1	2	3	4	5	6	7	8	1

TABLE 9. Cayley subtable for G_8^5.

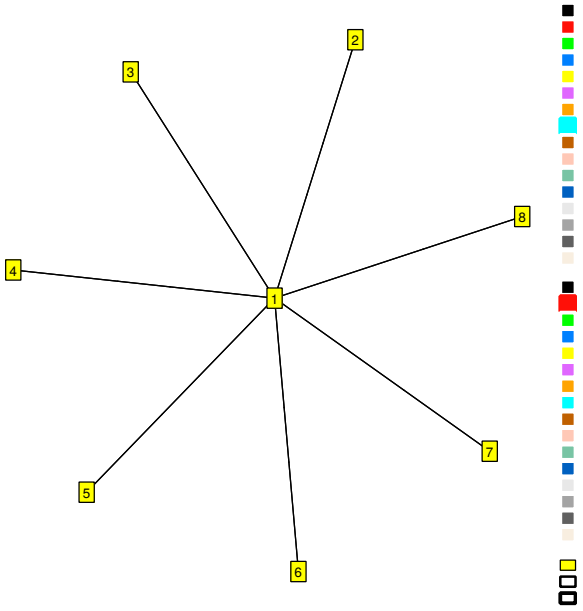

FIGURE 11. $G_8^5 \cong C_2 \times C_2 \times C_2$, subgroup of S_6. (2^7). $(x_1^6 + 3x_1^4x_2 + 3x_1^2x_2^2 + x_2^3)/8$.

	2	3	5	7	1
2	4	5	7	3	2
3	5	6	8	9	3
5	7	8	9	6	5
7	3	9	6	8	7
1	2	3	5	7	1

TABLE 10. Cayley subtable for G_9^2.

The graph of the group $G_9^1 \cong C_9$ is the simple cyclic ring with 9 vertices and not shown for that reason. The group is a subgroup of S_9 with cycle index $(x_1^9 + 2x_3^3 + 6x_9)/9$.

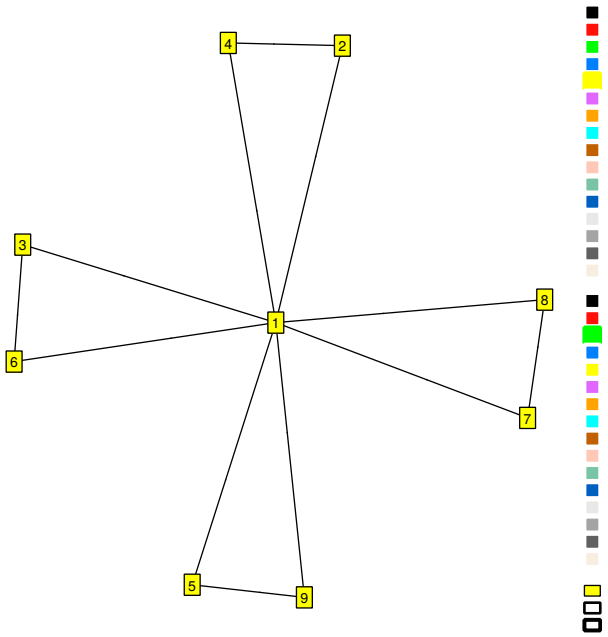

FIGURE 12. $G_9^2 \cong C_3 \times C_3$, subgroup of S_6. (3^4). $(x_1^6 + 4x_1^3x_3 + 4x_3^2)/9$.

	3	2	4	6	8	10	1
3	5	10	2	4	6	8	3
2	4	1	3	5	7	9	2
4	6	9	1	3	5	7	4
6	8	7	9	1	3	5	6
8	10	5	7	9	1	3	8
10	2	3	5	7	9	1	10
1	3	2	4	6	8	10	1

TABLE 11. Cayley subtable for G_{10}^1.

5. ORDER 10 . . . 12

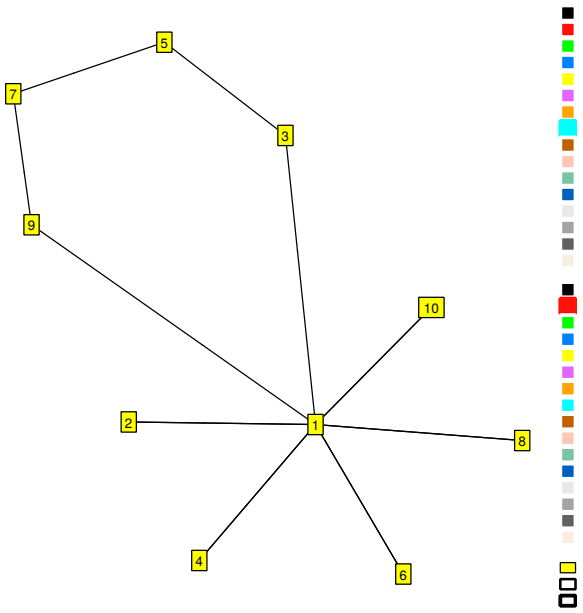

FIGURE 13. $G_{10}^1 \cong D_{10}$, subgroup of S_5. $(2^5, 5^1)$. $(x_1^{10} + 5x_2^5 + 4x_5^2)/10$.

	7	2	6	10	1
7	8	12	5	9	7
2	9	3	7	11	2
6	12	11	3	7	6
10	5	7	11	3	10
1	7	2	6	10	1

TABLE 12. Cayley subtable for G_{12}^1.

The graph of the group $G_{10}^2 \cong C_{10}$ is the simple cyclic ring with 10 elements and not shown for that reason. The group is a subgroup of S_7 with cycle index $(x_1^7 + x_1^5 x_2 + 4x_1^2 x_5 + 4x_2 x_5)/10$.

The graph of the group $G_{11}^1 \cong C_{11}$ is the simple cyclic ring with 11 elements and not shown for that reason. The group is a subgroup of S_{11} with cycle index $(x_1^{11} + 10x_{11})/11$.

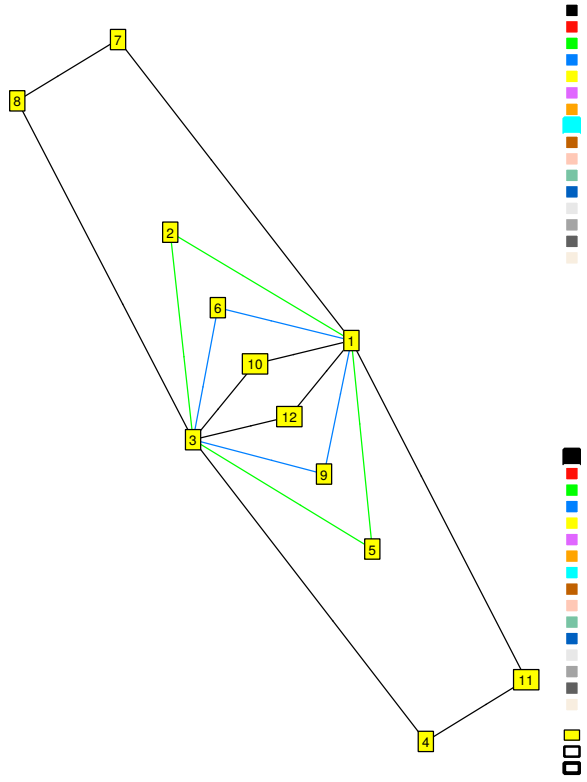

FIGURE 14. $G_{12}^1 \cong C_3 \rtimes C_4$, subgroup of S_7. $(4^3, 6^1)$. $(x_1^{12} + x_2^6 + 2x_3^4 + 6x_4^3 + 2x_6^2)/12$.

	2	6	7	10	3	4	8	1
2	5	9	10	4	6	7	11	2
6	10	12	5	3	2	11	7	6
7	12	10	9	8	11	2	6	7
10	8	4	3	11	12	5	9	10
3	7	11	2	9	1	8	4	3
4	11	7	6	12	8	1	3	4
8	6	2	11	5	4	3	1	8
1	2	6	7	10	3	4	8	1

TABLE 13. Cayley subtable for G_{12}^3.

The graph of the group $G_{12}^2 \cong C_{12}$ is the simple cyclic ring with 12 elements and not shown for that reason. The group is a subgroup of S_7 with cycle index $(x_1^7 + 2x_1^4x_3 + x_1^3x_2^2 + 2x_1^3x_4 + 2x_2^2x_3 + 4x_3x_4)/12$.

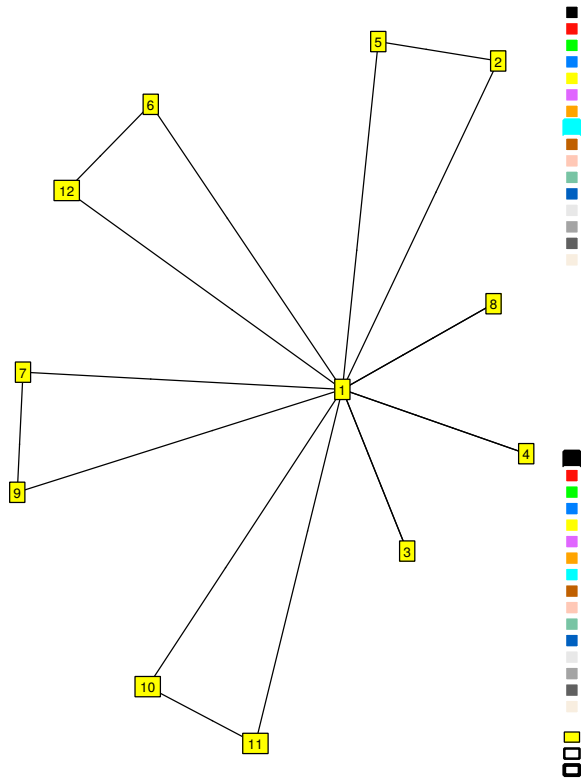

FIGURE 15. $G_{12}^3 \cong A_4$, subgroup of S_4. $(2^3, 3^4)$. $(x_1^{12} + 3x_2^6 + 8x_3^4)/12$.

	7	2	5	6	9	10	12	1
7	8	12	10	5	2	9	6	7
2	9	1	3	4	7	8	11	2
5	6	3	1	7	4	11	8	5
6	12	8	11	1	3	4	7	6
9	10	11	8	3	1	7	4	9
10	5	4	7	8	11	1	3	10
12	2	7	4	11	8	3	1	12
1	7	2	5	6	9	10	12	1

TABLE 14. Cayley subtable for G_{12}^4.

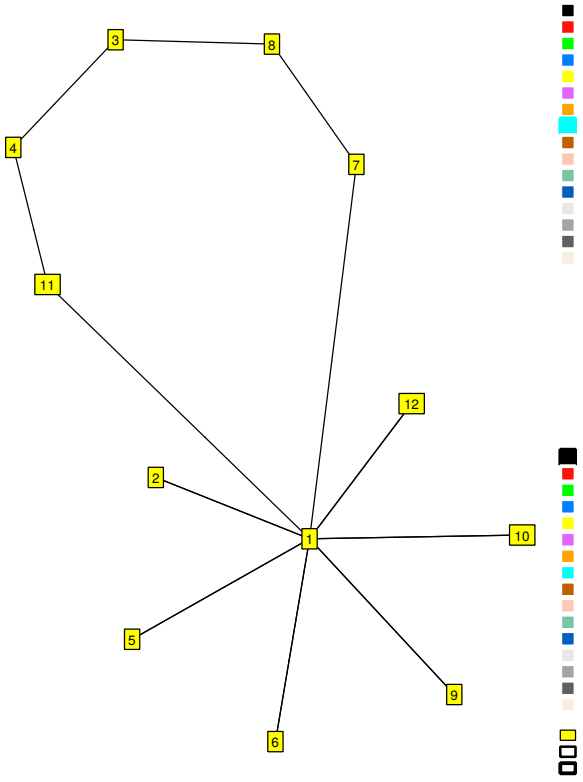

FIGURE 16. $G_{12}^4 \cong D_{12}$, subgroup of S_5. $(2^6, 6^1)$. $(x_1^{12} + 7x_2^6 + 2x_3^4 + 2x_6^2)/12$.

	6	7	9	1
6	8	12	11	6
7	12	8	10	7
9	11	10	8	9
1	6	7	9	1

TABLE 15. Cayley subtable for G_{12}^5.

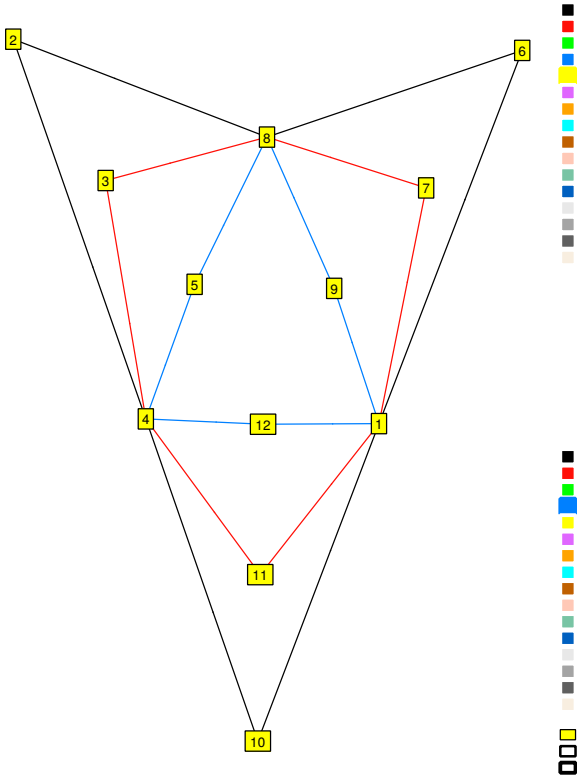

FIGURE 17. $G_{12}^5 \cong C_6 \times C_2$, subgroup of S_7. (6^3). $(x_1^7 + 2x_1^5x_2 + 2x_1^4x_3 + x_1^3x_2^2 + 4x_1^2x_2x_3 + 2x_2^2x_3)/12$.

The graph of the group $G_{13}^1 \cong C_{13}$ is the simple cyclic ring with 13 elements and not shown for that reason. The group is a subgroup of S_{13} with cycle index $(x_1^{13} + 12x_{13})/13$.

6. Order 14

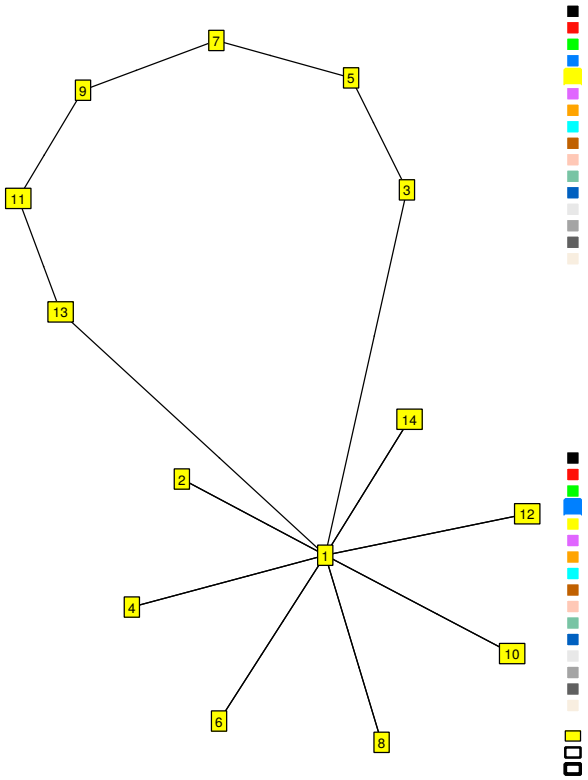

FIGURE 18. $G_{14}^1 \cong D_{14}$, subgroup of S_7. $(2^7, 7^1)$. $(x_1^{14} + 7x_2^7 + 6x_7^2)/14$.

	3	2	4	6	8	10	12	14	1
3	5	14	2	4	6	8	10	12	3
2	4	1	3	5	7	9	11	13	2
4	6	13	1	3	5	7	9	11	4
6	8	11	13	1	3	5	7	9	6
8	10	9	11	13	1	3	5	7	8
10	12	7	9	11	13	1	3	5	10
12	14	5	7	9	11	13	1	3	12
14	2	3	5	7	9	11	13	1	14
1	3	2	4	6	8	10	12	14	1

TABLE 16. Cayley subtable for G_{14}^1.

The graph of the group $G_{14}^2 \cong C_{14}$ is the simple cyclic ring with 14 elements and not shown for that reason. The group is a subgroup of S_9 with cycle index $(x_1^9 + x_1^7 x_2 + 6x_1^2 x_7 + 6x_2 x_7)/14$.

The graph of the group $G_{15}^1 \cong C_{15}$ is the simple cyclic ring with 15 elements and not shown for that reason. The group is a subgroup of S_8 with cycle index $(x_1^8 + 2x_1^5 x_3 + 4x_1^3 x_5 + 8x_3 x_5)/15$.

7. Order 16

The graph of the group $G_{16}^1 \cong C_{16}$ is the simple cyclic ring with 16 elements and not shown for that reason. The group is a subgroup of S_{16}, with cycle index $(x_1^{16} + x_2^8 + 2x_4^4 + 4x_8^2 + 8x_{16})/16$.

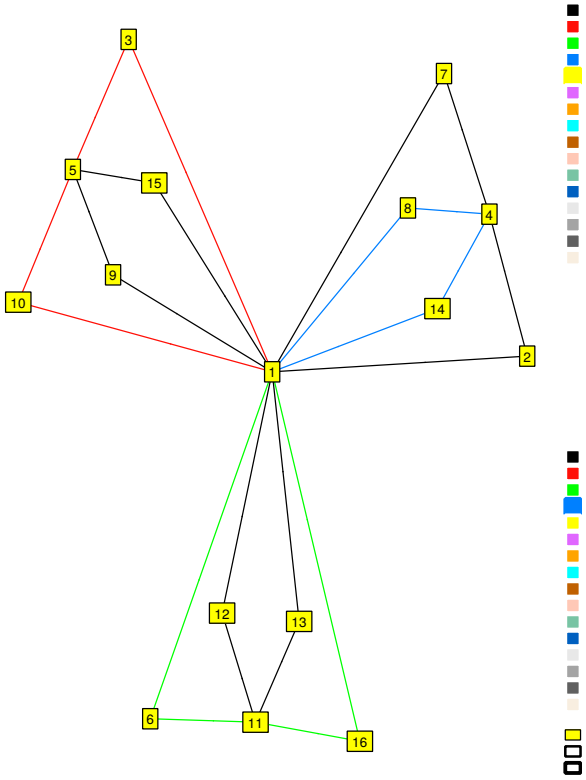

Figure 19. $G_{16}^2 \cong C_4 \times C_4$, subgroup of S_8. (4^6). $(x_1^8 + 2x_1^4 x_2^2 + 4x_1^4 x_4 + x_2^4 + 4x_2^2 x_4 + 4x_4^2)/16$.

	2	3	6	8	9	12	1
2	4	6	9	11	12	3	2
3	6	5	8	13	11	14	3
6	9	8	11	15	14	5	6
8	11	13	15	4	16	10	8
9	12	11	14	16	5	8	9
12	3	14	5	10	8	11	12
1	2	3	6	8	9	12	1

TABLE 17. Cayley subtable for G_{16}^2.

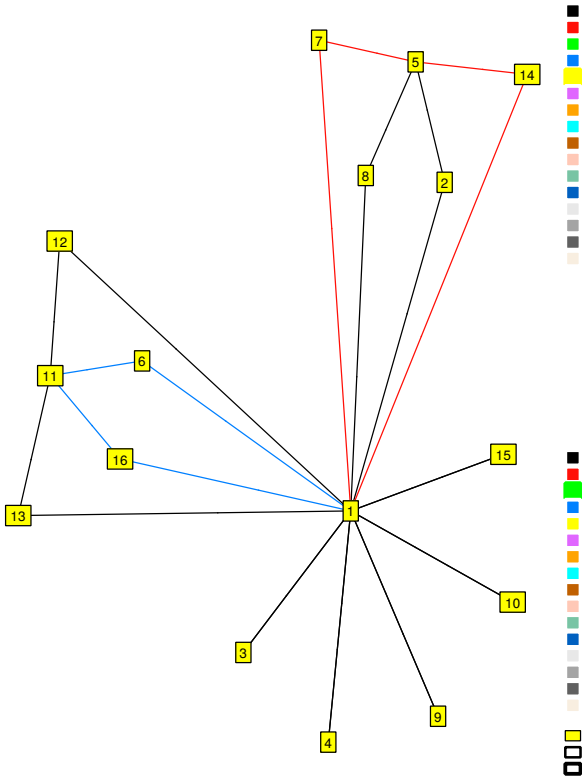

FIGURE 20. $G_{16}^3 \cong (C_4 \times C_2) \rtimes C_2$, subgroup of S_8. $(2^5, 4^4)$. $(x_1^{16} + 7x_2^8 + 8x_4^4)/16$.

	2	6	7	12	3	4	9	10	15	1
2	5	10	11	15	6	7	12	13	16	2
6	15	11	10	5	2	12	7	8	14	6
7	11	15	5	10	12	2	6	16	13	7
12	10	5	15	11	7	6	2	14	8	12
3	12	7	6	2	1	9	4	5	11	3
4	7	12	2	6	9	1	3	15	10	4
9	6	2	12	7	4	3	1	11	5	9
10	16	14	13	8	5	15	11	1	4	10
15	13	8	16	14	11	10	5	4	1	15
1	2	6	7	12	3	4	9	10	15	1

TABLE 18. Cayley subtable for G_{16}^3.

	2	3	6	7	10	12	11	1
2	5	6	10	11	13	15	14	2
3	12	4	2	6	11	7	15	3
6	15	7	5	10	14	11	16	6
7	11	12	15	5	16	10	8	7
10	16	11	8	13	4	14	9	10
12	10	2	11	15	8	5	13	12
11	14	15	16	8	9	13	1	11
1	2	3	6	7	10	12	11	1

TABLE 19. Cayley subtable for G_{16}^4.

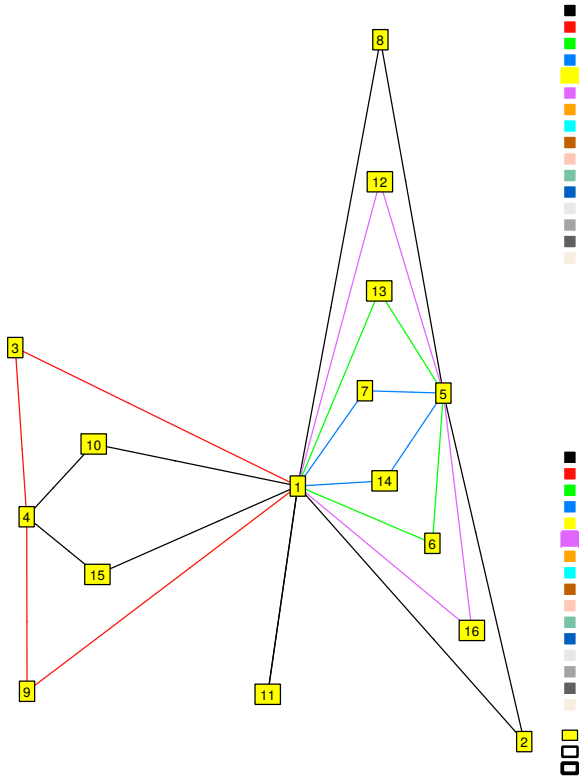

FIGURE 21. $G_{16}^4 \cong C_4 \rtimes C_4$, subgroup of S_8. $(2^1, 4^6)$. $(x_1^{16} + 3x_2^8 + 12x_4^4)/16$.

	2	6	9	3	10	1
2	4	9	12	6	13	2
6	9	4	7	2	8	6
9	12	7	5	4	11	9
3	6	2	4	1	5	3
10	13	8	11	5	1	10
1	2	6	9	3	10	1

TABLE 20. Cayley subtable for G_{16}^5.

The groups in Figures 22 and 23 show the first example of two groups with topologically identical (unlabeled) cycle graphs.

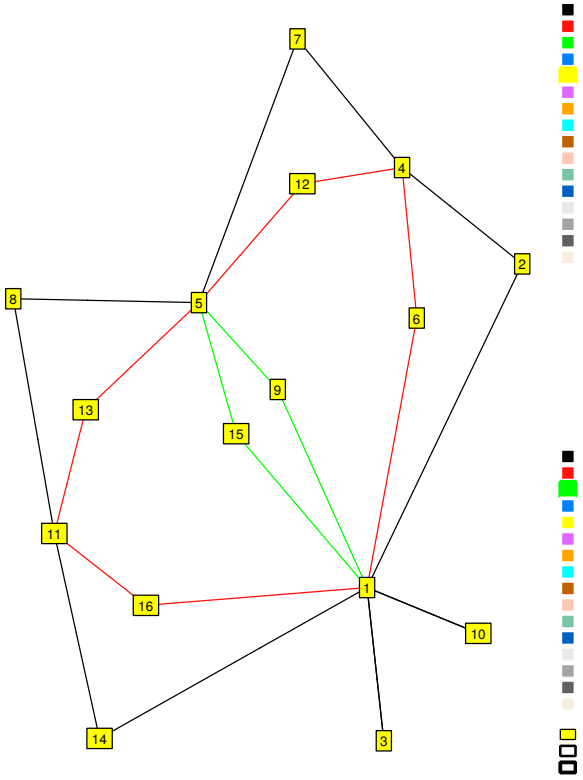

FIGURE 22. $G_{16}^5 \cong C_8 \times C_2$. $(2^2, 4^1, 8^2)$. $(x_1^{10} + x_1^8 x_2 + x_1^2 x_2^4 + x_2^5 + 2x_1^2 x_4^2 + 4x_1^2 x_8 + 2x_2 x_4^2 + 4x_2 x_8)/16$.

	2	6	9	3	10	1
2	4	9	12	6	13	2
6	15	11	7	2	8	6
9	16	14	5	4	11	9
3	13	8	4	1	5	3
10	6	2	11	5	1	10
1	2	6	9	3	10	1

TABLE 21. Cayley subtable for G_{16}^{6}.

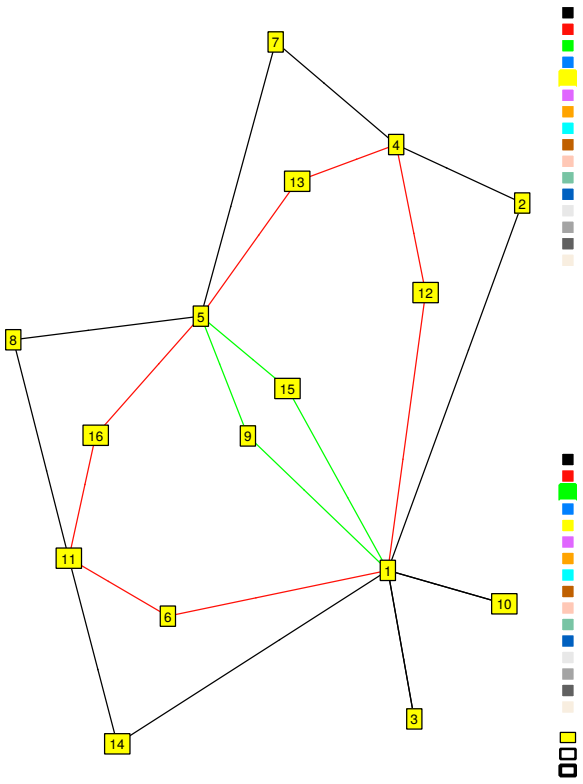

FIGURE 23. $G_{16}^{6} \cong C_8 \rtimes C_2$, subgroup of S_8. $(2^2, 4^1, 8^2)$. $(x_1^{16} + 3x_2^8 + 4x_4^4 + 8x_8^2)/16$.

	6	2	3	7	8	9	10	14	15	1
6	11	9	2	10	15	7	8	3	14	6
2	3	1	6	4	5	12	13	11	16	2
3	14	12	1	13	16	4	5	6	11	3
7	9	11	16	1	4	6	12	5	13	7
8	10	5	13	11	1	16	6	4	12	8
9	2	6	11	12	13	1	4	16	5	9
10	7	16	5	6	12	11	1	13	4	10
14	15	4	12	5	11	13	16	1	6	14
15	8	13	4	16	6	5	11	12	1	15
1	6	2	3	7	8	9	10	14	15	1

TABLE 22. Cayley subtable for G_{16}^{7}.

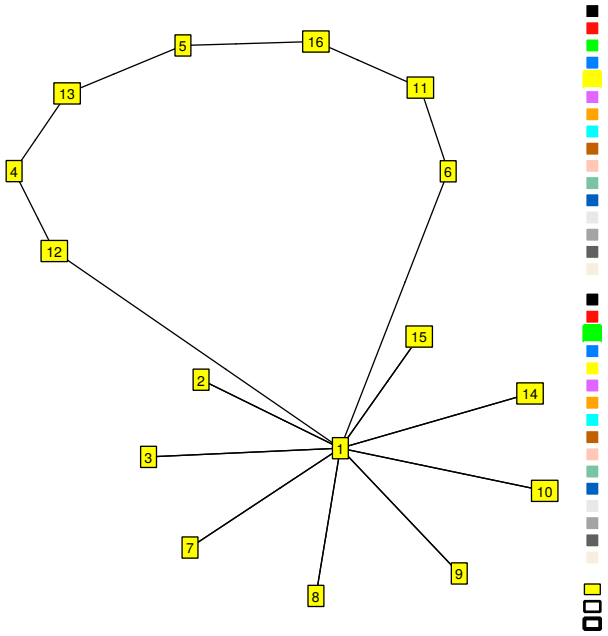

FIGURE 24. $G_{16}^{7} \cong D_{16}$, subgroup of S_8. $(2^8, 8^1)$. $(x_1^{16} + 9x_2^8 + 2x_4^4 + 4x_8^2)/16$.

	6	2	7	3	9	10	15	1
6	4	15	3	2	7	8	14	6
2	10	5	11	6	12	13	16	2
7	15	4	5	16	6	12	13	7
3	14	12	13	1	4	5	11	3
9	2	6	12	11	1	4	5	9
10	7	16	6	5	11	1	4	10
15	8	13	16	4	5	11	1	15
1	6	2	7	3	9	10	15	1

TABLE 23. Cayley subtable for G_{16}^{8}.

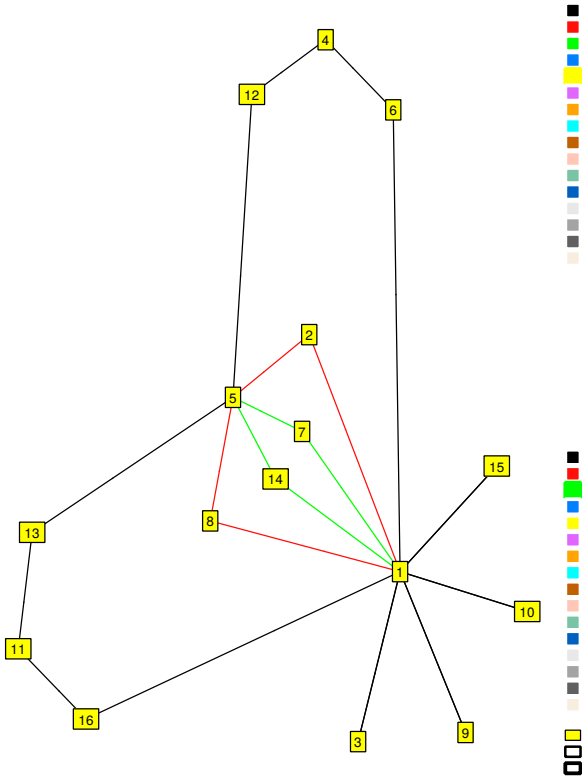

FIGURE 25. $G_{16}^{8} \cong QD_{16}$, subgroup of S_8. $(2^4, 4^2, 8^1)$. $(x_1^{16} + 5x_2^8 + 6x_4^4 + 4x_8^2)/16$.

	6	2	3	7	9	1
6	11	15	8	3	14	6
2	10	5	6	11	12	2
3	7	12	5	13	11	3
7	15	4	16	5	6	7
9	8	6	4	12	5	9
1	6	2	3	7	9	1

TABLE 24. Cayley subtable for G_{16}^{9}.

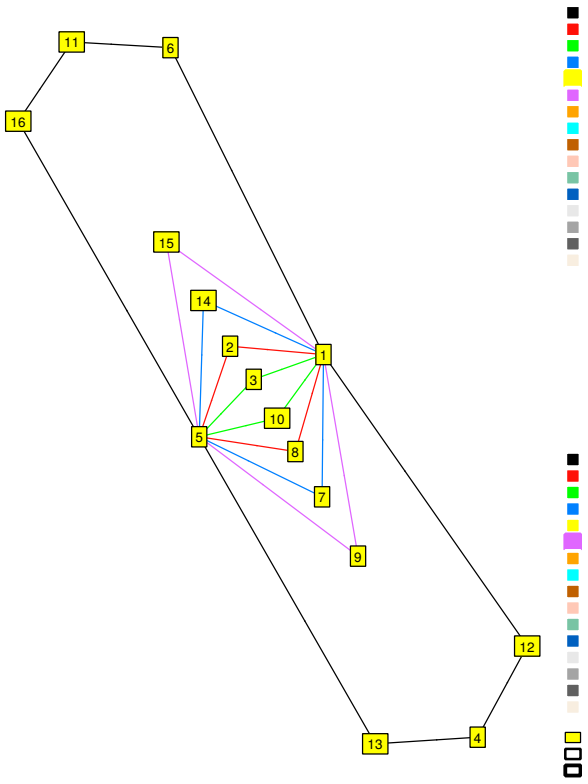

FIGURE 26. $G_{16}^{9} \cong Q_{16}$. $(4^4, 8^1)$. $(x_1^{16} + x_2^8 + 10x_4^4 + 4x_8^2)/16$.

	2	6	7	12	3	4	9	10	11	15	1
2	5	10	11	15	6	7	12	13	14	16	2
6	10	5	15	11	2	12	7	8	16	14	6
7	11	15	5	10	12	2	6	16	8	13	7
12	15	11	10	5	7	6	2	14	13	8	12
3	6	2	12	7	1	9	4	5	15	11	3
4	7	12	2	6	9	1	3	15	5	10	4
9	12	7	6	2	4	3	1	11	10	5	9
10	13	8	16	14	5	15	11	1	9	4	10
11	14	16	8	13	15	5	10	9	1	3	11
15	16	14	13	8	11	10	5	4	3	1	15
1	2	6	7	12	3	4	9	10	11	15	1

TABLE 25. Cayley subtable for G_{16}^{10}.

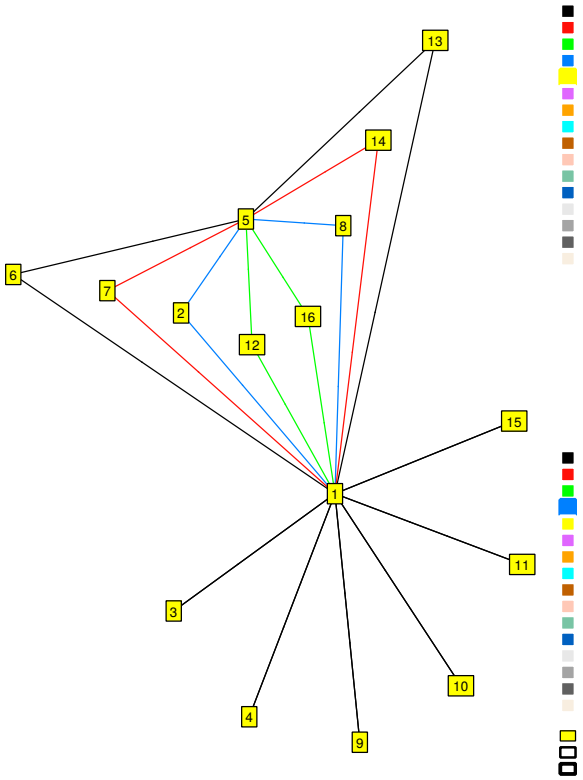

FIGURE 27. $G_{16}^{10} \cong C_4 \times C_2 \times C_2$, subgroup of S_8. $(2^6, 4^4)$. $(x_1^8 + 2x_1^6x_2 + 2x_1^4x_2^2 + 2x_1^4x_4 + 2x_1^2x_2^3 + 4x_1^2x_2x_4 + x_2^4 + 2x_2^2x_4)/16$.

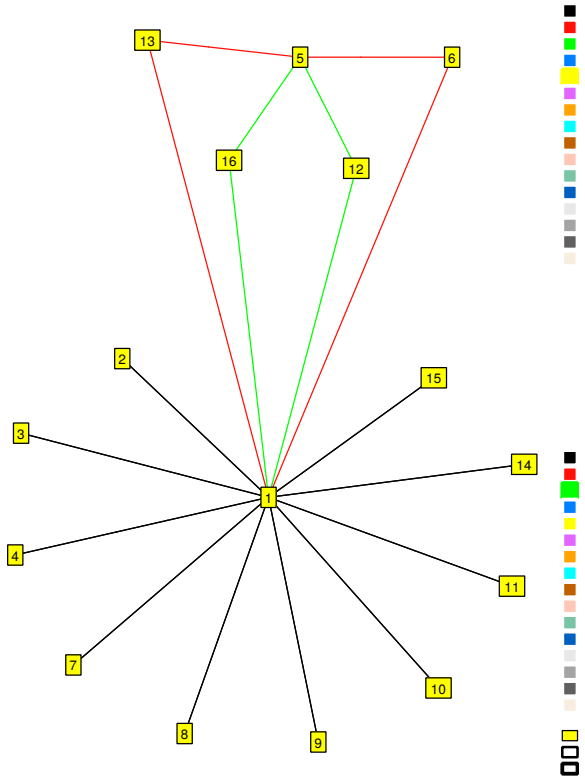

FIGURE 28. $G_{16}^{11} \cong C_2 \times D_8$, subgroup of S_6. $(2^{10}, 4^2)$. $(x_1^{16} + 11x_2^8 + 4x_4^4)/16$.

	6	12	2	3	4	7	8	9	10	11	14	15	1
6	5	11	10	2	12	15	3	7	8	16	9	14	6
12	11	5	15	7	6	10	9	2	14	13	3	8	12
2	3	9	1	6	7	4	5	12	13	14	11	16	2
3	8	14	13	1	9	16	6	4	5	15	12	11	3
4	12	6	7	9	1	2	14	3	15	5	8	10	4
7	9	3	4	12	2	1	11	6	16	8	5	13	7
8	10	15	5	13	14	11	1	16	6	7	4	12	8
9	14	8	16	4	3	13	12	1	11	10	6	5	9
10	2	7	6	5	15	12	13	11	1	9	16	4	10
11	16	13	14	15	5	8	7	10	9	1	2	3	11
14	15	10	11	16	8	5	4	13	12	2	1	6	14
15	7	2	12	11	10	6	16	5	4	3	13	1	15
1	6	12	2	3	4	7	8	9	10	11	14	15	1

TABLE 26. Cayley subtable for G_{16}^{11}.

	2	3	6	7	9	12	4	11	1
2	5	6	10	11	12	15	7	14	2
3	13	5	2	16	11	7	9	15	3
6	3	8	5	9	14	11	12	16	6
7	11	12	15	5	6	10	2	8	7
9	16	11	7	13	5	2	3	10	9
12	9	14	11	3	8	5	6	13	12
4	7	9	12	2	3	6	1	5	4
11	14	15	16	8	10	13	5	1	11
1	2	3	6	7	9	12	4	11	1

TABLE 27. Cayley subtable for G_{16}^{12}.

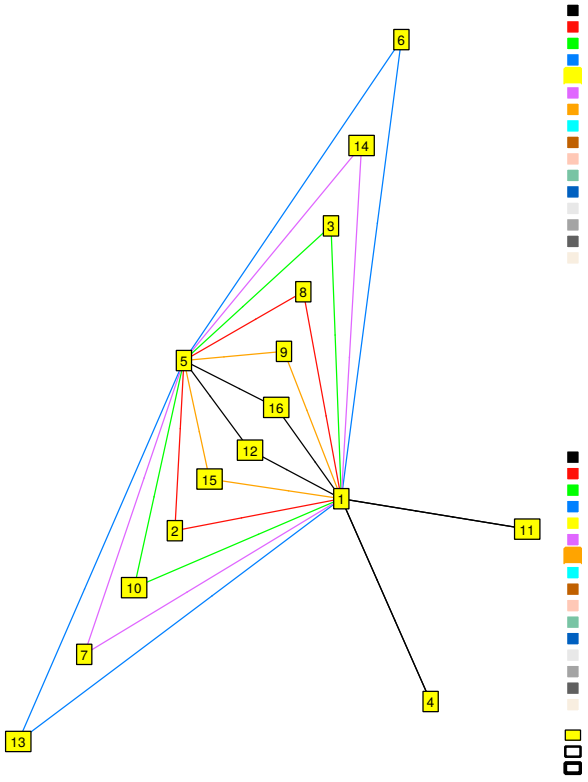

FIGURE 29. $G_{16}^{12} \cong C_2 \times Q_8$, $(2^2, 4^6)$. $(x_1^{16} + 3x_2^8 + 12x_4^4)/16$.

	4	6	7	9	2	3	8	10	12	16	1
4	5	12	8	10	7	9	14	15	13	6	4
6	12	5	15	7	10	2	3	8	11	4	6
7	8	9	5	13	4	12	11	16	10	3	7
9	10	14	6	5	16	4	12	11	2	8	9
2	7	3	4	12	1	6	5	13	9	15	2
3	9	8	16	4	13	1	6	5	14	7	3
8	14	10	11	16	5	13	1	6	15	9	8
10	15	2	12	11	6	5	13	1	7	14	10
12	13	11	3	8	15	7	9	14	1	5	12
16	6	4	10	2	9	14	15	7	5	1	16
1	4	6	7	9	2	3	8	10	12	16	1

TABLE 28. Cayley subtable for G_{16}^{13}.

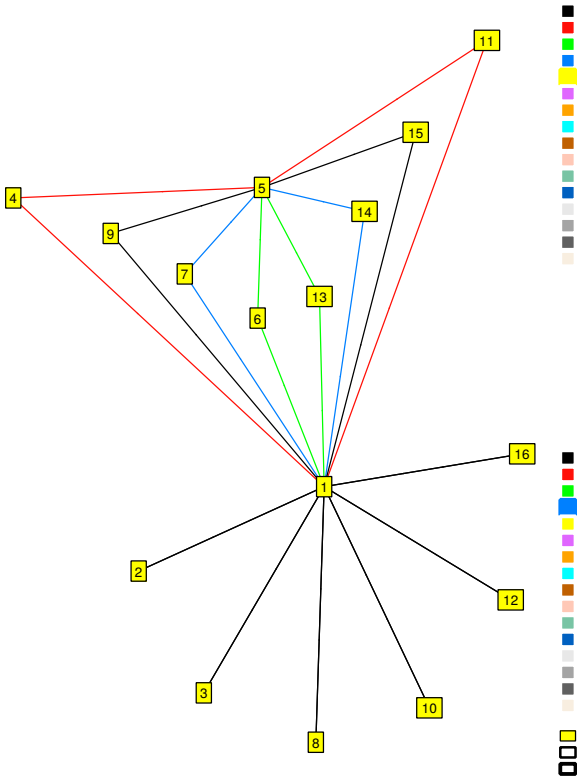

FIGURE 30. $G_{16}^{13} \cong (C_4 \times C_2) \rtimes C_2$, subgroup of S_8. $(2^6, 4^4)$. $(x_1^{16} + 7x_2^8 + 8x_4^4)/16$.

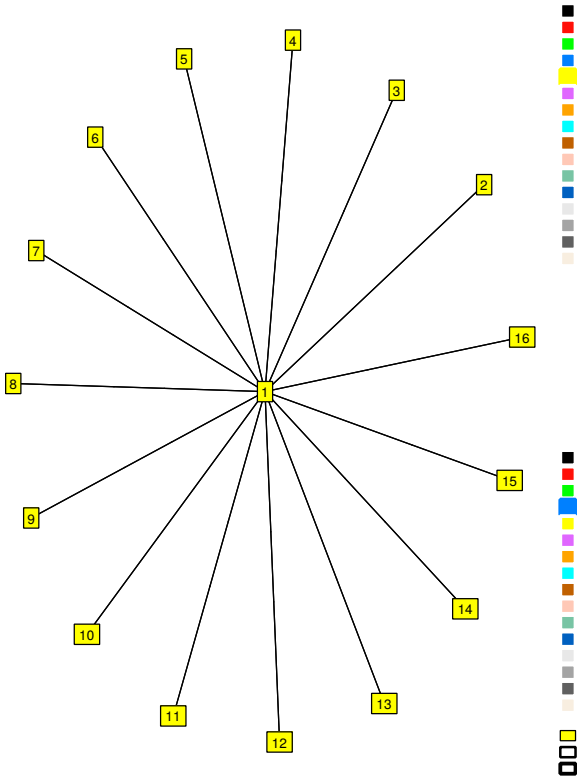

FIGURE 31. $G_{16}^{14} \cong C_2 \times C_2 \times C_2 \times C_2$, subgroup of S_8. (2^{15}). $(x_1^8 + 4x_1^6x_2 + 6x_1^4x_2^2 + 4x_1^2x_2^3 + x_2^4)/16$.

The graph of the group $G_{17}^{1} \cong C_{17}$ is the simple cyclic ring with 17 elements and not shown for that reason. The cycle index is $(x_1^{17} + 16x_{17})/17$.

8. ORDER 18

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1
2	1	6	7	8	3	4	5	12	13	14	9	10	11	16	15	2
3	6	1	9	10	2	12	13	4	5	15	7	8	16	11	14	3
4	7	9	1	11	12	2	14	3	15	5	6	16	8	10	13	4
5	8	10	11	1	13	14	2	15	3	4	16	6	7	9	12	5
6	3	2	12	13	1	9	10	7	8	16	4	5	15	14	11	6
7	4	12	2	14	9	1	11	6	16	8	3	15	5	13	10	7
8	5	13	14	2	10	11	1	16	6	7	15	3	4	12	9	8
9	12	4	3	15	7	6	16	1	11	10	2	14	13	5	8	9
10	13	5	15	3	8	16	6	11	1	9	14	2	12	4	7	10
11	14	15	5	4	16	8	7	10	9	1	13	12	2	3	6	11
12	9	7	6	16	4	3	15	2	14	13	1	11	10	8	5	12
13	10	8	16	6	5	15	3	14	2	12	11	1	9	7	4	13
14	11	16	8	7	15	5	4	13	12	2	10	9	1	6	3	14
15	16	11	10	9	14	13	12	5	4	3	8	7	6	1	2	15
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	16
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1

TABLE 29. Cayley subtable for G_{16}^{14}.

	3	2	5	6	10	11	12	15	16	18	1
3	7	18	2	10	5	6	15	11	12	16	3
2	5	1	3	4	7	8	9	13	14	17	2
5	10	17	1	7	3	4	13	8	9	14	5
6	11	9	14	1	17	3	4	7	8	13	6
10	6	14	17	3	1	7	8	4	13	9	10
11	15	13	9	17	14	1	7	3	4	8	11
12	16	4	8	9	13	14	1	17	3	7	12
15	12	8	13	14	9	17	3	1	7	4	15
16	18	7	4	13	8	9	17	14	1	3	16
18	2	3	7	8	4	13	14	9	17	1	18
1	3	2	5	6	10	11	12	15	16	18	1

TABLE 30. Cayley subtable for G_{18}^{1}.

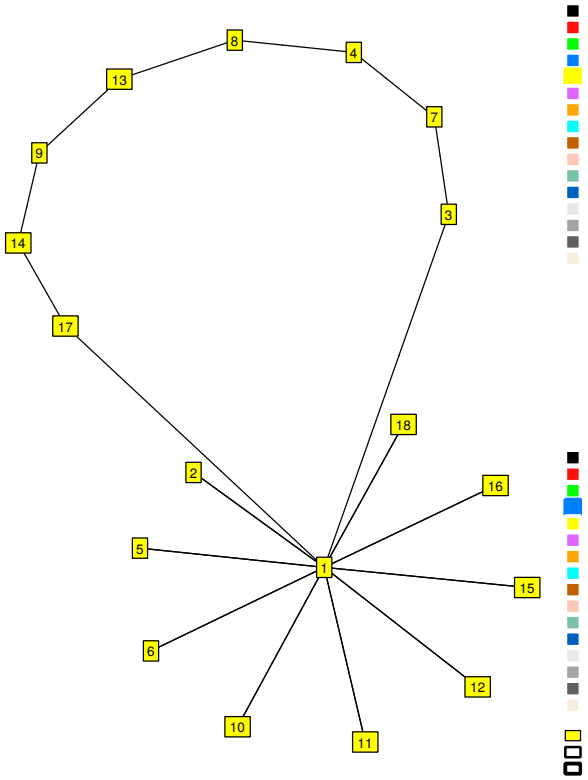

FIGURE 32. $G_{18}^1 \cong D_{18}$, subgroup of S_9. $(2^9, 9^1)$. $(x_1^{18} + 9x_2^9 + 2x_3^6 + 6x_9^2)/18$.

	5	11	16	4	8	13	1
5	7	13	17	11	15	6	5
11	17	7	13	16	18	12	11
16	13	17	7	5	10	2	16
4	16	5	11	9	14	17	4
8	18	10	15	14	17	9	8
13	12	2	6	17	9	14	13
1	5	11	16	4	8	13	1

TABLE 31. Cayley subtable for G_{18}^3.

The graph of the group $G_{18}^2 \cong C_{18}$ is the simple cyclic ring with 18 elements and not shown for that reason. The group is a subgroup of S_{11} with cycle index $(x_1^{11} + x_1^9 x_2 + 2x_1^2 x_3^3 + 2x_2 x_3^3 + 6x_1^2 x_9 + 6x_2 x_9)/18$.

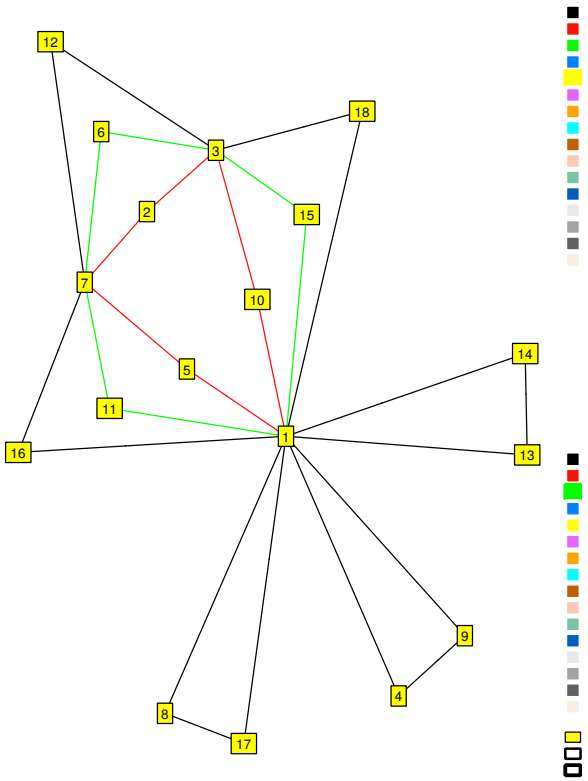

FIGURE 33. $G_{18}^3 \cong C_3 \times S_3$, subgroup of S_6. $(3^3, 6^3)$. $(x_1^{18} + 3x_2^9 + 8x_3^6 + 6x_6^3)/18$.

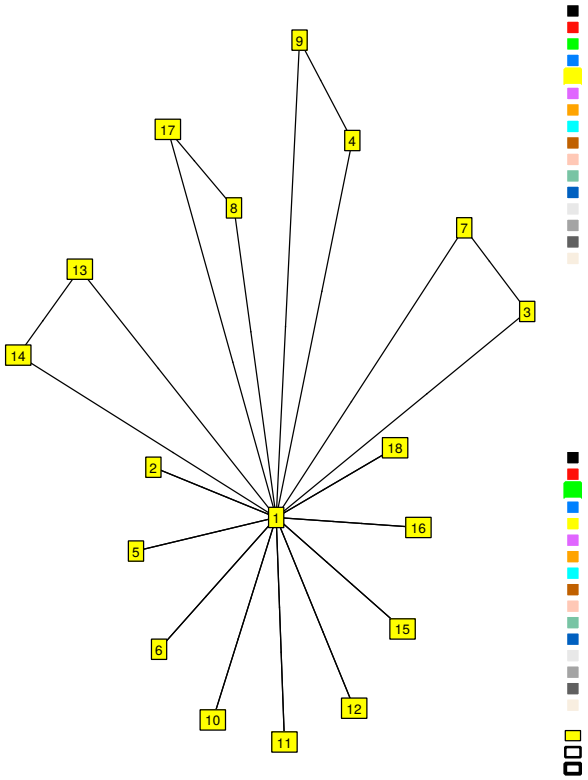

FIGURE 34. $G_{18}^4 \cong (C_3 \times C_3) \rtimes C_2$, subgroup of S_6. $(2^9, 3^4)$. $(x_1^{18} + 9x_2^9 + 8x_3^6)/18$.

	3	4	8	13	2	5	6	10	11	12	15	16	18	1
3	7	8	13	4	10	2	15	5	6	18	11	12	16	3
4	8	9	14	17	12	16	2	18	5	6	10	11	15	4
8	13	14	17	9	18	12	10	16	2	15	5	6	11	8
13	4	17	9	14	16	18	5	12	10	11	2	15	6	13
2	5	6	11	15	1	3	4	7	8	9	13	14	17	2
5	10	11	15	6	7	1	13	3	4	17	8	9	14	5
6	11	12	16	18	9	14	1	17	3	4	7	8	13	6
10	2	15	6	11	3	7	8	1	13	14	4	17	9	10
11	15	16	18	12	17	9	7	14	1	13	3	4	8	11
12	16	2	5	10	4	8	9	13	14	1	17	3	7	12
15	6	18	12	16	14	17	3	9	7	8	1	13	4	15
16	18	5	10	2	13	4	17	8	9	7	14	1	3	16
18	12	10	2	5	8	13	14	4	17	3	9	7	1	18
1	3	4	8	13	2	5	6	10	11	12	15	16	18	1

TABLE 32. Cayley subtable for G_{18}^4.

	5	6	11	15	1
5	7	8	13	4	5
6	8	9	14	17	6
11	13	14	17	9	11
15	4	17	9	14	15
1	5	6	11	15	1

TABLE 33. Cayley subtable for G_{18}^5.

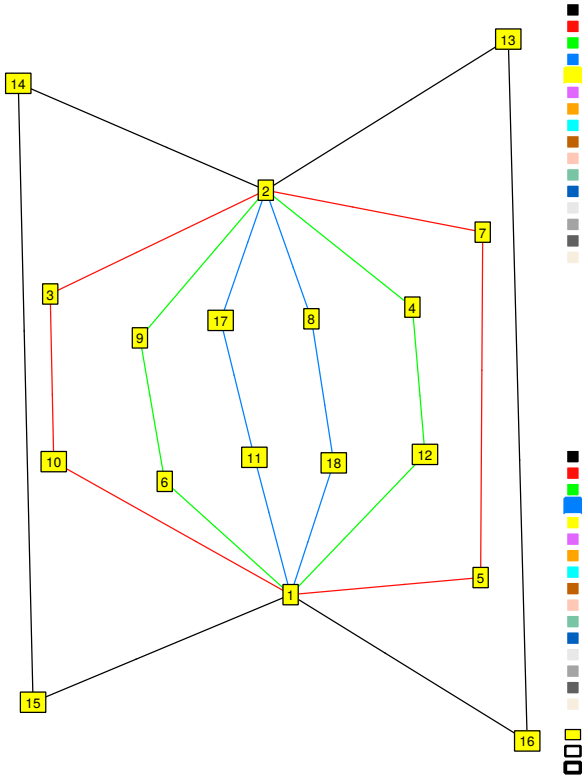

FIGURE 35. $G_{18}^5 \cong C_6 \times C_3$, subgroup of S_8. (6^4). $(x_1^8 + x_1^6x_2 + 4x_1^5x_3 + 4x_1^3x_2x_3 + 4x_1^2x_3^2 + 4x_2x_3^2)/18$.

	7	2	6	10	14	18	1
7	8	20	5	9	13	17	7
2	9	3	7	11	15	19	2
6	13	19	3	7	11	15	6
10	17	15	19	3	7	11	10
14	20	11	15	19	3	7	14
18	5	7	11	15	19	3	18
1	7	2	6	10	14	18	1

TABLE 34. Cayley subtable for G_{20}^1.

The graph of the group $G_{19}^1 \cong C_{19}$ is the simple cyclic ring with 19 elements and not shown for that reason. The group is a subgroup of S_{19} with cycle index $(x_1^{19} + 18x_{19})/19$.

9. ORDER 20

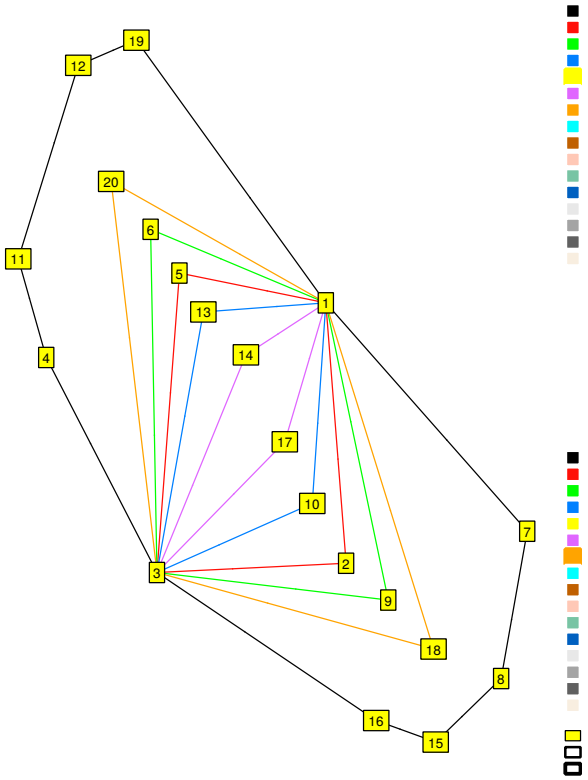

FIGURE 36. $G_{20}^1 \cong C_5 \rtimes C_4$, subgroup of S_9. $(4^5, 10^1)$. $(x_1^{20} + x_2^{10} + 10x_4^5 + 4x_5^4 + 4x_{10}^2)/20$.

	4	2	6	9	10	17	1
4	8	10	14	20	18	9	4
2	6	3	7	4	11	12	2
6	10	11	15	16	19	4	6
9	13	8	12	19	16	7	9
10	14	19	3	8	7	16	10
17	20	4	8	3	12	11	17
1	4	2	6	9	10	17	1

TABLE 35. Cayley subtable for G_{20}^3.

The graph of the group $G_{20}^2 \cong C_{20}$ is the simple cyclic ring with 20 elements and not shown for that reason. The group is a subgroup of S_9 with cycle index $(x_1^9 + x_1^5x_2^2 + 2x_1^5x_4 + 4x_1^4x_5 + 4x_2^2x_5 + 8x_4x_5)/20$.

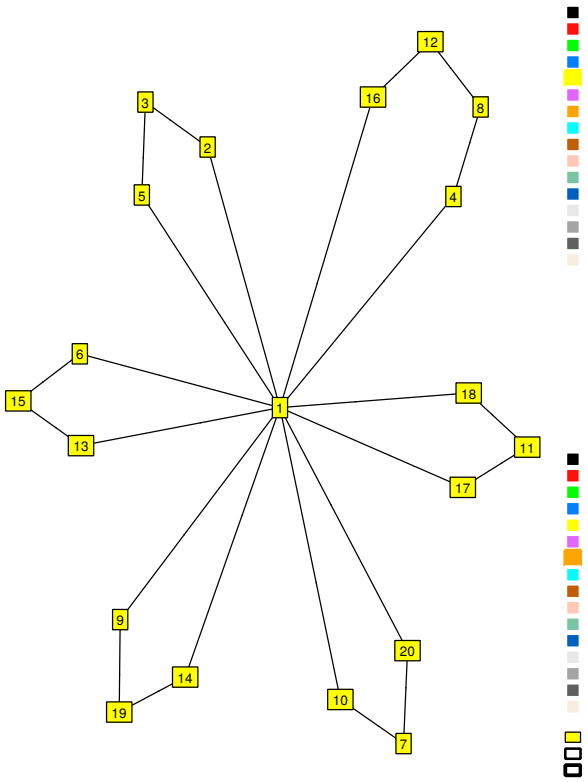

FIGURE 37. $G_{20}^3 \cong C_5 \rtimes C_4$, subgroup of S_5. $(4^5, 5^1)$. Note that the $\mathrm{Aut}(C_5)$, which is C_4, offers two nontrivial homomorphisms, so this semidirect product is not the same as the one generating G_{20}^1. $(x_1^{20} + 5x_2^{10} + 10x_4^5 + 4x_5^4)/20$.

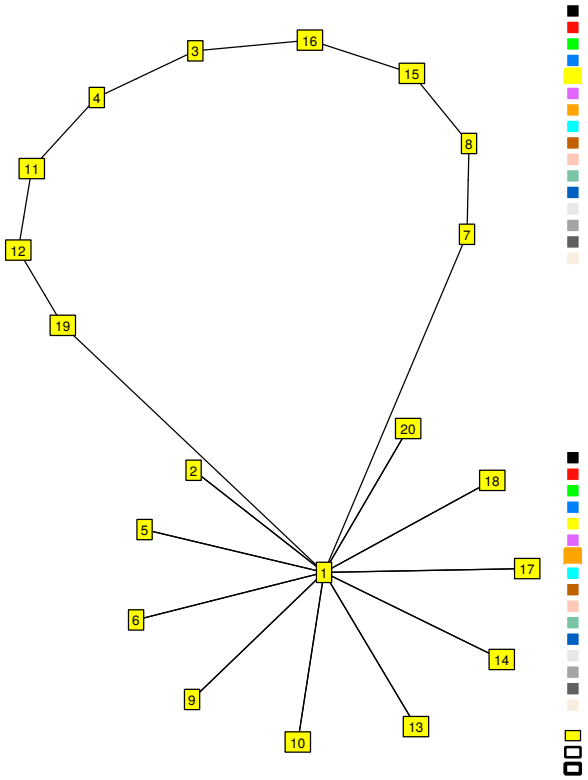

FIGURE 38. $G_{20}^4 \cong D_{20}$, subgroup of S_7. $(2^{10}, 10^1)$. $(x_1^{20} + 11x_2^{10} + 4x_5^4 + 4x_{10}^2)/20$.

	7	2	5	6	9	10	13	14	17	18	20	1
7	8	20	18	5	2	9	6	13	10	17	14	7
2	9	1	3	4	7	8	11	12	15	16	19	2
5	6	3	1	7	4	11	8	15	12	19	16	5
6	13	16	19	1	3	4	7	8	11	12	15	6
9	10	19	16	3	1	7	4	11	8	15	12	9
10	17	12	15	16	19	1	3	4	7	8	11	10
13	14	15	12	19	16	3	1	7	4	11	8	13
14	20	8	11	12	15	16	19	1	3	4	7	14
17	18	11	8	15	12	19	16	3	1	7	4	17
18	5	4	7	8	11	12	15	16	19	1	3	18
20	2	7	4	11	8	15	12	19	16	3	1	20
1	7	2	5	6	9	10	13	14	17	18	20	1

TABLE 36. Cayley subtable for G_{20}^4.

	6	7	9	1
6	8	13	11	6
7	13	8	10	7
9	11	10	8	9
1	6	7	9	1

TABLE 37. Cayley subtable for G_{20}^5.

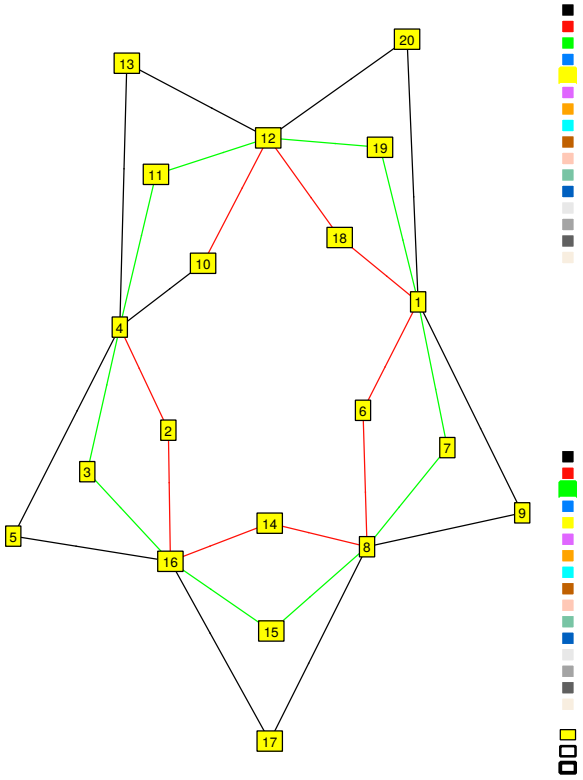

FIGURE 39. $G_{20}^5 \cong C_{10} \times C_2$, subgroup of S_9. (10^3). $(x_1^9+2x_1^7x_2+x_1^5x_2^2+4x_1^4x_5+8x_1^2x_2x_5+4x_2^2x_5)/20$. .

	3	2	5	7	8	10	14	16	1
3	6	8	11	19	14	21	20	7	3
2	5	4	7	3	10	6	16	12	2
5	8	10	13	15	16	18	21	3	5
7	10	6	9	17	12	20	18	5	7
8	11	16	19	6	21	9	7	15	8
10	13	12	15	8	18	11	3	17	10
14	17	7	10	9	13	12	19	18	14
16	19	3	6	11	9	14	15	20	16
1	3	2	5	7	8	10	14	16	1

TABLE 38. Cayley subtable for G_{21}^1.

10. ORDER 21

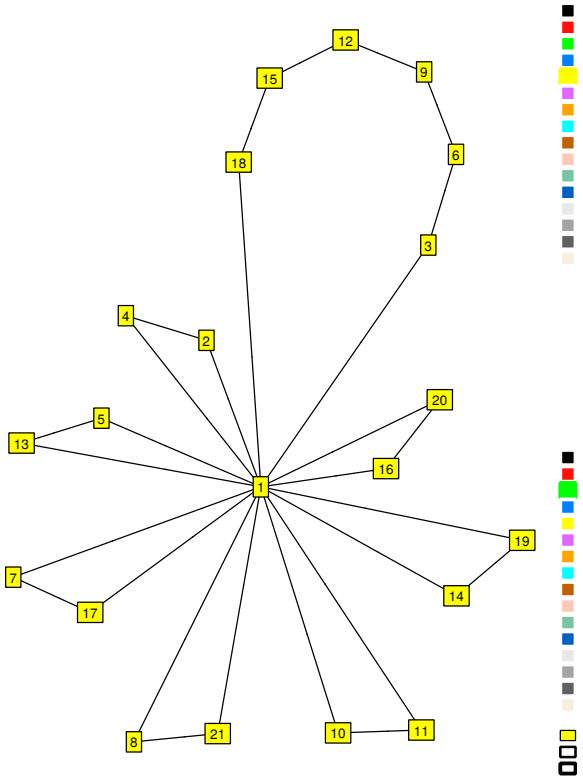

FIGURE 40. $G_{21}^1 \cong C_7 \rtimes C_3$, subgroup of S_7. $(3^7, 7^1)$. $(x_1^{21} + 14x_3^7 + 6x_7^3)/21$.

The graph of the group $G_{21}^2 \cong C_{21}$ is the simple cyclic ring with 21 elements and not shown for that reason. The group is a subgroup of S_{10} with cycle index $(x_1^{10} + 2x_1^7x_3 + 6x_1^3x_7 + 12x_3x_7)/21$.

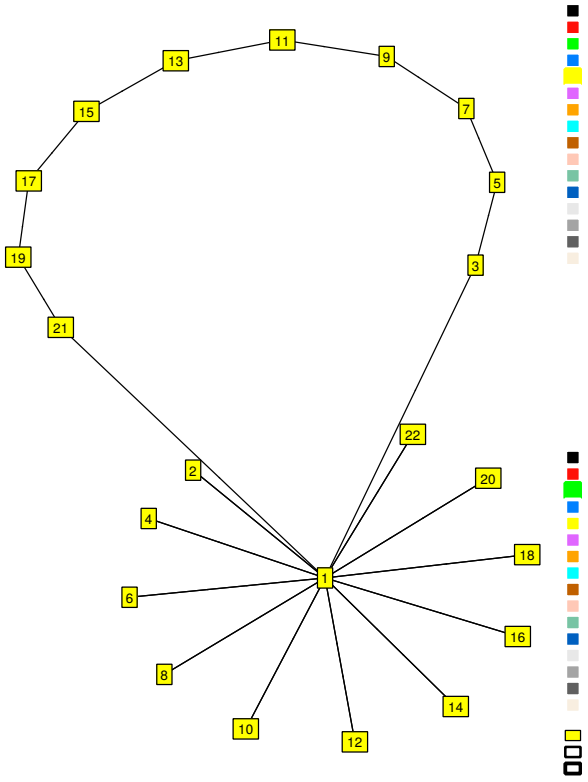

FIGURE 41. $G_{22}^1 \cong D_{22}$. $(2^{11}, 11^1)$. $(x_1^{22} + 11x_2^{11} + 10x_{11}^2)/22$.

The graph of the group $G_{22}^2 \cong C_{22}$ is the simple cyclic ring with 22 elements and not shown for that reason. The group is a subgroup of S_{13} with cycle index $(x_1^{13} + x_1^{11}x_2 + 10x_1^2x_{11} + 10x_2x_{11})/22$.

The graph of the group $G_{23}^1 \cong C_{23}$ is the simple cyclic ring with 23 elements and not shown for that reason. The group is a subgroup of S_{23} with cycle index $(x_1^{23} + 22x_{23})/23$.

11. ORDER 24

	3	2	4	6	8	10	12	14	16	18	20	22	1
3	5	22	2	4	6	8	10	12	14	16	18	20	3
2	4	1	3	5	7	9	11	13	15	17	19	21	2
4	6	21	1	3	5	7	9	11	13	15	17	19	4
6	8	19	21	1	3	5	7	9	11	13	15	17	6
8	10	17	19	21	1	3	5	7	9	11	13	15	8
10	12	15	17	19	21	1	3	5	7	9	11	13	10
12	14	13	15	17	19	21	1	3	5	7	9	11	12
14	16	11	13	15	17	19	21	1	3	5	7	9	14
16	18	9	11	13	15	17	19	21	1	3	5	7	16
18	20	7	9	11	13	15	17	19	21	1	3	5	18
20	22	5	7	9	11	13	15	17	19	21	1	3	20
22	2	3	5	7	9	11	13	15	17	19	21	1	22
1	3	2	4	6	8	10	12	14	16	18	20	22	1

TABLE 39. Cayley subtable for G_{22}^1.

	10	2	8	16	1
10	19	21	6	14	10
2	14	3	10	18	2
8	21	18	3	10	8
16	6	10	18	3	16
1	10	2	8	16	1

TABLE 40. Cayley subtable for G_{24}^1.

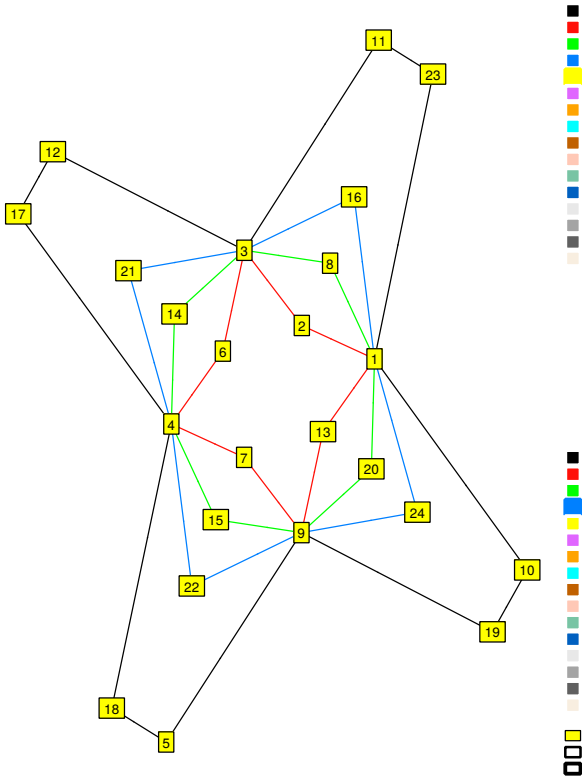

FIGURE 42. $G_{24}^1 \cong C_3 \rtimes C_8$. $(8^3, 12^1)$. $(x_1^{24} + x_2^{12} + 2x_3^8 + 2x_4^6 + 2x_6^4 + 12x_8^3 + 4x_{12}^2)/24$.

	9	13	14	17	3	4	10	1
9	6	11	12	13	17	18	23	9
13	12	8	17	10	15	20	22	13
14	19	9	16	12	24	15	13	14
17	14	12	3	24	2	23	8	17
3	18	14	21	16	5	10	12	3
4	23	15	20	18	19	5	3	4
10	17	24	15	2	4	11	5	10
1	9	13	14	17	3	4	10	1

TABLE 41. Cayley subtable for G_{24}^3.

The graph of the group $G_{24}^2 \cong C_{24}$ is the simple cyclic ring with 24 elements and not shown for that reason. The group is a subgroup of S_{11} with cycle index $(x_1^{11} + 2x_1^8x_3 + x_1^3x_2^4 + 2x_1^3x_4^2 + 2x_2^4x_3 + 4x_1^3x_8 + 4x_3x_4^2 + 8x_3x_8)/24$.

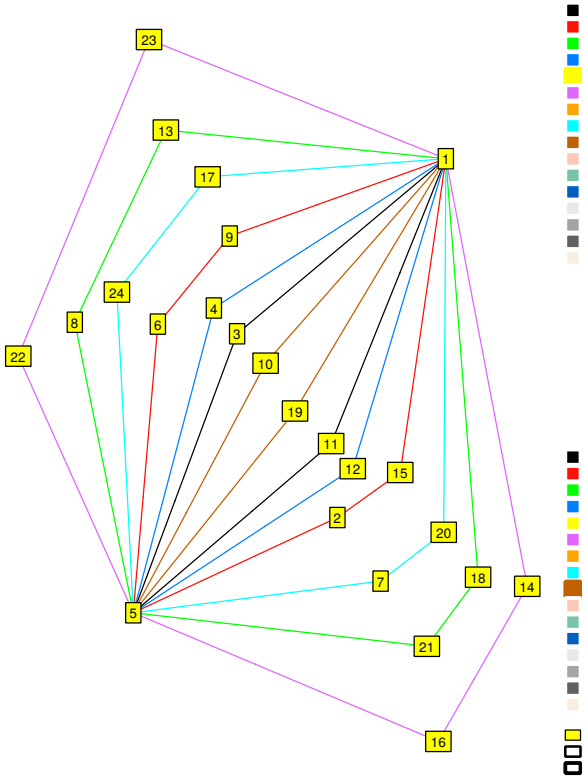

FIGURE 43. $G_{24}^3 \cong SL(2,3)$, subgroup of S_8. $(4^3, 6^4)$. $(x_1^{24} + x_2^{12} + 8x_3^8 + 6x_4^6 + 8x_6^4)/24$.

	10	2	6	8	14	16	21	1
10	19	24	16	13	2	20	8	10
2	14	4	9	11	17	19	23	2
6	15	3	4	10	11	18	19	6
8	21	19	23	4	9	11	17	8
14	22	18	19	3	4	10	11	14
16	6	11	17	19	23	4	9	16
21	7	10	11	18	19	3	4	21
1	10	2	6	8	14	16	21	1

TABLE 42. Cayley subtable for G_{24}^4.

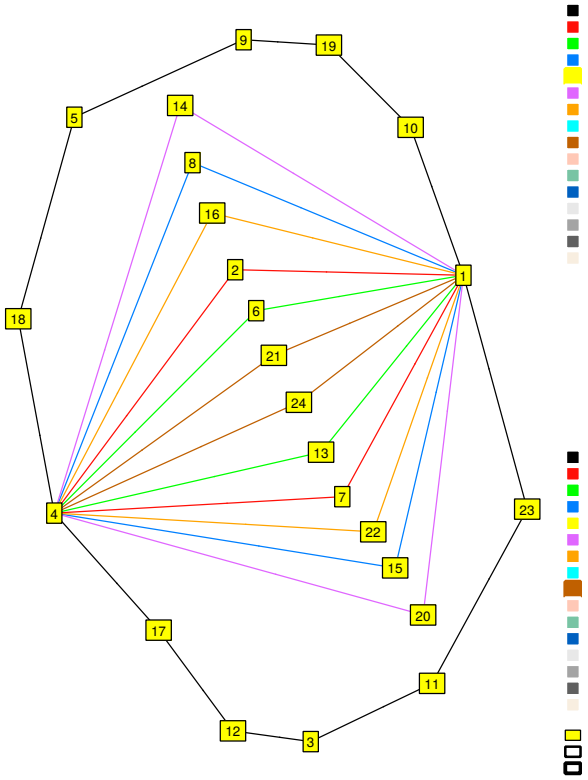

FIGURE 44. $G_{24}^4 \cong C_3 \rtimes Q_8$. $(4^6, 12^1)$. $(x_1^{24} + x_2^{12} + 2x_3^8 + 14x_4^6 + 2x_6^4 + 4x_{12}^2)/24$.

	10	6	14	21	2	7	8	15	16	22	1
10	19	22	7	15	21	24	6	13	14	20	10
6	15	4	11	19	3	9	10	17	18	23	6
14	22	19	4	11	18	23	3	9	10	17	14
21	7	11	19	4	10	17	18	23	3	9	21
2	14	3	10	18	1	4	5	11	12	19	2
7	20	9	17	23	4	1	11	5	19	12	7
8	21	18	3	10	12	19	1	4	5	11	8
15	24	23	9	17	19	12	4	1	11	5	15
16	6	10	18	3	5	11	12	19	1	4	16
22	13	17	23	9	11	5	19	12	4	1	22
1	10	6	14	21	2	7	8	15	16	22	1

TABLE 43. Cayley subtable for G_{24}^5.

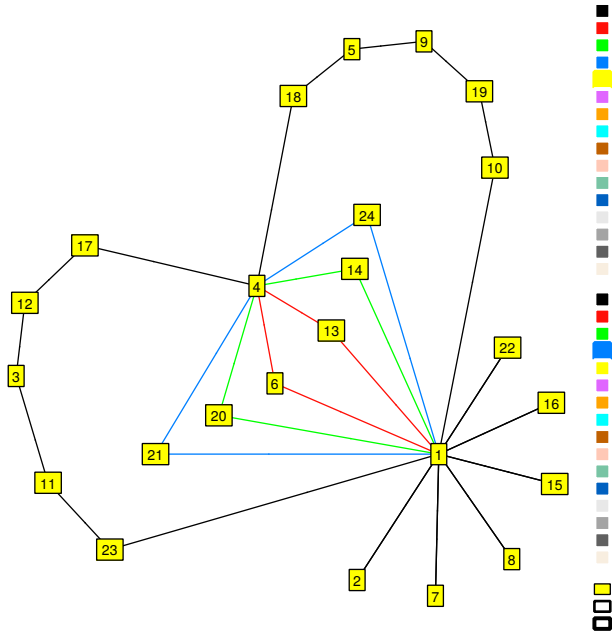

FIGURE 45. $G_{24}^5 \cong C_4 \times S_3$, subgroup of S_7. $(2^6, 4^3, 12^1)$. $(x_1^{24} + 7x_2^{12} + 2x_3^8 + 8x_4^6 + 2x_6^4 + 4x_{12}^2)/24$.

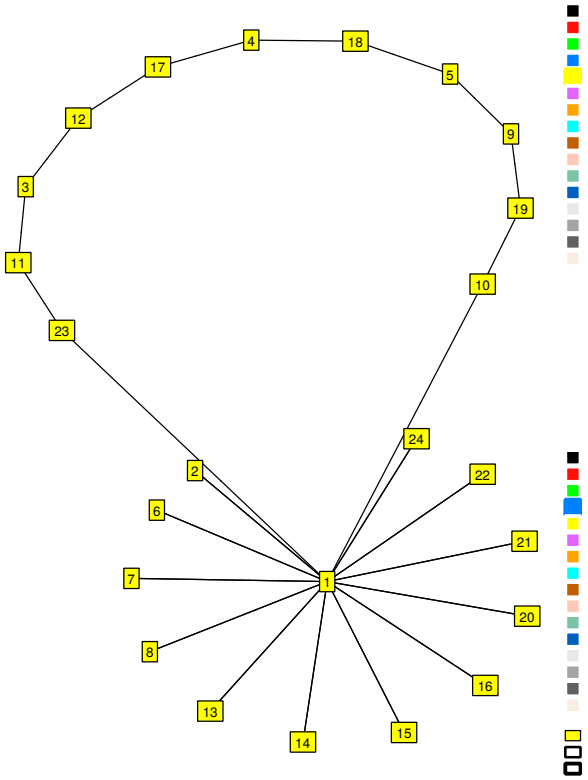

FIGURE 46. $G_{24}^{6} \cong D_{24}$, subgroup of S_7. $(2^{12}, 12^1)$. $(x_1^{24} + 13x_2^{12} + 2x_3^8 + 2x_4^6 + 2x_6^4 + 4x_{12}^2)/24$.

	10	2	6	7	8	13	14	15	16	20	21	22	24	1
10	19	24	16	21	13	22	2	6	20	7	8	14	15	10
2	14	1	3	4	5	9	10	11	12	17	18	19	23	2
6	15	9	1	3	17	4	5	10	23	11	12	18	19	6
7	20	4	9	1	11	3	17	5	19	10	23	12	18	7
8	21	12	18	19	1	23	3	4	5	9	10	11	17	8
13	8	3	4	9	10	1	11	17	18	5	19	23	12	13
14	22	23	12	18	9	19	1	3	17	4	5	10	11	14
15	24	19	23	12	4	18	9	1	11	3	17	5	10	15
16	6	5	10	11	12	17	18	19	1	23	3	4	9	16
20	16	18	19	23	3	12	4	9	10	1	11	17	5	20
21	7	17	5	10	23	11	12	18	9	19	1	3	4	21
22	13	11	17	5	19	10	23	12	4	18	9	1	3	22
24	2	10	11	17	18	5	19	23	3	12	4	9	1	24
1	10	2	6	7	8	13	14	15	16	20	21	22	24	1

TABLE 44. Cayley subtable for G_{24}^{6}.

	10	11	17	2	6	8	14	16	21	1
10	12	23	19	21	16	6	2	14	8	10
11	23	12	18	22	24	7	13	15	20	11
17	19	18	12	24	22	13	7	20	15	17
2	14	15	20	4	9	11	17	19	23	2
6	8	20	15	9	4	17	11	23	19	6
8	21	22	24	19	23	4	9	11	17	8
14	16	24	22	23	19	9	4	17	11	14
16	6	7	13	11	17	19	23	4	9	16
21	2	13	7	17	11	23	19	9	4	21
1	10	11	17	2	6	8	14	16	21	1

TABLE 45. Cayley subtable for G_{24}^7.

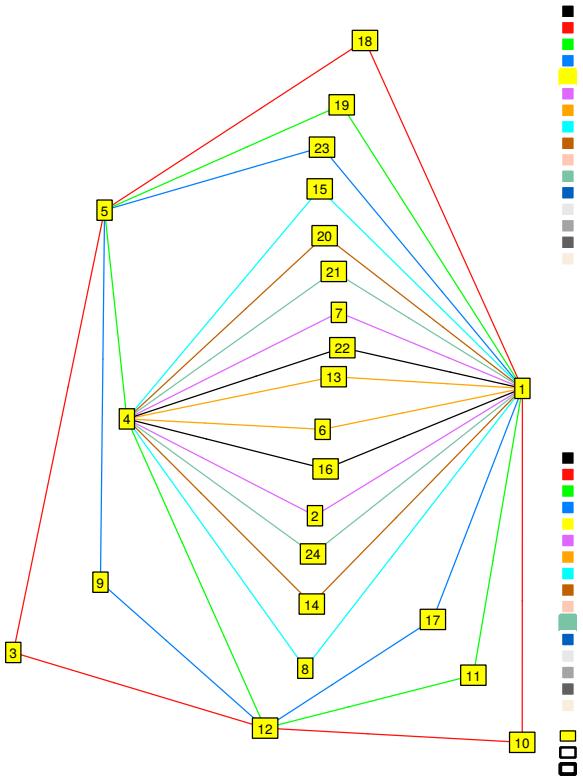

FIGURE 47. $G_{24}^7 \cong C_2 \times (C_3 \rtimes C_4)$, subgroup of S_9. $(4^6, 6^3)$. $(x_1^{24} + 3x_2^{12} + 2x_3^8 + 12x_4^6 + 6x_6^4)/24$.

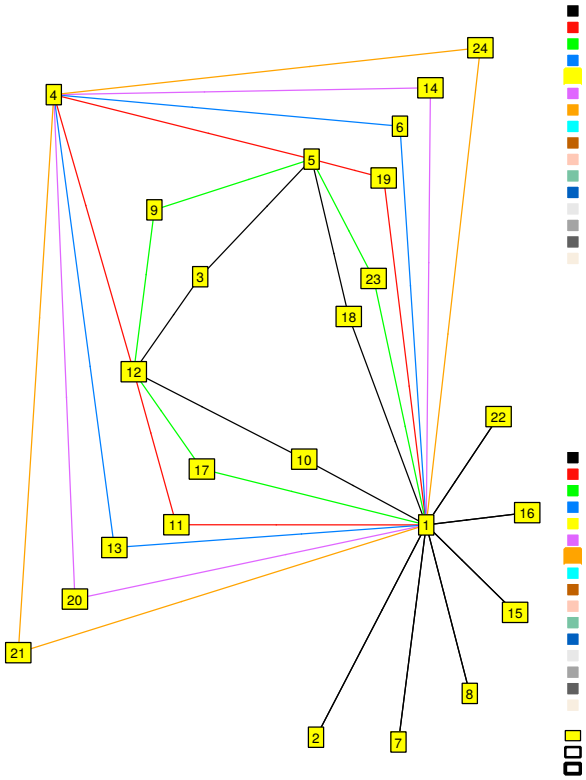

FIGURE 48. $G_{24}^{8} \cong (C_6 \times C_2) \rtimes C_2$, subgroup of S_7. $(2^6, 4^3, 6^3)$. $(x_1^{24} + 9x_2^{12} + 2x_3^8 + 6x_4^6 + 6x_6^4)/24$.

	10	11	17	6	14	21	2	7	8	15	16	22	1
10	12	23	19	22	7	15	24	21	13	6	20	14	10
11	23	12	18	24	13	20	22	16	7	2	15	8	11
17	19	18	12	16	2	8	21	24	6	13	14	20	17
6	8	20	15	4	11	19	9	3	17	10	23	18	6
14	16	24	22	19	4	11	23	18	9	3	17	10	14
21	2	13	7	11	19	4	17	10	23	18	9	3	21
2	14	15	20	3	10	18	1	4	5	11	12	19	2
7	20	8	14	9	17	23	4	1	11	5	19	12	7
8	21	22	24	18	3	10	12	19	1	4	5	11	8
15	24	16	21	23	9	17	19	12	4	1	11	5	15
16	6	7	13	10	18	3	5	11	12	19	1	4	16
22	13	2	6	17	23	9	11	5	19	12	4	1	22
1	10	11	17	6	14	21	2	7	8	15	16	22	1

TABLE 46. Cayley subtable for G_{24}^{8}.

	7	13	9	18	1
7	19	23	20	24	7
13	23	19	15	22	13
9	20	15	11	19	9
18	24	22	19	11	18
1	7	13	9	18	1

TABLE 47. Cayley subtable for G_{24}^{9}.

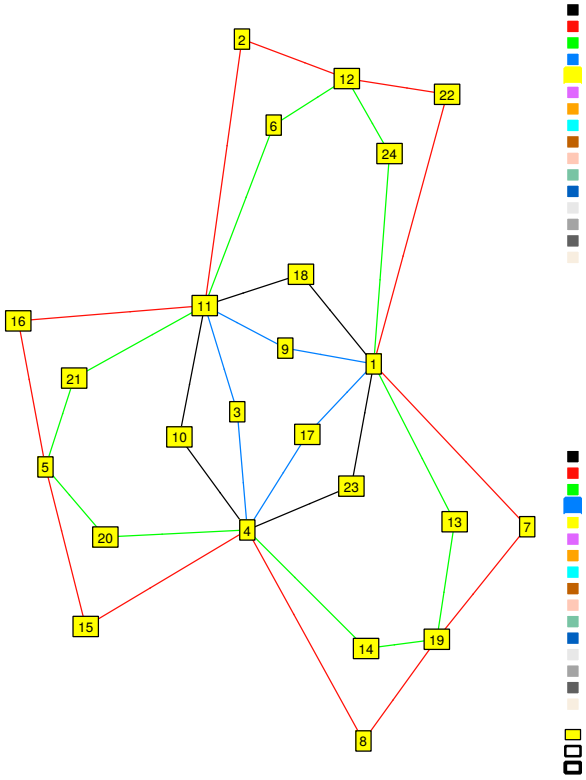

FIGURE 49. $G_{24}^{9} \cong C_{12} \times C_2$, subgroup of S_9. $(6^2, 12^2)$. $(x_1^9 + x_1^7x_2 + 2x_1^6x_3 + x_1^5x_2^2 + 2x_1^5x_4 + 2x_1^4x_2x_3 + x_1^3x_2^3 + 2x_1^3x_2x_4 + 2x_1^2x_2^2x_3 + 4x_1^2x_3x_4 + 2x_2^3x_3 + 4x_2x_3x_4)/24$.

	13	7	9	16	18	1
13	19	23	15	17	22	13
7	17	11	20	19	24	7
9	22	24	11	20	19	9
16	23	19	24	11	20	16
18	15	20	19	24	11	18
1	13	7	9	16	18	1

TABLE 48. Cayley subtable for G_{24}^{10}.

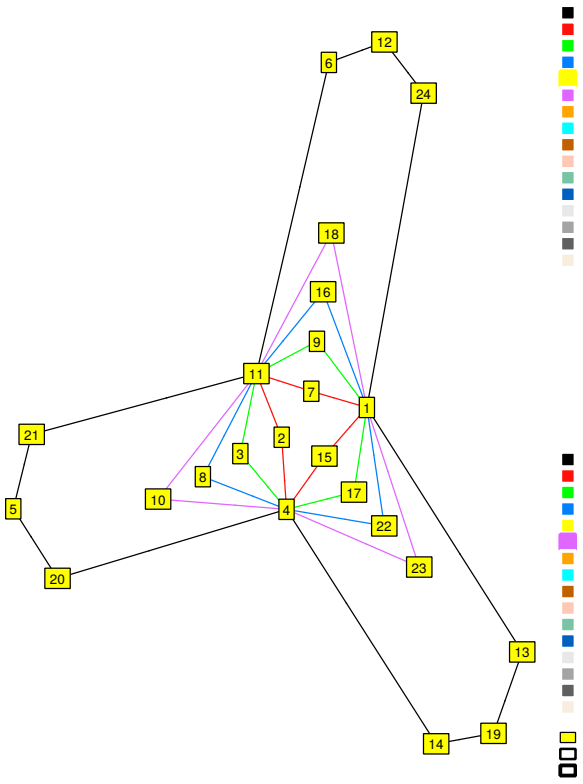

FIGURE 50. $G_{24}^{10} \cong C_3 \times D_8$, subgroup of S_7. $(6^4, 12^1)$. $(x_1^{24} + 5x_2^{12} + 2x_3^8 + 2x_4^6 + 10x_6^4 + 4x_{12}^2)/24$.

	7	9	13	1
7	19	20	23	7
9	24	19	15	9
13	17	22	19	13
1	7	9	13	1

TABLE 49. Cayley subtable for G_{24}^{11}.

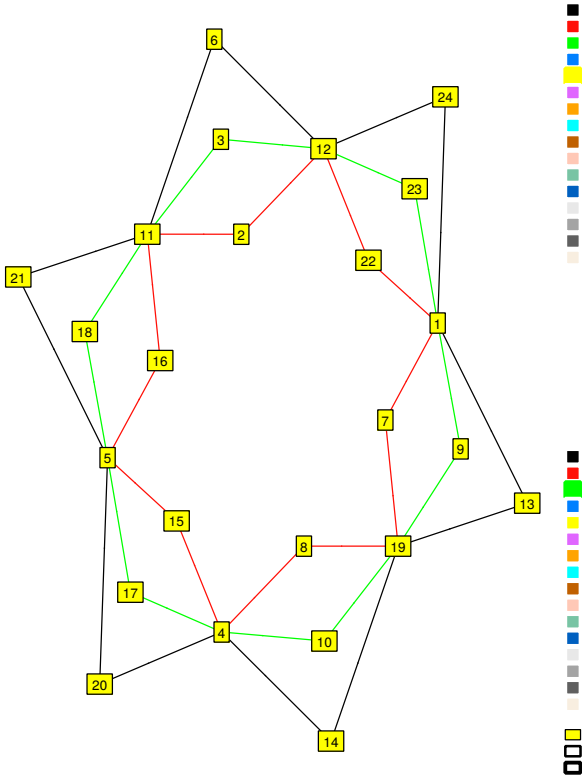

FIGURE 51. $G_{24}^{11} \cong C_3 \times Q_8$. (12^3). $(x_1^{24} + x_2^{12} + 2x_3^8 + 6x_4^6 + 2x_6^4 + 12x_{12}^2)/24$.

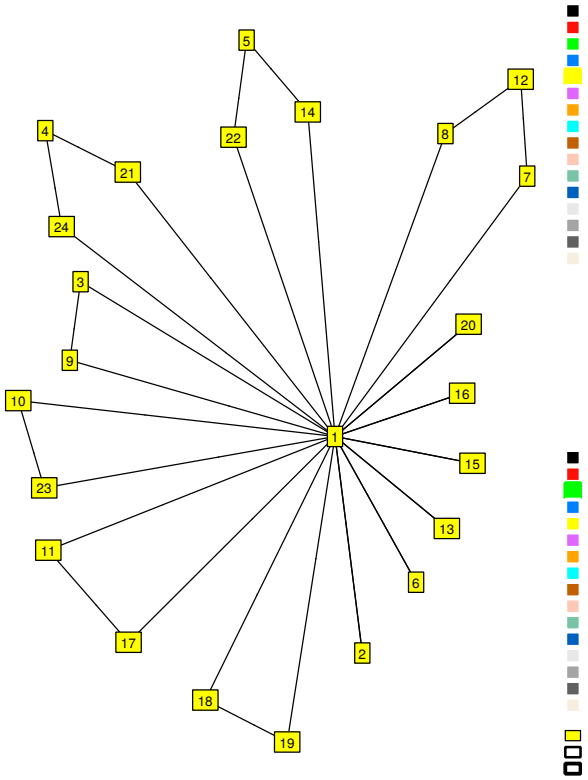

FIGURE 52. $G_{24}^{12} \cong S_4$. $(2^6, 3^4, 4^3)$. $(x_1^{24} + 9x_2^{12} + 8x_3^8 + 6x_4^6)/24$.

	7	14	21	3	10	11	18	2	6	13	15	16	20	1
7	12	11	23	15	22	6	20	5	19	17	10	4	9	7
14	23	5	19	21	24	13	7	18	12	10	4	17	3	14
21	3	23	4	16	8	7	22	10	18	12	9	11	5	21
3	20	7	15	9	17	18	5	13	2	6	8	24	14	3
10	24	8	22	18	23	9	4	21	16	14	7	20	6	10
11	13	16	14	23	18	17	12	20	8	22	2	21	15	11
18	6	24	7	12	5	4	19	14	21	16	13	15	8	18
2	4	10	18	6	14	15	21	1	3	9	11	12	17	2
6	17	4	11	13	20	21	8	9	1	3	5	23	10	6
13	10	17	5	2	7	8	15	3	9	1	18	19	4	13
15	9	12	10	24	21	20	16	17	5	19	1	18	11	15
16	5	3	9	14	6	22	13	12	10	18	19	1	23	16
20	19	18	12	8	16	2	14	11	23	4	17	10	1	20
1	7	14	21	3	10	11	18	2	6	13	15	16	20	1

TABLE 50. Cayley subtable for G_{24}^{12}.

	6	14	15	21	4	5	7	8	12	16	1
6	9	17	18	5	14	15	10	11	22	19	6
14	18	23	9	4	6	22	3	19	15	11	14
15	23	18	17	12	22	6	19	3	14	10	15
21	12	5	4	19	24	13	23	9	20	17	21
4	15	22	6	20	1	12	2	16	5	8	4
5	22	15	14	24	12	1	16	2	4	7	5
7	11	19	3	17	2	16	1	12	8	5	7
8	19	11	10	23	16	2	12	1	7	4	8
12	14	6	22	13	5	4	8	7	1	2	12
16	10	3	19	9	8	7	5	4	2	1	16
1	6	14	15	21	4	5	7	8	12	16	1

TABLE 51. Cayley subtable for G_{24}^{13}.

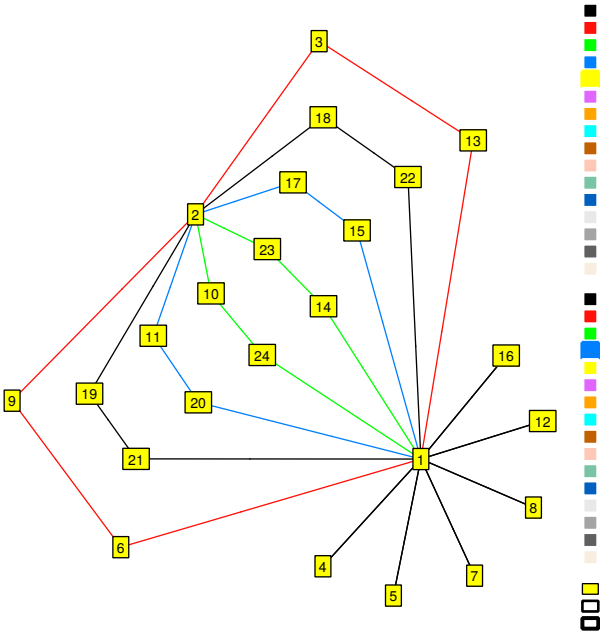

FIGURE 53. $G_{24}^{13} \cong C_2 \times A_4$, subgroup of S_6. $(2^6, 6^4)$. $(x_1^{24} + 7x_2^{12} + 8x_3^8 + 8x_6^4)/24$.

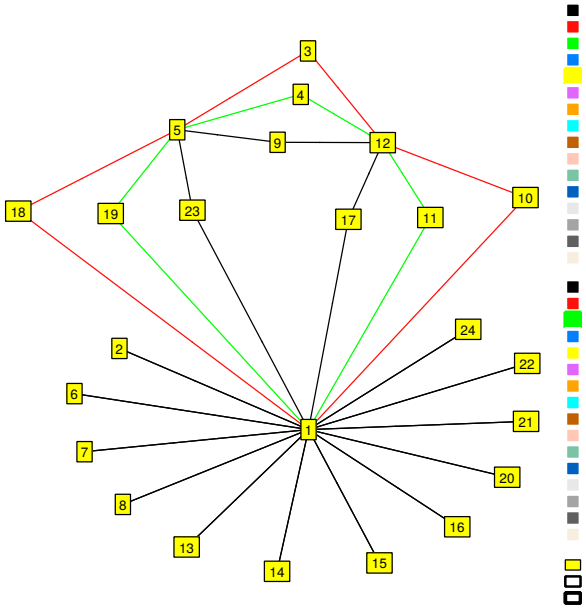

FIGURE 54. $G_{24}^{14} \cong C_2 \times C_2 \times S_3$, subgroup of S_7. $(2^{12}, 6^3)$. $(x_1^{24} + 15x_2^{12} + 2x_3^8 + 6x_6^4)/24$.

	10	11	17	2	6	7	8	13	14	15	16	20	21	22	24	1
10	12	23	19	21	16	24	6	22	2	13	14	7	8	20	15	10
11	23	12	18	22	24	16	7	21	13	2	15	6	20	8	14	11
17	19	18	12	24	22	21	13	16	7	6	20	2	15	14	8	17
2	14	15	20	1	3	4	5	9	10	11	12	17	18	19	23	2
6	8	20	15	3	1	9	10	4	5	17	18	11	12	23	19	6
7	20	8	14	4	9	1	11	3	17	5	19	10	23	12	18	7
8	21	22	24	12	18	19	1	23	3	4	5	9	10	11	17	8
13	15	14	8	9	4	3	17	1	11	10	23	5	19	18	12	13
14	16	24	22	18	12	23	3	19	1	9	10	4	5	17	11	14
15	24	16	21	19	23	12	4	18	9	1	11	3	17	5	10	15
16	6	7	13	5	10	11	12	17	18	19	1	23	3	4	9	16
20	22	21	16	23	19	18	9	12	4	3	17	1	11	10	5	20
21	2	13	7	10	5	17	18	11	12	23	3	19	1	9	4	21
22	13	2	6	11	17	5	19	10	23	12	4	18	9	1	3	22
24	7	6	2	17	11	10	23	5	19	18	9	12	4	3	1	24
1	10	11	17	2	6	7	8	13	14	15	16	20	21	22	24	1

TABLE 52. Cayley subtable for G_{24}^{14}.

	8	10	11	14	15	17	20	1
8	12	21	22	18	19	24	23	8
10	21	12	23	16	24	19	22	10
11	22	23	12	24	16	18	21	11
14	18	16	24	12	23	22	19	14
15	19	24	16	23	12	21	18	15
17	24	19	18	22	21	12	16	17
20	23	22	21	19	18	16	12	20
1	8	10	11	14	15	17	20	1

TABLE 53. Cayley subtable for G_{24}^{15}.

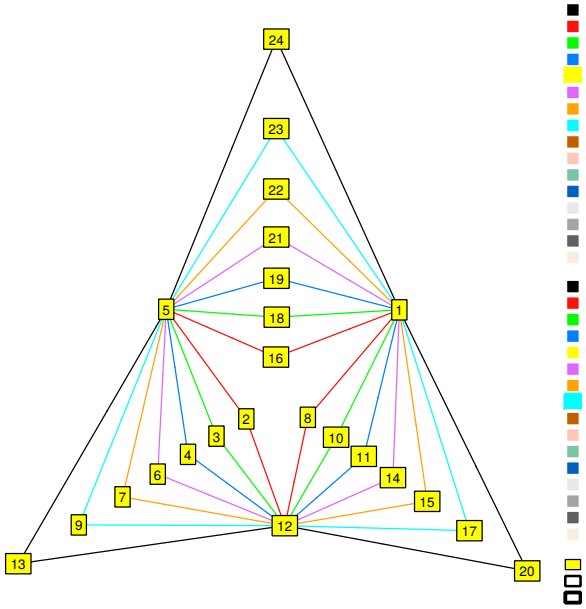

FIGURE 55. $G_{24}^{15} \cong C_6 \times C_2 \times C_2$, subgroup of S_9. (6^7). $(x_1^9 + 3x_1^7x_2 + 2x_1^6x_3 + 3x_1^5x_2^2 + 6x_1^4x_2x_3 + x_1^3x_2^3 + 6x_1^2x_2^2x_3 + 2x_2^3x_3)/24$.

	2	3	5	8	9	16	1
2	4	5	8	12	13	3	2
3	5	6	9	13	14	20	3
5	8	9	13	17	18	6	5
8	12	13	17	20	21	9	8
9	13	14	18	21	22	10	9
16	3	20	6	9	10	17	16
1	2	3	5	8	9	16	1

TABLE 54. Cayley subtable for G_{25}^2.

12. ORDER 25

The graph of the group $G_{25}^1 \cong C_{25}$ is the simple cyclic ring with 25 elements and not shown for that reason. The group is a subgroup of S_{25} with cycle index $(x_1^{25} + 4x_5^5 + 20x_{25})/25$.

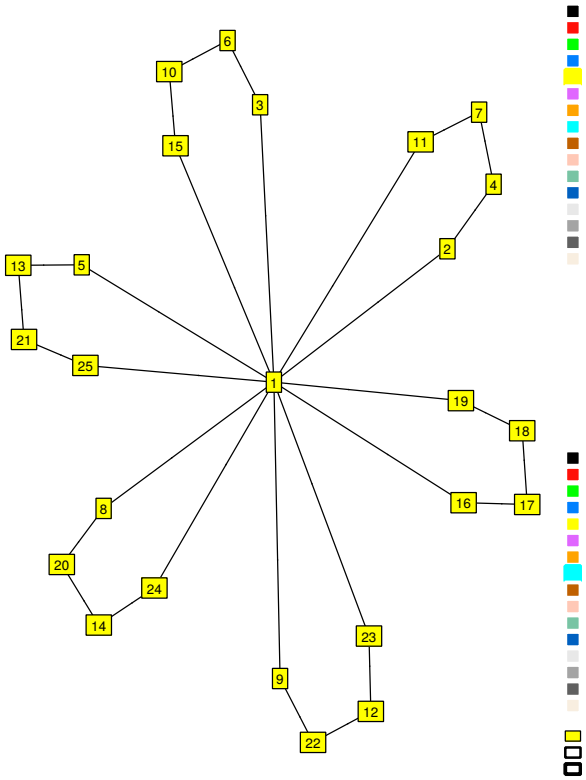

FIGURE 56. $G_{25}^2 \cong C_5 \times C_5$. (5^6). $(x_1^{10} + 8x_1^5x_5 + 16x_5^2)/25$.

13. Order 26

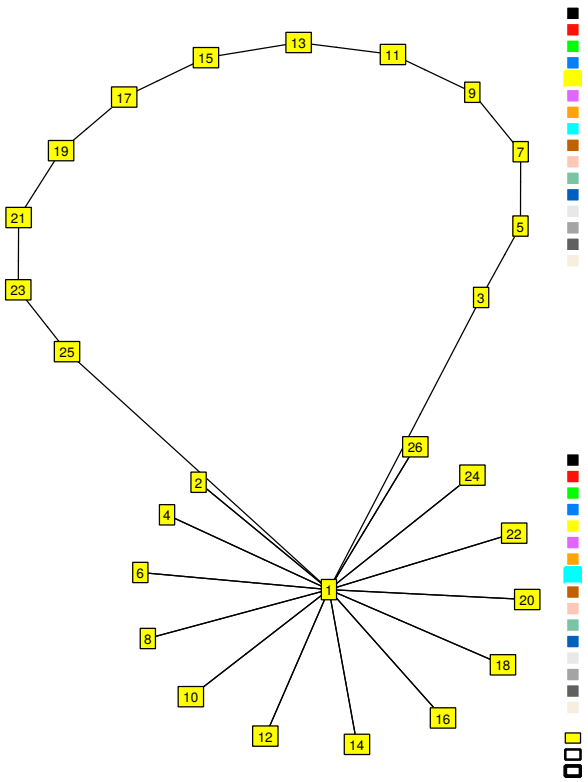

FIGURE 57. $G_{26}^{1} \cong D_{26}$. $(2^{13}, 13^{1})$. $(x_1^{26} + 13x_2^{13} + 12x_{13}^{2})/26$.

	3	2	4	6	8	10	12	14	16	18	20	22	24	26	1
3	5	26	2	4	6	8	10	12	14	16	18	20	22	24	3
2	4	1	3	5	7	9	11	13	15	17	19	21	23	25	2
4	6	25	1	3	5	7	9	11	13	15	17	19	21	23	4
6	8	23	25	1	3	5	7	9	11	13	15	17	19	21	6
8	10	21	23	25	1	3	5	7	9	11	13	15	17	19	8
10	12	19	21	23	25	1	3	5	7	9	11	13	15	17	10
12	14	17	19	21	23	25	1	3	5	7	9	11	13	15	12
14	16	15	17	19	21	23	25	1	3	5	7	9	11	13	14
16	18	13	15	17	19	21	23	25	1	3	5	7	9	11	16
18	20	11	13	15	17	19	21	23	25	1	3	5	7	9	18
20	22	9	11	13	15	17	19	21	23	25	1	3	5	7	20
22	24	7	9	11	13	15	17	19	21	23	25	1	3	5	22
24	26	5	7	9	11	13	15	17	19	21	23	25	1	3	24
26	2	3	5	7	9	11	13	15	17	19	21	23	25	1	26
1	3	2	4	6	8	10	12	14	16	18	20	22	24	26	1

TABLE 55. Cayley subtable for G_{26}^{1}.

The graph of the group $G_{26}^2 \cong C_{26}$ is the simple cyclic ring with 26 elements and not shown for that reason. The group is a subgroup of S_{15} with cycle index $(x_1^{15} + x_1^{13}x_2 + 12x_1^2x_{13} + 12x_2x_{13})/26$.

14. Order 27

The graph of the group $G_{27}^1 \cong C_{27}$ is the simple cyclic ring with 27 elements and not shown for that reason. The group is a subgroup of S_{27} with cycle index $(x_1^{27} + 2x_3^9 + 6x_9^3 + 18x_{27})/27$.

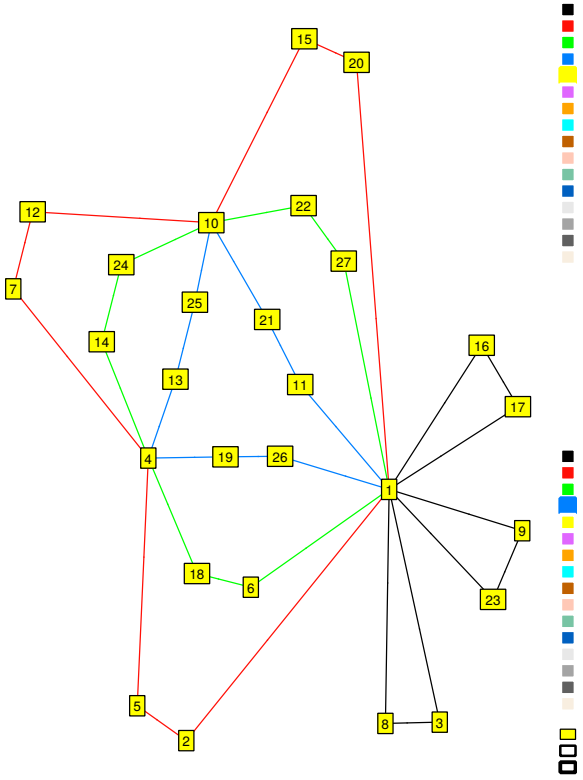

Figure 58. $G_{27}^2 \cong C_9 \times C_3$. $(3^3, 9^3)$. $(x_1^{12} + 2x_1^9x_3 + 2x_1^3x_3^3 + 6x_1^3x_9 + 4x_3^4 + 12x_3x_9)/27$.

	2	6	11	3	9	16	1
2	5	11	9	6	14	21	2
6	11	18	16	13	21	7	6
11	9	16	21	18	24	12	11
3	6	13	18	8	16	4	3
9	14	21	24	16	23	10	9
16	21	7	12	4	10	17	16
1	2	6	11	3	9	16	1

TABLE 56. Cayley subtable for G_{27}^2.

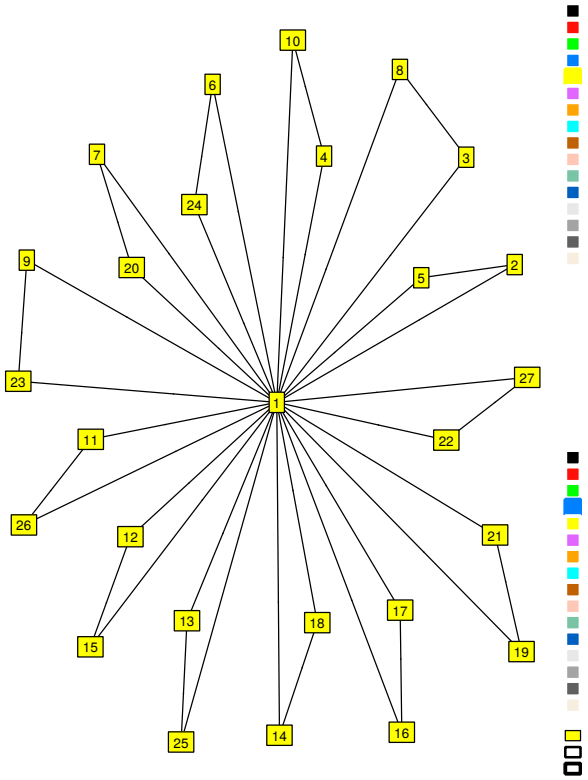

FIGURE 59. $G_{27}^3 \cong (C_3 \times C_3) \rtimes C_3$. (3^{13}). $(x_1^{27} + 26x_3^9)/27$.

	2	3	4	6	7	9	11	12	13	14	16	19	22	1
2	5	6	7	11	12	14	3	4	18	19	21	9	25	2
3	14	8	9	21	22	16	27	11	7	26	4	18	13	3
4	7	9	10	14	15	17	19	20	21	22	23	25	6	4
6	19	13	14	24	25	21	23	3	12	27	7	8	18	6
7	12	14	15	19	20	22	9	10	24	25	26	17	11	7
9	22	16	17	26	6	23	18	19	15	13	10	24	21	9
11	9	18	19	16	17	24	26	6	4	23	12	13	8	11
12	4	19	20	9	10	25	14	15	16	17	27	22	3	12
13	27	2	21	20	18	7	4	23	25	5	14	10	12	13
14	25	21	22	27	11	26	8	9	20	18	15	16	24	14
16	13	4	23	2	21	10	20	18	6	7	17	5	15	16
19	17	24	25	23	3	27	13	14	10	8	20	21	16	19
22	11	26	6	18	19	13	16	17	5	24	2	23	27	22
1	2	3	4	6	7	9	11	12	13	14	16	19	22	1

TABLE 57. Cayley subtable for G_{27}^3.

	2	6	11	3	9	16	1
2	5	11	9	6	14	21	2
6	19	24	8	13	21	7	6
11	17	23	13	18	24	12	11
3	14	21	27	8	16	4	3
9	22	26	18	16	23	10	9
16	13	2	20	4	10	17	16
1	2	6	11	3	9	16	1

TABLE 58. Cayley subtable for G_{27}^4.

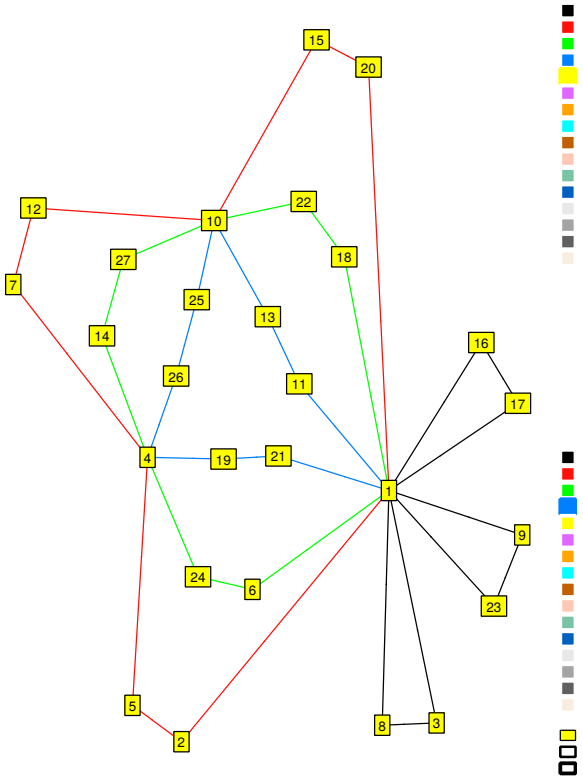

FIGURE 60. $G_{27}^4 \cong C_9 \rtimes C_3$. $(3^3, 9^3)$. $(x_1^{27} + 8x_3^9 + 18x_9^3)/27$.

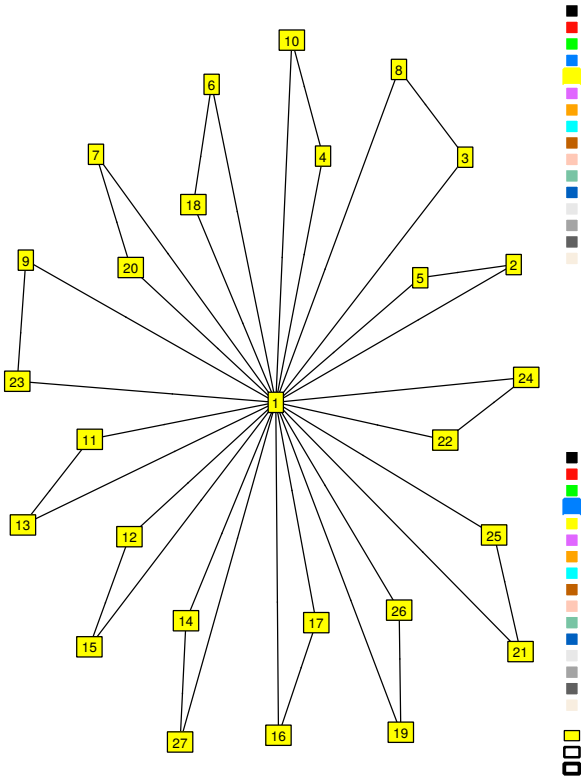

FIGURE 61. $G_{27}^{5} \cong C_3 \times C_3 \times C_3$. (3^{13}). $(x_1^9 + 6x_1^6x_3 + 12x_1^3x_3^2 + 8x_3^3)/27$.

	2	3	4	6	7	9	11	12	14	16	19	21	22	1
2	5	6	7	11	12	14	3	4	19	21	9	24	25	2
3	6	8	9	13	14	16	18	19	21	4	24	7	26	3
4	7	9	10	14	15	17	19	20	22	23	25	26	6	4
6	11	13	14	18	19	21	8	9	24	7	16	12	27	6
7	12	14	15	19	20	22	9	10	25	26	17	27	11	7
9	14	16	17	21	22	23	24	25	26	10	27	15	13	9
11	3	18	19	8	9	24	13	14	16	12	21	4	23	11
12	4	19	20	9	10	25	14	15	17	27	22	23	3	12
14	19	21	22	24	25	26	16	17	27	15	23	20	18	14
16	21	4	23	7	26	10	12	27	15	17	20	22	2	16
19	9	24	25	16	17	27	21	22	23	20	26	10	8	19
21	24	7	26	12	27	15	4	23	20	22	10	25	5	21
22	25	26	6	27	11	13	23	3	18	2	8	5	24	22
1	2	3	4	6	7	9	11	12	14	16	19	21	22	1

TABLE 59. Cayley subtable for G_{27}^5.

	7	2	6	10	14	18	22	26	1
7	8	28	5	9	13	17	21	25	7
2	9	3	7	11	15	19	23	27	2
6	13	27	3	7	11	15	19	23	6
10	17	23	27	3	7	11	15	19	10
14	21	19	23	27	3	7	11	15	14
18	25	15	19	23	27	3	7	11	18
22	28	11	15	19	23	27	3	7	22
26	5	7	11	15	19	23	27	3	26
1	7	2	6	10	14	18	22	26	1

TABLE 60. Cayley subtable for G_{28}^1.

15. ORDER 28

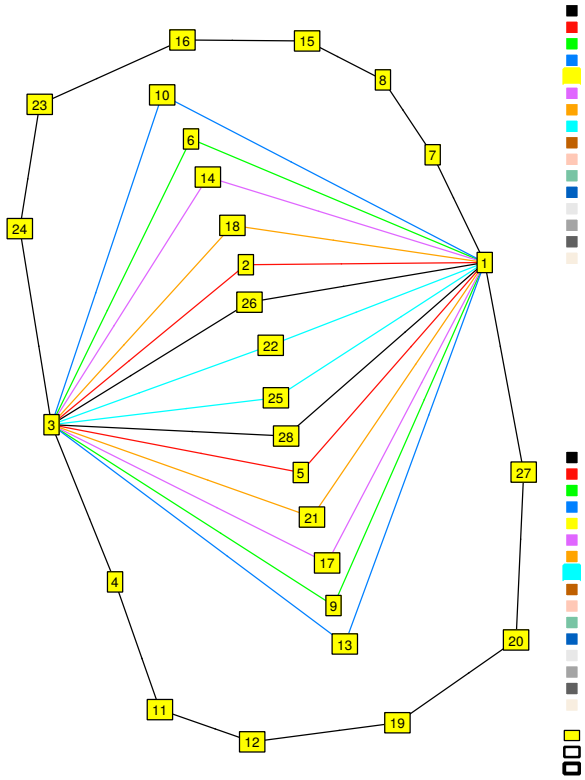

FIGURE 62. $G_{28}^1 \cong C_7 \rtimes C_4$. $(4^7, 14^1)$. $(x_1^{28} + x_2^{14} + 14x_4^7 + 6x_7^4 + 6x_{14}^2)/28$.

The graph of the group $G_{28}^2 \cong C_{28}$ is the simple cyclic ring with 28 elements and not shown for that reason. The group is a subgroup of S_{11} with cycle index $(x_1^{11} + x_1^7x_2^2 + 2x_1^7x_4 + 6x_1^4x_7 + 6x_2^2x_7 + 12x_4x_7)/28$.

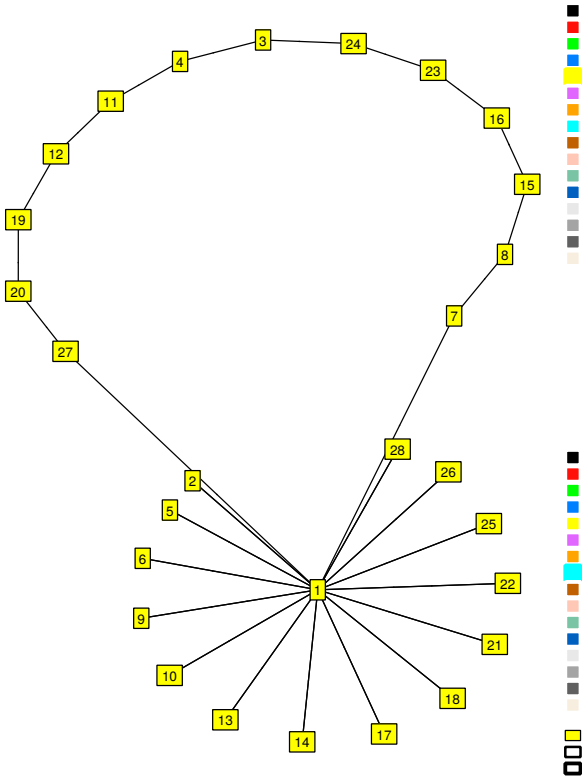

FIGURE 63. $G_{28}^3 \cong D_{28}$. $(2^{14}, 14^1)$. $(x_1^{28} + 15x_2^{14} + 6x_7^4 + 6x_{14}^2)/28$.

	7	2	5	6	9	10	13	14	17	18	21	22	25	26	28	1
7	8	28	26	5	2	9	6	13	10	17	14	21	18	25	22	7
2	9	1	3	4	7	8	11	12	15	16	19	20	23	24	27	2
5	6	3	1	7	4	11	8	15	12	19	16	23	20	27	24	5
6	13	24	27	1	3	4	7	8	11	12	15	16	19	20	23	6
9	10	27	24	3	1	7	4	11	8	15	12	19	16	23	20	9
10	17	20	23	24	27	1	3	4	7	8	11	12	15	16	19	10
13	14	23	20	27	24	3	1	7	4	11	8	15	12	19	16	13
14	21	16	19	20	23	24	27	1	3	4	7	8	11	12	15	14
17	18	19	16	23	20	27	24	3	1	7	4	11	8	15	12	17
18	25	12	15	16	19	20	23	24	27	1	3	4	7	8	11	18
21	22	15	12	19	16	23	20	27	24	3	1	7	4	11	8	21
22	28	8	11	12	15	16	19	20	23	24	27	1	3	4	7	22
25	26	11	8	15	12	19	16	23	20	27	24	3	1	7	4	25
26	5	4	7	8	11	12	15	16	19	20	23	24	27	1	3	26
28	2	7	4	11	8	15	12	19	16	23	20	27	24	3	1	28
1	7	2	5	6	9	10	13	14	17	18	21	22	25	26	28	1

TABLE 61. Cayley subtable for G_{28}^{3}.

	6	7	9	1
6	8	13	11	6
7	13	8	10	7
9	11	10	8	9
1	6	7	9	1

TABLE 62. Cayley subtable for G_{28}^4.

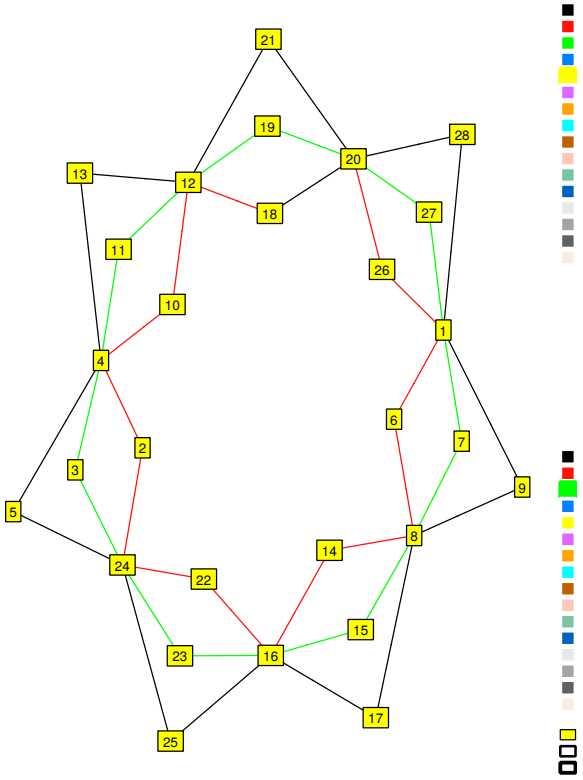

FIGURE 64. $G_{28}^4 \cong C_{14} \times C_2$. (14^3). $(x_1^{11}+2x_1^9x_2+x_1^7x_2^2+6x_1^4x_7+12x_1^2x_2x_7+6x_2^2x_7)/28$.

	8	5	11	18	1
8	21	24	10	17	8
5	17	7	14	21	5
11	24	21	7	14	11
18	10	14	21	7	18
1	8	5	11	18	1

TABLE 63. Cayley subtable for G_{30}^1.

16. Order 29

The graph of the group $G_{29}^1 \cong C_{29}$ is the simple cyclic ring with 29 elements and not shown for that reason. The group is a subgroup of S_{29} with cycle index $(x_1^{29} + 28x_{29})/29$.

17. Order 30

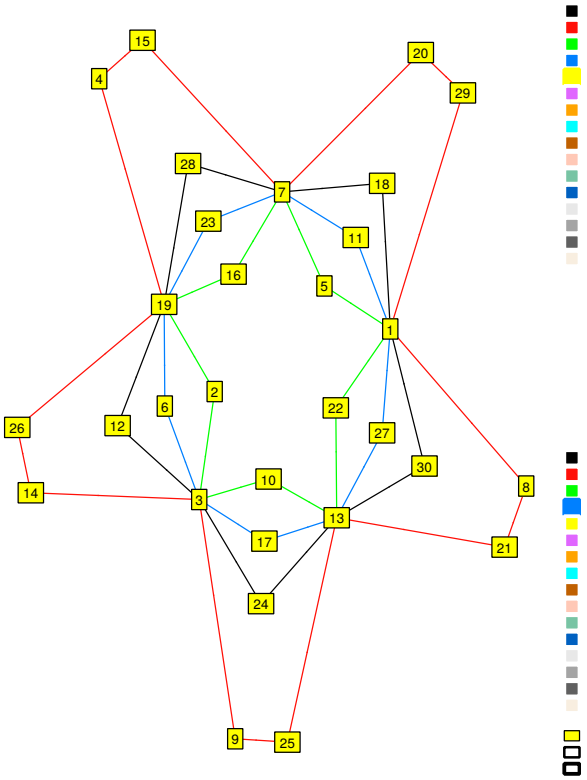

FIGURE 65. $G_{30}^1 \cong C_5 \times S_3$. $(10^3, 15^1)$. $(x_1^{30} + 3x_2^{15} + 2x_3^{10} + 4x_5^6 + 12x_{10}^3 + 8x_{15}^2)/30$.

	8	5	11	17	23	28	1
8	19	30	10	16	22	27	8
5	16	7	13	19	25	29	5
11	22	29	7	13	19	25	11
17	27	25	29	7	13	19	17
23	30	19	25	29	7	13	23
28	10	13	19	25	29	7	28
1	8	5	11	17	23	28	1

TABLE 64. Cayley subtable for G_{30}^2.

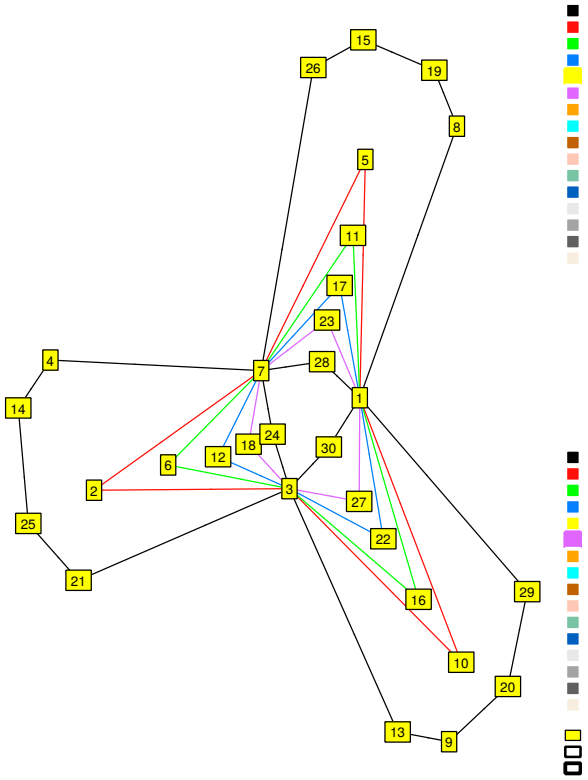

FIGURE 66. $G_{30}^2 \cong C_3 \times D_{10}$. $(6^5, 15^1)$. $(x_1^{30} + 5x_2^{15} + 2x_3^{10} + 4x_5^6 + 10x_6^5 + 8x_{15}^2)/30$.

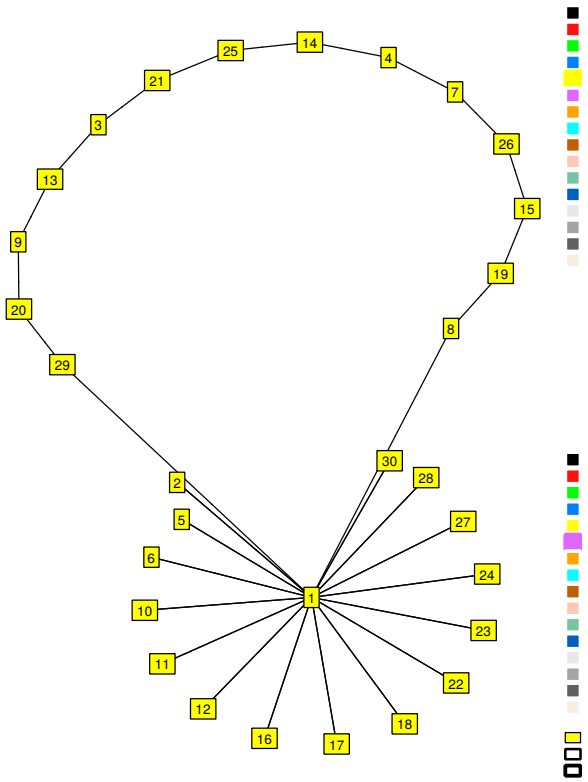

FIGURE 67. $G_{30}^{3} \cong D_{30}$. $(2^{15}, 15^{1})$. $(x_1^{30} + 15x_2^{15} + 2x_3^{10} + 4x_5^6 + 8x_{15}^2)/30$.

The graph of the group $G_{30}^{4} \cong C_{30}$ is the simple cyclic ring with 30 elements and not shown for that reason. The group is a subgroup of S_{10} with cycle index $(x_1^{10} + x_1^8x_2 + 2x_1^7x_3 + 2x_1^5x_2x_3 + 4x_1^5x_5 + 4x_1^3x_2x_5 + 8x_1^2x_3x_5 + 8x_2x_3x_5)/30$.

18. Order 31

The graph of the group $G_{31}^{1} \cong C_{31}$ is the simple cyclic ring with 31 elements and not shown for that reason. The group is a subgroup of S_{31} with cycle index $(x_1^{31} + 30x_{31})/31$.

19. Order 32

The graph of the group $G_{32}^{1} \cong C_{32}$ is the simple cyclic ring with 32 elements and not shown for that reason. The group is a subgroup of S_{32} with cycle index $(x_1^{32} + x_2^{16} + 2x_4^8 + 4x_8^4 + 8x_{16}^2 + 16x_{32})/32$.

	8	2	5	6	10	11	12	16	17	18	22	23	24	27	28	30	1
8	19	30	24	10	28	2	16	5	6	22	11	12	27	17	18	23	8
2	11	1	3	4	7	8	9	13	14	15	19	20	21	25	26	29	2
5	16	7	1	13	3	4	19	8	9	25	14	15	29	20	21	26	5
6	17	21	26	1	29	3	4	7	8	9	13	14	15	19	20	25	6
10	6	3	7	8	1	13	14	4	19	20	9	25	26	15	29	21	10
11	22	29	21	7	26	1	13	3	4	19	8	9	25	14	15	20	11
12	23	15	20	21	25	26	1	29	3	4	7	8	9	13	14	19	12
16	12	26	29	3	21	7	8	1	13	14	4	19	20	9	25	15	16
17	27	25	15	29	20	21	7	26	1	13	3	4	19	8	9	14	17
18	28	9	14	15	19	20	21	25	26	1	29	3	4	7	8	13	18
22	18	20	25	26	15	29	3	21	7	8	1	13	14	4	19	9	22
23	30	19	9	25	14	15	29	20	21	7	26	1	13	3	4	8	23
24	5	4	8	9	13	14	15	19	20	21	25	26	1	29	3	7	24
27	24	14	19	20	9	25	26	15	29	3	21	7	8	1	13	4	27
28	10	13	4	19	8	9	25	14	15	29	20	21	7	26	1	3	28
30	2	8	13	14	4	19	20	9	25	26	15	29	3	21	7	1	30
1	8	2	5	6	10	11	12	16	17	18	22	23	24	27	28	30	1

TABLE 65. Cayley subtable for G_{30}^3.

	2	3	7	8	10	11	12	17	18	19	21	23	4	14	15	16	1
2	5	7	12	14	16	17	18	23	3	25	26	27	8	20	21	22	2
3	17	6	21	7	28	15	16	10	30	8	19	26	11	23	24	25	3
7	23	10	26	12	31	21	22	16	15	14	25	30	17	27	28	29	7
8	14	17	23	5	26	7	27	12	11	31	16	18	2	9	10	30	8
10	16	19	25	26	5	28	29	31	13	12	14	32	21	30	8	9	10
11	7	15	10	17	19	6	26	21	22	2	28	16	3	12	13	31	11
12	27	16	30	18	32	26	6	22	21	20	29	15	23	11	31	13	12
17	12	21	16	23	25	10	30	26	6	5	31	22	7	18	19	32	17
18	11	22	15	3	24	30	10	6	26	4	13	21	27	17	32	19	18
19	31	2	14	25	23	8	9	5	4	26	12	20	28	32	17	18	19
21	26	28	31	16	14	19	32	25	24	23	5	29	10	22	2	20	21
23	18	26	22	27	29	16	15	30	10	9	32	6	12	3	25	24	23
4	8	11	17	2	21	3	23	7	27	28	10	12	1	5	6	26	4
14	20	23	27	9	30	12	11	18	17	32	22	3	5	1	16	15	14
15	21	24	28	10	8	13	31	19	32	17	2	25	6	16	1	14	15
16	22	25	29	30	9	31	13	32	19	18	20	24	26	15	14	1	16
1	2	3	7	8	10	11	12	17	18	19	21	23	4	14	15	16	1

TABLE 66. Cayley subtable for G_{32}^2.

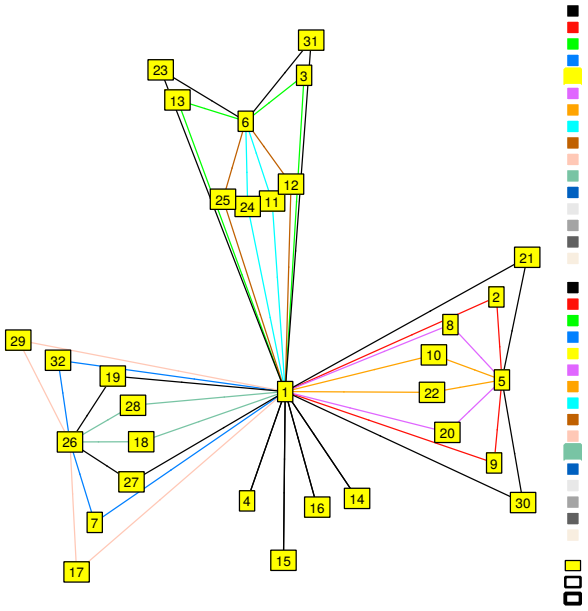

FIGURE 68. $G_{32}^2 \cong (C_4 \times C_2) \rtimes C_4$. $(2^4, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

	2	7	9	17	3	11	13	23	1
2	4	11	14	13	7	17	19	27	2
7	11	14	23	16	9	20	22	8	7
9	14	23	4	25	18	27	29	17	9
17	13	16	25	26	20	22	30	10	17
3	7	9	18	20	5	14	16	4	3
11	17	20	27	22	14	16	26	6	11
13	19	22	29	30	16	26	5	15	13
23	27	8	17	10	4	6	15	16	23
1	2	7	9	17	3	11	13	23	1

TABLE 67. Cayley subtable for G_{32}^3.

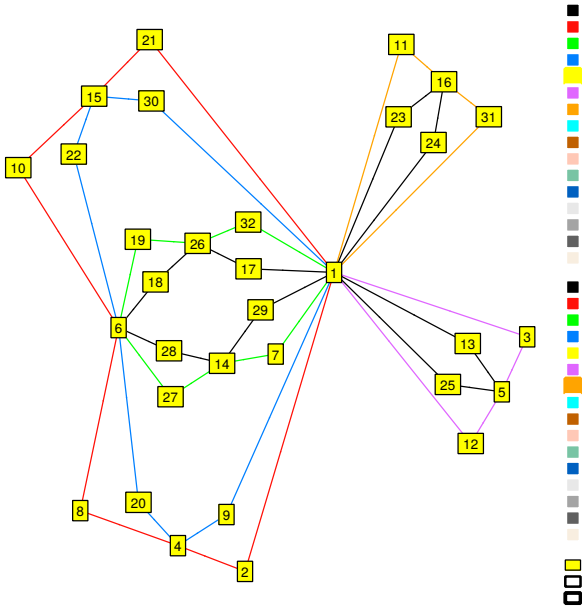

FIGURE 69. $G_{32}^3 \cong C_8 \times C_4$. $(4^4, 8^4)$. $(x_1^{12} + x_1^8 x_2^2 + 2x_1^8 x_4 + x_1^4 x_2^4 + 2x_1^4 x_4^2 + x_2^6 + 4x_1^4 x_8 + 2x_2^4 x_4 + 2x_2^2 x_4^2 + 4x_2^2 x_8 + 4x_4^3 + 8x_4 x_8)/32$.

	2	7	9	17	3	11	13	23	1
2	4	11	14	13	7	17	19	27	2
7	24	26	31	5	9	20	22	8	7
9	14	23	4	25	18	27	29	17	9
17	3	5	12	14	20	22	30	10	17
3	19	22	29	30	5	14	16	4	3
11	28	30	32	9	14	16	26	6	11
13	7	9	18	20	16	26	5	15	13
23	32	21	28	2	4	6	15	16	23
1	2	7	9	17	3	11	13	23	1

TABLE 68. Cayley subtable for G_{32}^4.

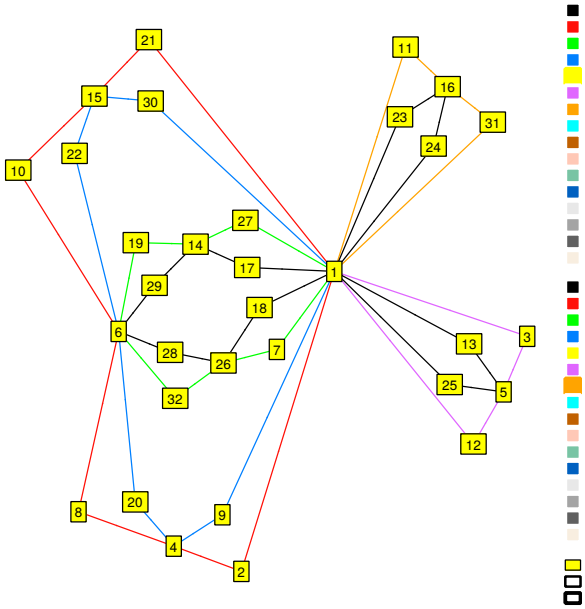

FIGURE 70. $G_{32}^4 \cong C_8 \rtimes C_4$. $(4^4, 8^4)$. $(x_1^{32} + 3x_2^{16} + 12x_4^8 + 16x_8^4)/32$.

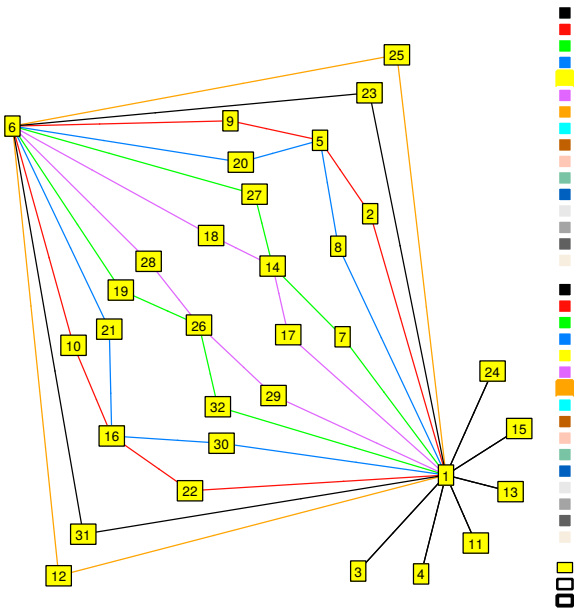

FIGURE 71. $G_{32}^5 \cong (C_8 \times C_2) \rtimes C_2$. $(2^6, 4^2, 8^4)$. $(x_1^{32} + 7x_2^{16} + 8x_4^8 + 16x_8^4)/32$.

	2	7	8	17	12	23	3	4	11	13	15	24	1
2	5	12	14	23	18	27	7	8	17	19	21	28	2
7	23	14	12	5	9	20	2	17	8	10	28	21	7
8	14	23	5	12	27	18	17	2	7	28	10	19	8
17	12	5	23	14	20	9	8	7	2	21	19	10	17
12	27	20	18	9	6	15	5	23	14	16	31	26	12
23	18	9	27	20	15	6	14	12	5	26	25	16	23
3	17	8	7	2	5	14	1	11	4	6	24	15	3
4	8	17	2	7	23	12	11	1	3	24	6	13	4
11	7	2	17	8	14	5	4	3	1	15	13	6	11
13	28	21	19	10	16	26	6	24	15	1	11	4	13
15	21	28	10	19	31	25	24	6	13	11	1	3	15
24	19	10	28	21	26	16	15	13	6	4	3	1	24
1	2	7	8	17	12	23	3	4	11	13	15	24	1

TABLE 69. Cayley subtable for G_{32}^{5}.

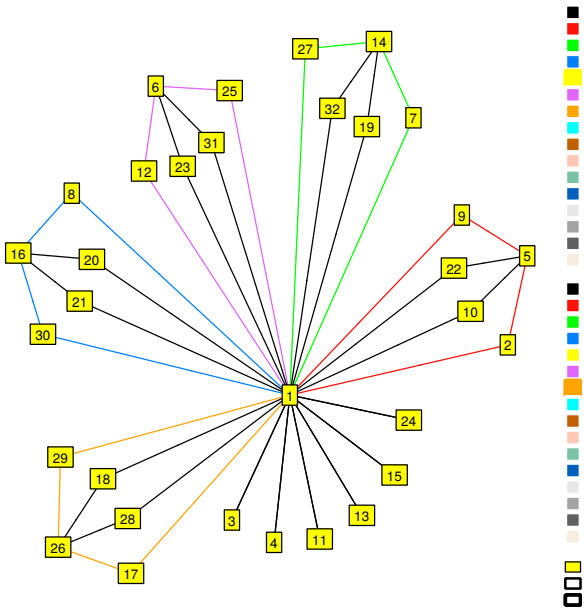

FIGURE 72. $G_{32}^{6} \cong ((C_4 \times C_2) \rtimes C_2) \rtimes C_2$. $(2^6, 4^{10})$. $(x_1^{32} + 11x_2^{16} + 20x_4^8)/32$.

	2	7	8	10	12	17	18	19	20	23	3	4	11	13	15	24	1
2	5	25	14	16	18	31	13	12	4	27	7	8	17	19	21	28	2
7	31	14	25	23	9	5	4	26	13	20	2	17	8	10	28	21	7
8	26	23	16	14	27	12	11	31	6	18	17	2	7	28	10	19	8
10	16	12	26	5	29	23	3	25	15	32	19	21	28	7	8	17	10
12	27	30	18	32	6	22	21	20	7	15	16	23	26	5	31	14	12
17	12	16	23	25	20	26	6	5	11	9	8	7	2	21	19	10	17
18	24	15	13	11	10	6	26	4	25	21	22	27	30	9	32	20	18
19	23	26	12	31	22	16	15	14	3	30	10	28	21	2	17	8	19
20	15	24	6	4	28	13	31	11	16	19	32	9	29	27	22	18	20
23	29	9	32	18	15	20	2	22	28	6	26	12	16	14	25	5	23
3	17	8	7	28	5	2	20	21	18	14	1	11	4	6	24	15	3
4	21	28	10	8	23	19	32	17	22	12	11	1	3	24	6	13	4
11	19	10	28	7	14	21	22	2	32	5	4	3	1	15	13	6	11
13	28	21	19	17	16	10	30	8	29	26	6	24	15	1	11	4	13
15	8	17	2	21	31	7	27	28	9	25	24	6	13	11	1	3	15
24	7	2	17	19	26	8	9	10	27	16	15	13	6	4	3	1	24
1	2	7	8	10	12	17	18	19	20	23	3	4	11	13	15	24	1

TABLE 70. Cayley subtable for G_{32}^{6}.

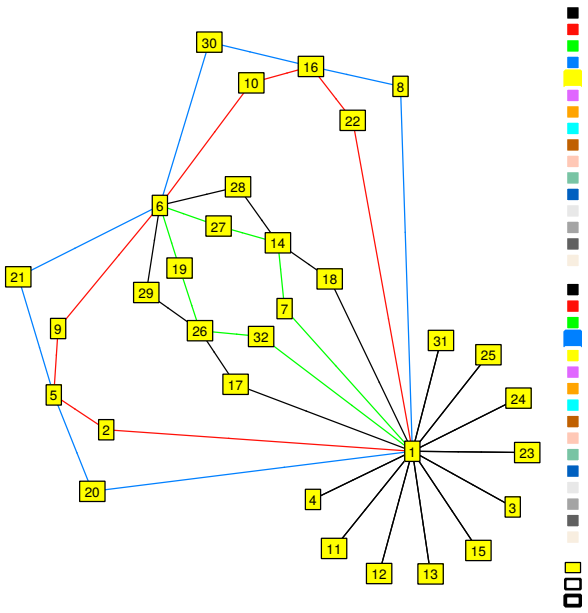

FIGURE 73. $G_{32}^{7} \cong (C_8 \rtimes C_2) \rtimes C_2$. $(2^{10}, 8^4)$. $(x_1^{32} + 11x_2^{16} + 4x_4^8 + 16x_8^4)/32$.

	2	7	8	17	3	4	11	12	13	15	23	24	25	31	1
2	5	25	14	31	7	8	17	18	19	21	27	28	29	32	2
7	31	14	25	5	2	17	8	9	10	28	20	21	22	30	7
8	26	23	16	12	17	2	7	27	28	10	18	19	32	29	8
17	12	16	23	26	8	7	2	20	21	19	9	10	30	22	17
3	17	8	7	2	1	11	4	5	6	24	14	15	16	26	3
4	21	28	10	19	11	1	3	23	24	6	12	13	31	25	4
11	19	10	28	21	4	3	1	14	15	13	5	6	26	16	11
12	27	30	18	22	16	23	26	1	5	31	4	14	6	15	12
13	28	21	19	10	6	24	15	16	1	11	26	4	5	14	13
15	8	17	2	7	24	6	13	31	11	1	25	3	23	12	15
23	29	9	32	20	26	12	16	4	14	25	1	5	15	6	23
24	7	2	17	8	15	13	6	26	4	3	16	1	14	5	24
25	32	20	29	9	5	31	14	6	16	23	15	26	1	4	25
31	18	22	27	30	14	25	5	15	26	12	6	16	4	1	31
1	2	7	8	17	3	4	11	12	13	15	23	24	25	31	1

TABLE 71. Cayley subtable for G_{32}^{7}.

	2	7	8	17	3	11	12	23	4	15	1
2	5	25	14	31	7	17	18	27	8	21	2
7	31	26	25	16	10	21	22	30	17	28	7
8	26	23	16	12	17	7	27	18	2	10	8
17	12	5	23	14	21	10	30	22	7	19	17
3	17	21	7	10	6	15	16	26	11	24	3
11	19	2	28	8	15	6	26	16	3	13	11
12	27	20	18	9	5	14	6	15	23	31	12
23	29	22	32	30	14	5	15	6	12	25	23
4	21	28	10	19	11	3	23	12	1	6	4
15	8	17	2	7	24	13	31	25	6	1	15
1	2	7	8	17	3	11	12	23	4	15	1

TABLE 72. Cayley subtable for G_{32}^{8}.

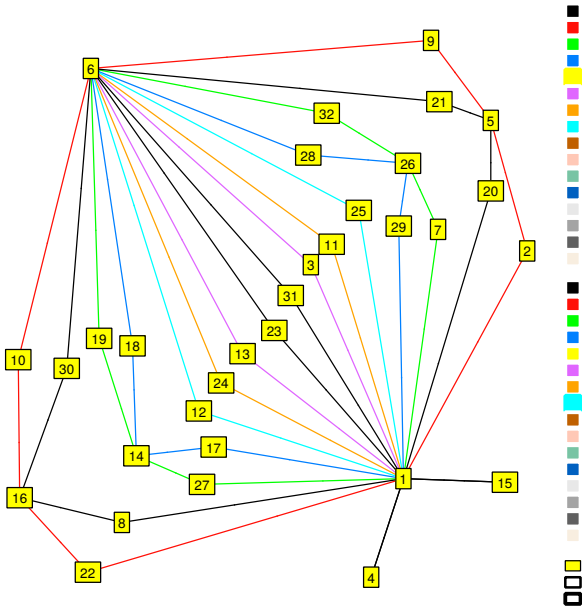

FIGURE 74. $G_{32}^{8} \cong C_8 \to ((C_4 \times C_2) \rtimes C_2)$. $(2^2, 4^4, 8^4)$. $(x_1^{32} + 3x_2^{16} + 12x_4^8 + 16x_8^4)/32$.

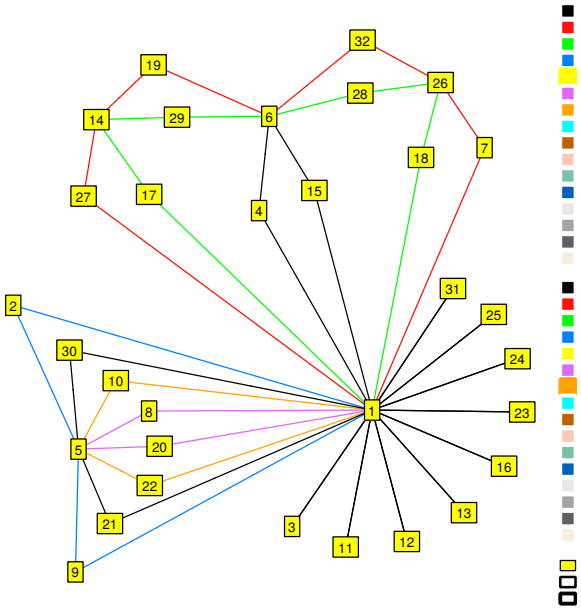

FIGURE 75. $G_{32}^{9} \cong (C_8 \times C_2) \rtimes C_2$. $(2^9, 4^5, 8^2)$. $(x_1^{32} + 11x_2^{16} + 12x_4^8 + 8x_8^4)/32$.

	7	17	2	4	8	10	21	3	11	12	13	16	23	24	25	31	1
7	26	5	23	17	25	31	12	2	8	9	10	29	20	21	22	30	7
17	5	14	12	19	23	25	31	21	2	30	8	32	9	10	20	22	17
2	12	23	5	8	14	16	26	7	17	18	19	22	27	28	29	32	2
4	17	19	21	6	2	8	10	24	3	31	11	26	12	13	23	25	4
8	23	25	26	10	5	14	16	28	7	32	17	30	18	19	27	29	8
10	25	31	16	21	26	5	14	19	28	29	7	9	32	17	18	27	10
21	31	12	14	2	16	26	5	17	19	27	28	20	29	7	32	18	21
3	21	2	17	11	19	28	7	1	4	5	6	25	14	15	16	26	3
11	2	8	7	13	17	19	28	15	1	26	4	31	5	6	14	16	11
12	30	9	27	23	29	32	18	5	14	1	16	13	4	26	6	15	12
13	8	10	28	24	7	17	19	6	15	16	1	12	26	4	5	14	13
16	29	32	22	26	30	9	20	25	31	13	12	1	24	23	3	11	16
23	9	20	18	25	27	29	32	26	5	15	14	24	1	16	4	6	23
24	10	21	19	3	28	7	17	4	6	14	15	23	16	1	26	5	24
25	20	22	32	31	18	27	29	16	26	6	5	3	15	14	1	4	25
31	22	30	29	12	32	18	27	14	16	4	26	11	6	5	15	1	31
1	7	17	2	4	8	10	21	3	11	12	13	16	23	24	25	31	1

TABLE 73. Cayley subtable for G_{32}^{9}.

	7	17	2	3	4	8	10	11	12	21	23	16	1
7	14	16	23	10	17	25	31	21	22	12	30	29	7
17	16	26	12	8	19	23	25	10	20	31	22	32	17
2	12	23	5	7	8	14	16	17	18	26	27	22	2
3	8	10	17	6	11	19	28	15	16	7	26	25	3
4	17	19	21	24	6	2	8	3	31	10	12	26	4
8	23	25	26	28	10	5	14	7	32	16	18	30	8
10	25	31	16	19	21	26	5	28	29	14	32	9	10
11	10	21	7	4	13	17	19	6	14	28	16	31	11
12	20	22	27	16	23	29	32	26	6	18	15	13	12
21	31	12	14	17	2	16	26	19	27	5	29	20	21
23	22	30	18	14	25	27	29	16	4	32	6	24	23
16	29	32	22	25	26	30	9	31	13	20	24	1	16
1	7	17	2	3	4	8	10	11	12	21	23	16	1

TABLE 74. Cayley subtable for G_{32}^{10}.

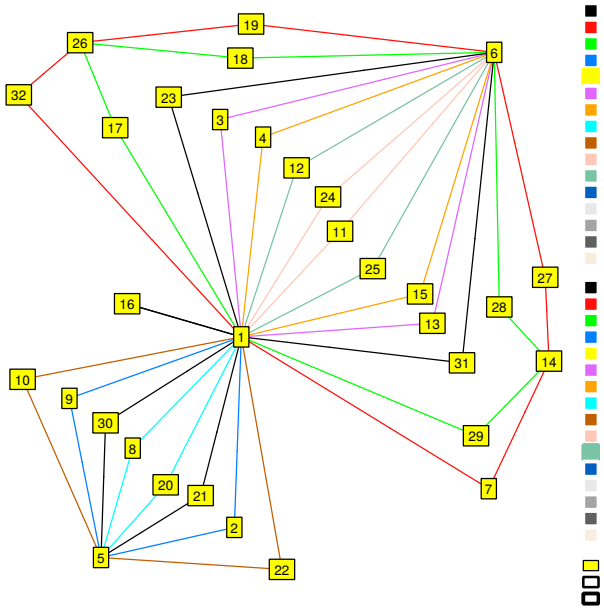

FIGURE 76. $G_{32}^{10} \cong Q_8 \rtimes C_4$. $(2^1, 4^9, 8^2)$. $(x_1^{32} + 3x_2^{16} + 20x_4^8 + 8x_8^4)/32$.

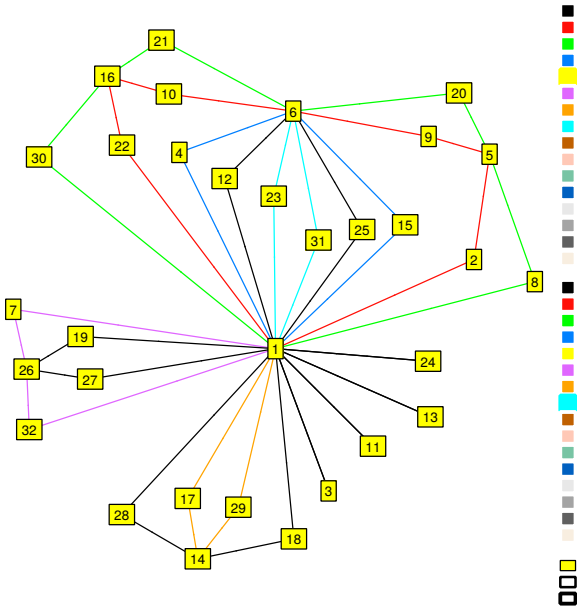

FIGURE 77. $G_{32}^{11} \cong (C_4 \times C_4) \rtimes C_2$. $(2^4, 4^7, 8^2)$. $(x_1^{32} + 7x_2^{16} + 16x_4^8 + 8x_8^4)/32$.

	2	8	4	7	12	17	18	19	23	3	11	13	24	1
2	5	14	8	12	18	23	13	25	27	7	17	19	28	2
8	26	5	10	23	32	25	24	31	18	28	7	17	19	8
4	21	2	6	17	31	19	27	28	12	24	3	11	13	4
7	23	25	17	26	9	5	4	14	20	2	8	10	21	7
12	27	29	23	30	6	9	8	20	15	5	14	16	26	12
17	12	23	19	5	30	14	6	16	9	21	2	8	10	17
18	24	3	27	4	10	6	14	15	21	9	20	22	30	18
19	31	12	28	14	22	16	15	26	30	10	21	2	8	19
23	18	27	25	9	4	20	10	22	6	26	5	14	16	23
3	17	19	11	21	5	2	30	8	14	1	4	6	15	3
11	7	17	13	2	26	8	9	10	5	15	1	4	6	11
13	28	7	24	8	16	10	20	21	26	6	15	1	4	13
24	19	28	3	10	14	21	22	2	16	4	6	15	1	24
1	2	8	4	7	12	17	18	19	23	3	11	13	24	1

TABLE 75. Cayley subtable for G_{32}^{11}.

	2	7	8	17	3	12	13	14	23	1
2	5	12	14	23	7	18	19	20	27	2
7	23	5	12	14	8	20	21	27	9	7
8	14	23	5	12	17	27	28	9	18	8
17	12	14	23	5	2	9	10	18	20	17
3	17	2	7	8	4	14	15	23	5	3
12	27	9	18	20	14	15	26	24	6	12
13	28	10	19	21	15	26	4	31	16	13
14	20	27	9	18	23	24	31	6	13	14
23	18	20	27	9	5	6	16	13	15	23
1	2	7	8	17	3	12	13	14	23	1

TABLE 76. Cayley subtable for G_{32}^{12}.

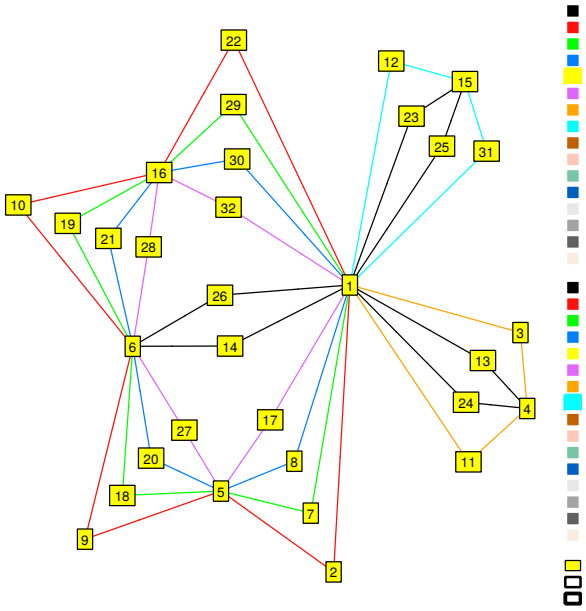

FIGURE 78. $G_{32}^{12} \cong C_4 \rtimes C_8$. $(4^5, 8^4)$. $(x_1^{32} + 3x_2^{16} + 12x_4^8 + 16x_8^4)/32$.

	3	12	2	7	8	10	14	17	18	21	27	1
3	4	14	17	10	19	28	23	21	22	7	30	3
12	14	4	27	22	29	32	11	30	10	18	21	12
2	7	18	5	12	14	16	20	23	3	26	11	2
7	8	20	23	16	25	31	27	26	6	12	15	7
8	17	27	26	31	5	14	22	12	24	16	3	8
10	19	29	16	25	26	5	30	31	13	14	24	10
14	23	11	30	32	9	20	6	18	28	22	7	14
17	10	22	12	14	23	25	29	16	4	31	6	17
18	20	8	11	6	13	24	17	15	16	3	26	18
21	28	32	14	23	16	26	9	25	11	5	13	21
27	22	10	3	4	11	13	19	6	14	24	16	27
1	3	12	2	7	8	10	14	17	18	21	27	1

TABLE 77. Cayley subtable for G_{32}^{13}.

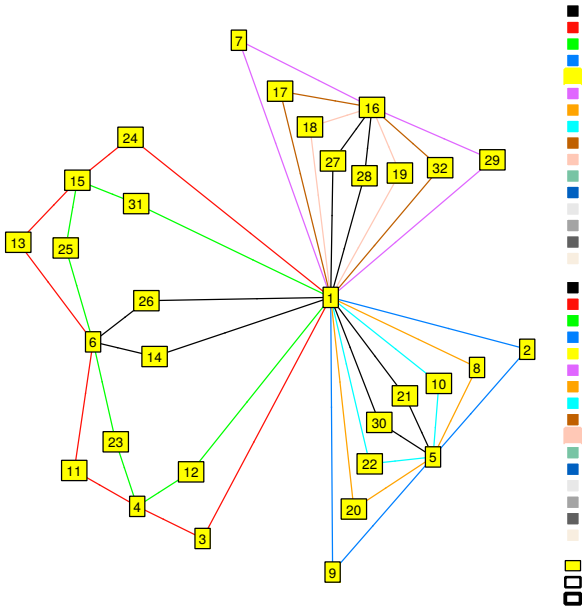

FIGURE 79. $G_{32}^{13} \cong C_8 \rtimes C_4$. $(4^9, 8^2)$. $(x_1^{32} + 3x_2^{16} + 20x_4^8 + 8x_8^4)/32$.

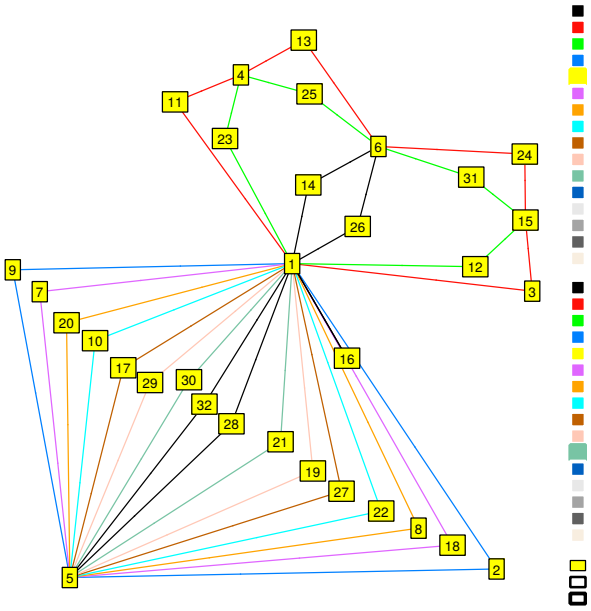

FIGURE 80. $G_{32}^{14} \cong C_8 \rtimes C_4$. $(2^1, 4^9, 8^2)$. $(x_1^{32} + 3x_2^{16} + 20x_4^8 + 8x_8^4)/32$.

	3	12	2	7	8	10	14	17	19	21	28	16	1
3	15	26	17	2	19	28	23	8	10	7	21	25	3
12	26	15	27	9	29	32	11	20	22	18	30	13	12
2	7	18	5	12	14	16	20	23	25	26	31	22	2
7	21	30	23	5	25	31	27	14	16	12	26	29	7
8	17	27	26	31	5	14	22	12	23	16	25	30	8
10	19	29	16	25	26	5	30	31	12	14	23	9	10
14	23	11	30	32	9	20	6	18	27	22	29	15	14
17	2	9	12	26	23	25	29	5	14	31	16	32	17
19	8	20	31	16	12	23	32	26	5	25	14	18	19
21	28	32	14	23	16	26	9	25	31	5	12	20	21
28	10	22	25	14	31	12	18	16	26	23	5	27	28
16	25	13	22	29	30	9	15	32	18	20	27	1	16
1	3	12	2	7	8	10	14	17	19	21	28	16	1

TABLE 78. Cayley subtable for G_{32}^{14}.

	2	3	7	8	12	17	14	26	1
2	5	7	12	14	18	23	20	30	2
3	17	4	10	19	14	21	23	31	3
7	23	8	16	25	20	26	27	32	7
8	26	17	31	5	27	12	22	9	8
12	27	14	22	29	15	30	24	11	12
17	12	10	14	23	22	16	29	18	17
14	30	23	32	9	24	18	1	6	14
26	20	31	27	22	11	29	6	1	26
1	2	3	7	8	12	17	14	26	1

TABLE 79. Cayley subtable for G_{32}^{15}.

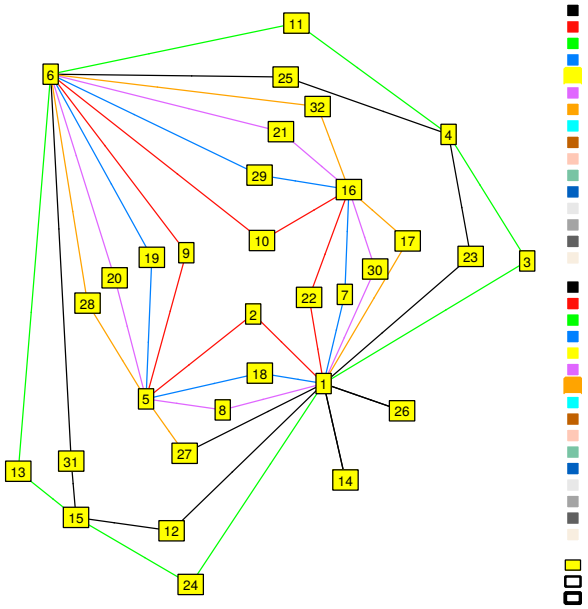

FIGURE 81. $G_{32}^{15} \cong C_4 \to D_8$. $(2^2, 8^6)$. $(x_1^{32} + 3x_2^{16} + 4x_4^8 + 24x_8^4)/32$.

	2	7	11	12	3	13	1
2	4	11	17	18	7	19	2
7	11	4	8	9	2	10	7
11	17	8	5	14	4	15	11
12	18	9	14	6	5	16	12
3	7	2	4	5	1	6	3
13	19	10	15	16	6	1	13
1	2	7	11	12	3	13	1

TABLE 80. Cayley subtable for G_{32}^{16}.

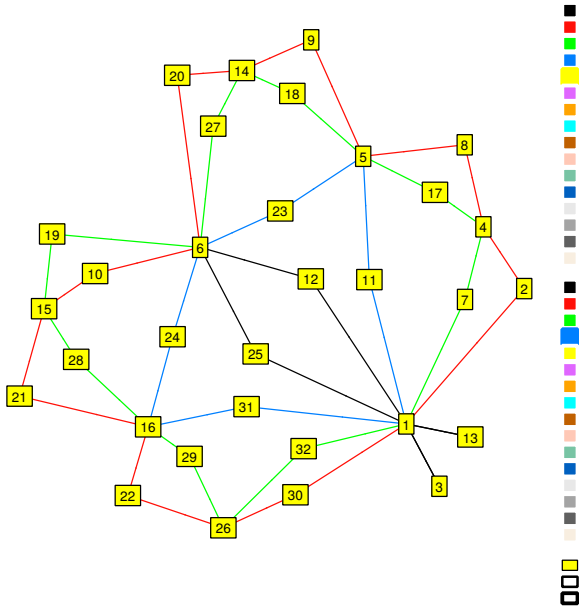

FIGURE 82. $G_{32}^{16} \cong C_{16} \rtimes C_2$. $(2^2, 4^1, 8^1, 16^2)$. $(x_1^{18} + x_1^{16}x_2 + x_1^2x_2^8 + x_2^9 + 2x_1^2x_4^4 + 2x_2x_4^4 + 4x_1^2x_8^2 + 8x_1^2x_{16} + 4x_2x_8^2 + 8x_2x_{16})/32$.

	2	7	11	12	3	13	1
2	4	11	17	18	7	19	2
7	24	15	8	9	2	10	7
11	28	21	5	14	4	15	11
12	29	22	14	6	5	16	12
3	19	10	4	5	1	6	3
13	7	2	15	16	6	1	13
1	2	7	11	12	3	13	1

TABLE 81. Cayley subtable for G_{32}^{17}.

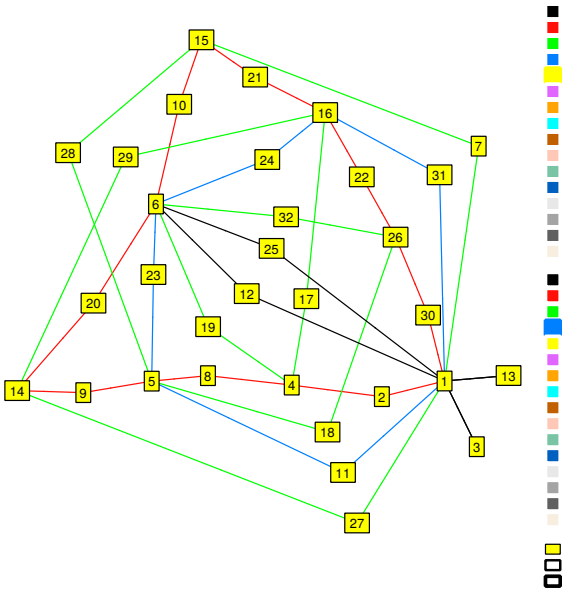

FIGURE 83. $G_{32}^{17} \cong C_{16} \rtimes C_2$. $(2^2, 4^1, 8^1, 16^2)$. $(x_1^{32} + 3x_2^{16} + 4x_4^8 + 8x_8^4 + 16x_{16}^2)/32$.

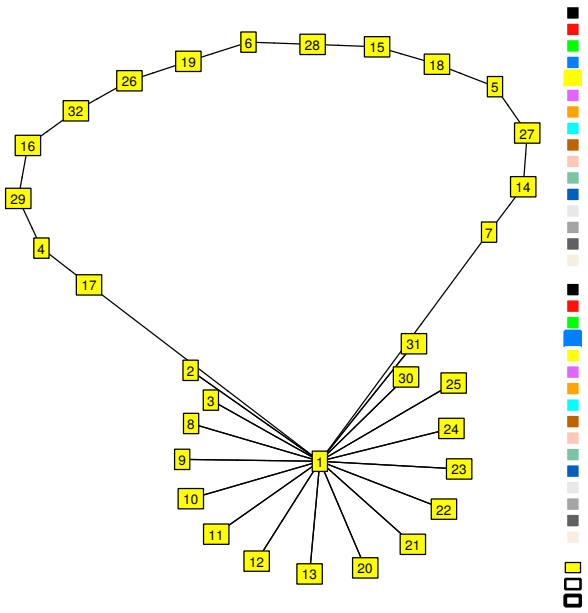

FIGURE 84. $G_{32}^{18} \cong D_{32}$. $(2^{16}, 16^1)$. $(x_1^{32} + 17x_2^{16} + 2x_4^8 + 4x_8^4 + 8x_{16}^2)/32$.

	7	2	3	8	9	10	11	12	13	20	21	22	23	24	25	30	31	1
7	14	11	2	25	23	24	8	9	10	3	12	31	20	21	22	13	30	7
2	3	1	7	4	5	6	17	18	19	14	15	16	27	28	29	26	32	2
3	20	17	1	29	27	28	4	5	6	7	18	32	14	15	16	19	26	3
8	11	14	27	1	15	26	7	28	32	5	6	4	18	19	17	16	29	8
9	12	16	29	26	1	5	32	7	18	4	14	6	17	27	19	15	28	9
10	13	6	19	15	16	1	28	29	7	26	4	5	32	17	18	14	27	10
11	2	7	14	17	18	19	1	15	26	27	28	29	5	6	4	32	16	11
12	21	32	16	19	17	27	26	1	5	29	7	28	4	14	6	18	15	12
13	30	28	6	18	32	17	15	16	1	19	29	27	26	4	5	7	14	13
20	23	4	17	16	14	15	29	27	28	1	5	26	7	18	32	6	19	20
21	24	26	32	6	4	14	19	17	27	16	1	15	29	7	28	5	18	21
22	25	5	18	14	6	16	27	19	29	15	26	1	28	32	7	4	17	22
23	9	29	4	32	7	18	16	14	15	17	27	19	1	5	26	28	6	23
24	10	19	26	28	29	7	6	4	14	32	17	18	16	1	15	27	5	24
25	8	27	5	7	28	32	14	6	16	18	19	17	15	26	1	29	4	25
30	31	15	28	5	26	4	18	32	17	6	16	14	19	29	27	1	7	30
31	22	18	15	27	19	29	5	26	4	28	32	7	6	16	14	17	1	31
1	7	2	3	8	9	10	11	12	13	20	21	22	23	24	25	30	31	1

TABLE 82. Cayley subtable for G_{32}^{18}.

	7	2	8	9	20	3	11	12	13	23	24	25	31	1
7	26	24	12	31	13	2	8	9	10	20	21	22	30	7
2	13	6	15	16	26	7	17	18	19	27	28	29	32	2
8	24	26	6	4	16	27	7	28	32	18	19	17	29	8
9	25	5	14	6	15	29	32	7	18	17	27	19	28	9
20	31	15	5	26	6	17	29	27	28	7	18	32	19	20
3	20	17	29	27	7	1	4	5	6	14	15	16	26	3
11	2	7	17	18	27	14	1	15	26	5	6	4	16	11
12	21	32	19	17	29	16	26	1	5	4	14	6	15	12
13	30	28	18	32	19	6	15	16	1	26	4	5	14	13
23	9	29	32	7	17	4	16	14	15	1	5	26	6	23
24	10	19	28	29	32	26	6	4	14	16	1	15	5	24
25	8	27	7	28	18	5	14	6	16	15	26	1	4	25
31	22	18	27	19	28	15	5	26	4	6	16	14	1	31
1	7	2	8	9	20	3	11	12	13	23	24	25	31	1

TABLE 83. Cayley subtable for G_{32}^{19}.

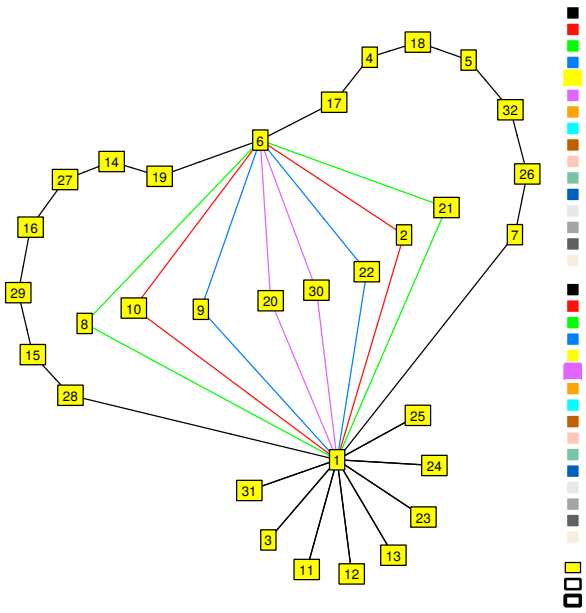

FIGURE 85. $G_{32}^{19} \cong QD_{32}$. $(2^8, 4^4, 16^1)$. $(x_1^{32} + 9x_2^{16} + 10x_4^8 + 4x_8^4 + 8x_{16}^2)/32$.

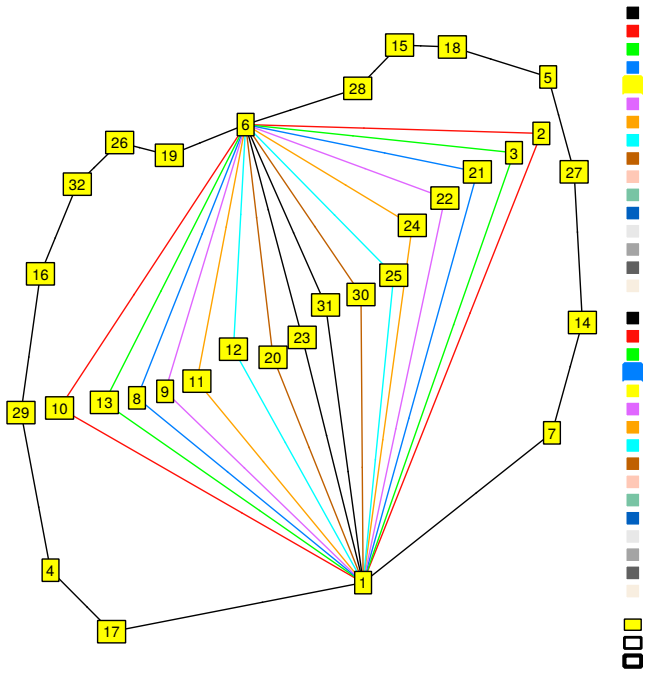

FIGURE 86. $G_{32}^{20} \cong Q_{32}$. $(4^8, 16^1)$. $(x_1^{32} + x_2^{16} + 18x_4^8 + 4x_8^4 + 8x_{16}^2)/32$.

	7	2	3	8	9	11	12	20	23	1
7	14	24	10	12	31	21	22	13	30	7
2	13	6	7	15	16	17	18	26	27	2
3	30	17	6	29	27	15	16	7	26	3
8	24	26	27	6	4	7	28	16	18	8
9	25	5	29	14	6	32	7	15	17	9
11	10	7	26	17	18	6	4	27	16	11
12	8	32	5	19	17	14	6	29	15	12
20	31	15	17	5	26	29	27	6	7	20
23	22	29	15	32	7	5	26	17	6	23
1	7	2	3	8	9	11	12	20	23	1

TABLE 84. Cayley subtable for G_{32}^{20}.

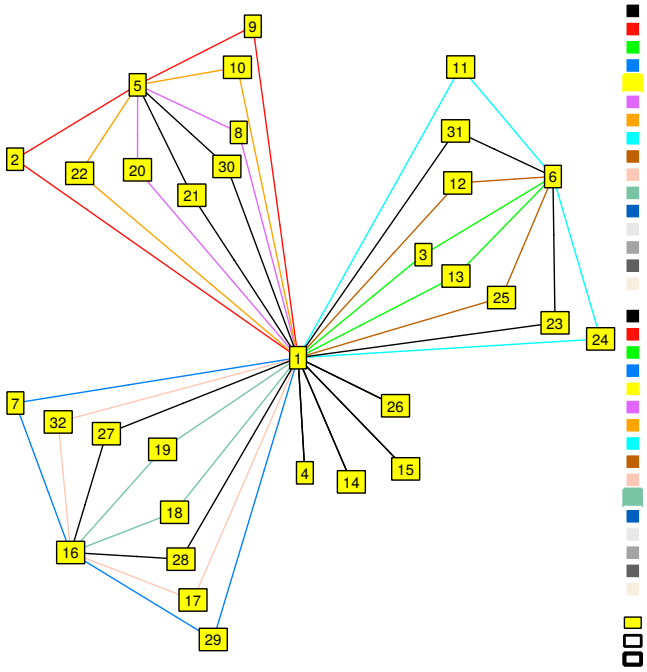

FIGURE 87. $G_{32}^{21} \cong C_4 \times C_4 \times C_2$. $(2^4, 4^{12})$. $(x_1^{10} + x_1^8 x_2 + 2x_1^6 x_2^2 + 4x_1^6 x_4 + 2x_1^4 x_2^3 + 4x_1^4 x_2 x_4 + x_1^2 x_2^4 + 4x_1^2 x_2^2 x_4 + x_2^5 + 4x_1^2 x_4^2 + 4x_2^3 x_4 + 4x_2 x_4^2)/32$.

	2	3	7	8	10	11	12	17	18	21	23	27	4	14	15	26	1
2	5	7	12	14	16	17	18	23	3	26	27	11	8	20	21	30	2
3	7	6	10	17	19	15	16	21	22	28	26	30	11	23	24	31	3
7	12	10	16	23	25	21	22	26	6	31	30	15	17	27	28	32	7
8	14	17	23	5	26	7	27	12	11	16	18	3	2	9	10	22	8
10	16	19	25	26	5	28	29	31	13	14	32	24	21	30	8	20	10
11	17	15	21	7	28	6	26	10	30	19	16	22	3	12	13	25	11
12	18	16	22	27	29	26	6	30	10	32	15	21	23	11	31	24	12
17	23	21	26	12	31	10	30	16	15	25	22	6	7	18	19	29	17
18	3	22	6	11	13	30	10	15	16	24	21	26	27	17	32	28	18
21	26	28	31	16	14	19	32	25	24	5	29	13	10	22	2	9	21
23	27	26	30	18	32	16	15	22	21	29	6	10	12	3	25	13	23
27	11	30	15	3	24	22	21	6	26	13	10	16	18	7	29	19	27
4	8	11	17	2	21	3	23	7	27	10	12	18	1	5	6	16	4
14	20	23	27	9	30	12	11	18	17	22	3	7	5	1	16	6	14
15	21	24	28	10	8	13	31	19	32	2	25	29	6	16	1	5	15
26	30	31	32	22	20	25	24	29	28	9	13	19	16	6	5	1	26
1	2	3	7	8	10	11	12	17	18	21	23	27	4	14	15	26	1

TABLE 85. Cayley subtable for G_{32}^{21}.

	2	7	8	9	17	18	20	27	3	4	5	11	12	13	14	15	23	24	25	26	31	1
2	6	13	15	16	24	25	26	31	7	8	9	17	18	19	20	21	27	28	29	30	32	2
7	25	16	31	13	26	6	24	15	2	17	18	8	9	10	27	28	20	21	22	32	30	7
8	15	24	6	26	13	31	16	25	17	2	20	7	27	28	9	10	18	19	32	22	29	8
9	16	25	26	6	31	13	15	24	18	20	2	27	7	29	8	30	17	32	19	21	28	9
17	31	26	25	24	16	15	13	6	8	7	27	2	20	21	18	19	9	10	30	29	22	17
18	13	6	24	25	15	16	31	26	9	27	7	20	2	22	17	32	8	30	10	28	21	18
20	26	31	16	15	25	24	6	13	27	9	8	18	17	32	2	22	7	29	28	10	19	20
27	24	15	13	31	6	26	25	16	20	18	17	9	8	30	7	29	2	22	21	19	10	27
3	18	9	27	7	20	2	17	8	1	11	12	4	5	6	23	24	14	15	16	31	26	3
4	8	17	2	20	7	27	9	18	11	1	14	3	23	24	5	6	12	13	31	16	25	4
5	9	18	20	2	27	7	8	17	12	14	1	23	3	25	4	26	11	31	13	15	24	5
11	27	20	18	17	9	8	7	2	4	3	23	1	14	15	12	13	5	6	26	25	16	11
12	7	2	17	18	8	9	27	20	5	23	3	14	1	16	11	31	4	26	6	24	15	12
13	29	22	32	19	30	10	28	21	6	24	25	15	16	1	31	11	26	4	5	23	14	13
14	20	27	9	8	18	17	2	7	23	5	4	12	11	31	1	16	3	25	24	6	13	14
15	21	28	10	30	19	32	22	29	24	6	26	13	31	11	16	1	25	3	23	5	12	15
23	17	8	7	27	2	20	18	9	14	12	11	5	4	26	3	25	1	16	15	13	6	23
24	32	30	29	28	22	21	19	10	15	13	31	6	26	4	25	3	16	1	14	12	5	24
25	19	10	28	29	21	22	32	30	16	31	13	26	6	5	24	23	15	14	1	11	4	25
26	30	32	22	21	29	28	10	19	31	16	15	25	24	23	6	5	13	12	11	1	3	26
31	28	21	19	32	10	30	29	22	26	25	24	16	15	14	13	12	6	5	4	3	1	31
1	2	7	8	9	17	18	20	27	3	4	5	11	12	13	14	15	23	24	25	26	31	1

TABLE 86. Cayley subtable for G_{32}^{22}.

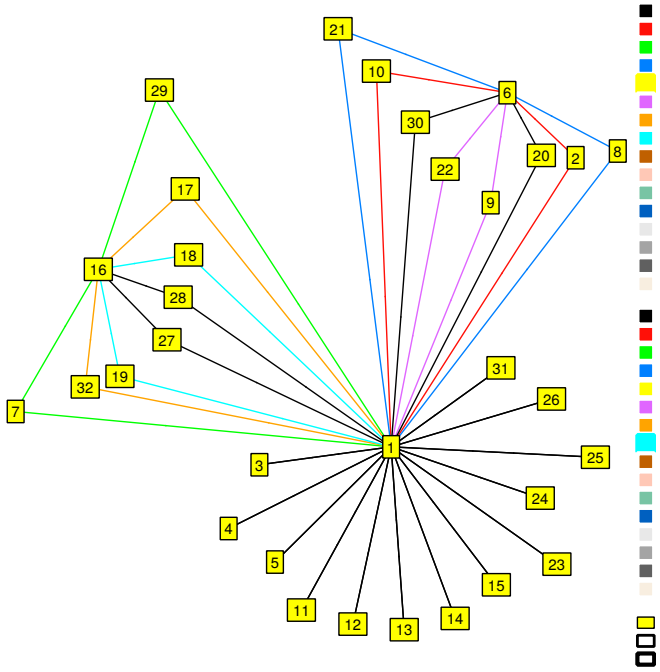

FIGURE 88. $G_{32}^{22} \cong C_2 \times ((C_4 \times C_2) \rtimes C_2)$. $(2^{13}, 4^8)$. $(x_1^{32} + 15x_2^{16} + 16x_4^8)/32$.

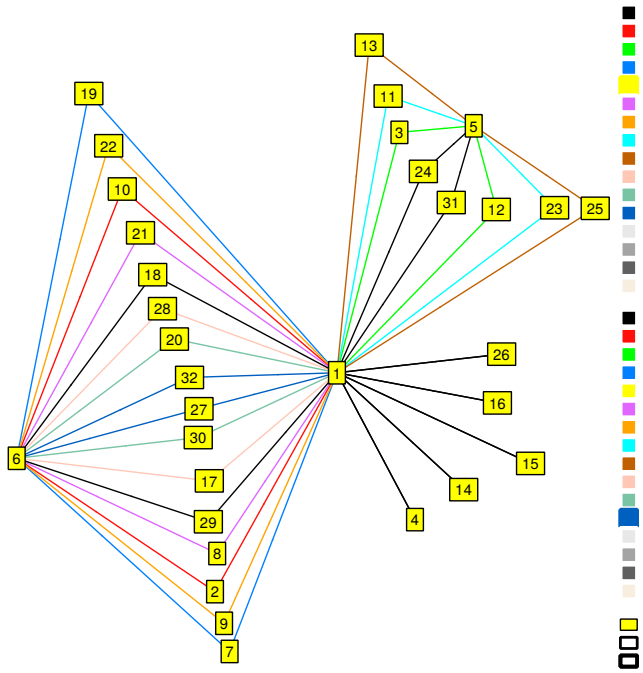

FIGURE 89. $G_{32}^{23} \cong C_2 \times (C_4 \rtimes C_4)$. $(2^5, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

	2	3	7	8	9	11	13	17	18	20	24	27	4	14	15	16	26	1
2	6	7	13	15	16	17	19	24	25	26	28	31	8	20	21	22	30	2
3	18	5	2	27	7	14	16	8	9	17	26	20	11	23	24	25	31	3
7	25	9	6	31	13	20	22	15	16	24	30	26	17	27	28	29	32	7
8	15	17	24	6	26	7	28	13	31	16	19	25	2	9	10	30	22	8
9	16	18	25	26	6	27	29	31	13	15	32	24	20	8	30	10	21	9
11	27	14	8	18	17	5	26	2	20	7	16	9	3	12	13	31	25	11
13	29	16	10	32	19	26	5	21	22	28	14	30	24	31	11	12	23	13
17	31	20	15	25	24	9	30	6	26	13	22	16	7	18	19	32	29	17
18	13	2	16	24	25	8	10	26	6	31	21	15	27	17	32	19	28	18
20	26	27	31	16	15	18	32	25	24	6	29	13	9	2	22	21	10	20
24	32	26	21	29	28	16	14	10	30	19	5	22	13	25	3	23	12	24
27	24	8	26	13	31	2	21	16	15	25	10	6	18	7	29	28	19	27
4	8	11	17	2	20	3	24	7	27	9	13	18	1	5	6	26	16	4
14	20	23	27	9	8	12	31	18	17	2	25	7	5	1	16	15	6	14
15	21	24	28	10	30	13	11	19	32	22	3	29	6	16	1	14	5	15
16	22	25	29	30	10	31	12	32	19	21	23	28	26	15	14	1	4	16
26	30	31	32	22	21	25	23	29	28	10	12	19	16	6	5	4	1	26
1	2	3	7	8	9	11	13	17	18	20	24	27	4	14	15	16	26	1

TABLE 87. Cayley subtable for G_{32}^{23}.

	2	4	7	8	9	11	15	17	18	20	24	27	3	12	13	25	1
2	6	8	13	15	16	17	21	24	25	26	28	31	7	18	19	29	2
4	8	5	17	9	20	12	16	18	27	2	25	7	11	23	24	31	4
7	25	17	16	31	13	8	28	26	6	24	21	15	2	9	10	22	7
8	15	9	24	16	26	18	22	25	31	6	29	13	17	27	28	32	8
9	16	20	25	26	6	27	30	31	13	15	32	24	18	7	29	19	9
11	27	12	20	7	17	5	25	2	8	18	16	9	4	14	15	26	11
15	21	16	28	22	30	25	5	29	32	10	12	19	24	31	11	23	15
17	31	18	26	13	24	9	29	6	15	25	22	16	8	20	21	30	17
18	13	27	6	24	25	20	32	15	16	31	30	26	9	2	22	10	18
20	26	2	31	6	15	7	10	13	24	16	19	25	27	17	32	28	20
24	32	25	30	19	28	16	12	10	21	29	5	22	15	26	4	14	24
27	24	7	15	25	31	2	19	16	26	13	10	6	20	8	30	21	27
3	18	11	9	27	7	4	24	20	2	17	15	8	1	5	6	16	3
12	7	23	2	17	18	14	31	8	9	27	26	20	5	1	16	6	12
13	29	24	22	32	19	15	11	30	10	28	4	21	6	16	1	5	13
25	19	31	10	28	29	26	23	21	22	32	14	30	16	6	5	1	25
1	2	4	7	8	9	11	15	17	18	20	24	27	3	12	13	25	1

TABLE 88. Cayley subtable for G_{32}^{24}.

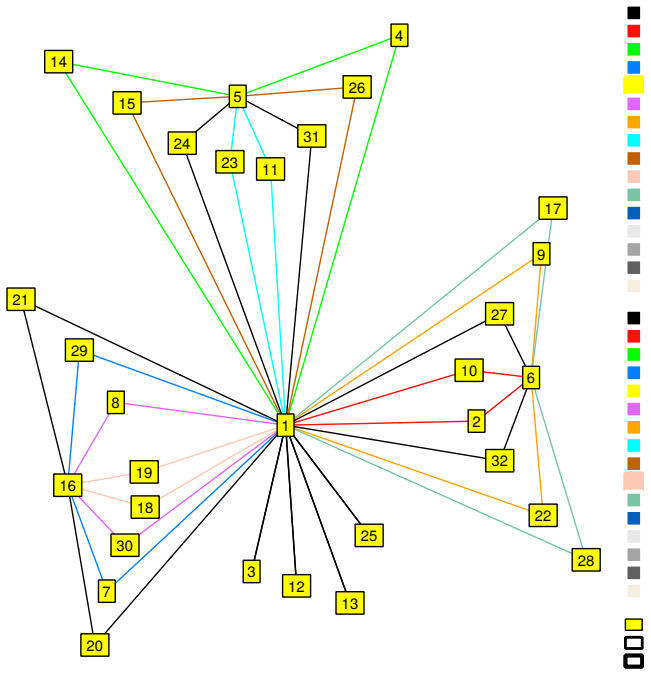

FIGURE 90. $G_{32}^{24} \cong (C_2 \times C_4) \rtimes C_2$. $(2^4, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

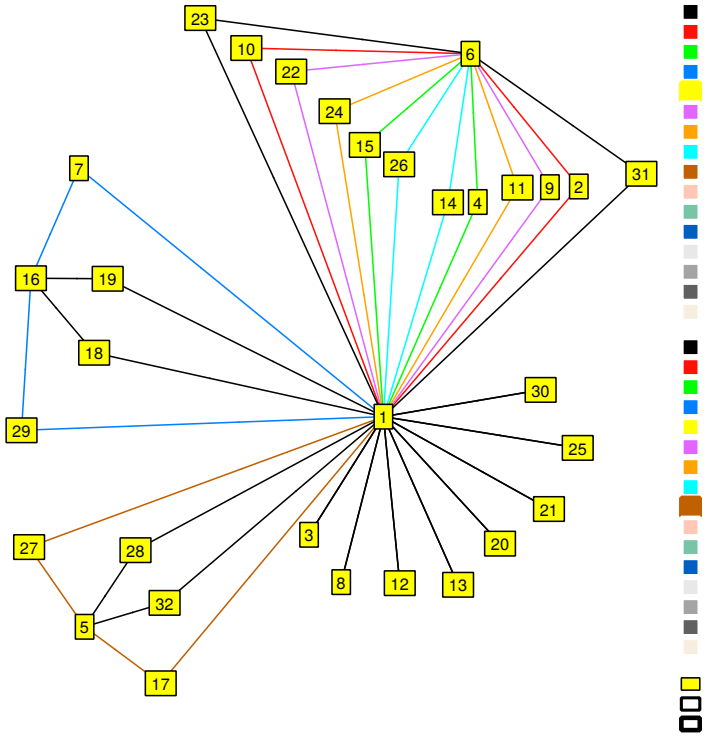

FIGURE 91. $G_{32}^{25} \cong C_2 \times D_8$. $(2^8, 4^{10})$. $(x_1^{32} + 11x_2^{16} + 20x_4^8)/32$.

	2	4	7	9	11	14	17	18	23	28	3	8	12	13	20	21	25	30	1
2	6	8	13	16	17	20	24	25	27	11	7	15	18	19	26	4	29	14	2
4	8	6	17	20	13	16	19	27	25	7	11	10	23	24	22	2	31	9	4
7	25	17	16	13	8	27	26	6	20	14	2	31	9	10	24	23	22	11	7
9	16	20	25	6	27	8	31	13	17	23	18	26	7	29	15	14	19	4	9
11	27	13	20	17	6	25	22	8	16	9	4	29	14	15	19	18	26	7	11
14	20	16	27	8	25	6	29	17	13	18	23	22	11	31	10	9	24	2	14
17	31	19	26	24	10	29	5	15	22	16	8	12	20	21	3	25	30	13	17
18	13	27	6	25	20	17	15	16	8	4	9	24	2	22	31	11	10	23	18
23	17	25	8	27	16	13	10	20	6	2	14	19	4	26	29	7	15	18	23
28	23	7	14	11	2	18	16	4	9	5	21	25	30	8	13	12	20	3	28
3	18	11	9	7	4	23	20	2	14	30	1	27	5	6	17	32	16	28	3
8	15	10	24	26	19	22	3	31	29	13	17	1	27	28	5	6	32	16	8
12	7	23	2	18	14	11	8	9	4	21	5	17	1	16	27	28	6	32	12
13	29	24	22	19	15	31	30	10	26	20	6	32	16	1	28	27	5	17	13
20	26	22	31	15	29	10	12	24	19	25	27	5	17	32	1	16	28	6	20
21	4	2	11	14	7	9	13	23	18	3	28	6	32	17	16	1	27	5	21
25	19	31	10	29	26	24	21	22	15	8	16	28	6	5	32	17	1	27	25
30	14	9	23	4	18	2	25	11	7	12	32	16	28	27	6	5	17	1	30
1	2	4	7	9	11	14	17	18	23	28	3	8	12	13	20	21	25	30	1

TABLE 89. Cayley subtable for G_{32}^{25}.

	2	3	4	7	8	9	11	13	14	17	18	21	23	28	1
2	6	7	8	13	15	16	17	19	20	24	25	4	27	11	2
3	18	5	11	2	27	7	14	16	23	8	9	32	4	21	3
4	8	11	16	17	22	20	25	24	6	29	27	9	13	18	4
7	25	9	17	6	31	13	20	22	27	15	16	23	8	4	7
8	15	17	22	24	5	26	29	28	10	12	31	16	19	25	8
9	16	18	20	25	26	6	27	29	8	31	13	14	17	23	9
11	27	14	25	8	19	17	6	26	13	22	20	7	16	9	11
13	29	16	24	10	32	19	26	5	31	21	22	27	15	8	13
14	20	23	6	27	10	8	13	31	16	19	17	2	25	7	14
17	31	20	29	15	3	24	10	30	19	5	26	13	22	16	17
18	13	2	27	16	24	25	8	10	17	26	6	11	20	14	18
21	4	28	9	11	16	14	18	17	2	25	23	5	7	12	21
23	17	4	13	20	29	27	16	15	25	10	8	18	6	2	23
28	23	30	18	4	13	11	2	20	7	16	14	3	9	5	28
1	2	3	4	7	8	9	11	13	14	17	18	21	23	28	1

TABLE 90. Cayley subtable for G_{32}^{26}.

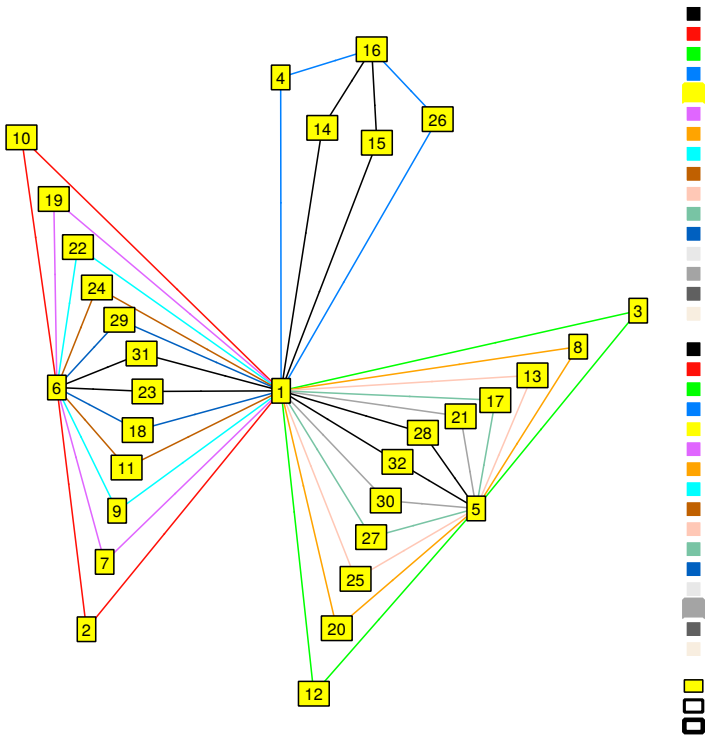

FIGURE 92. $G_{32}^{26} \cong C_4 \times Q_8$. (4^{14}). $(x_1^{32} + 3x_2^{16} + 28x_4^8)/32$.

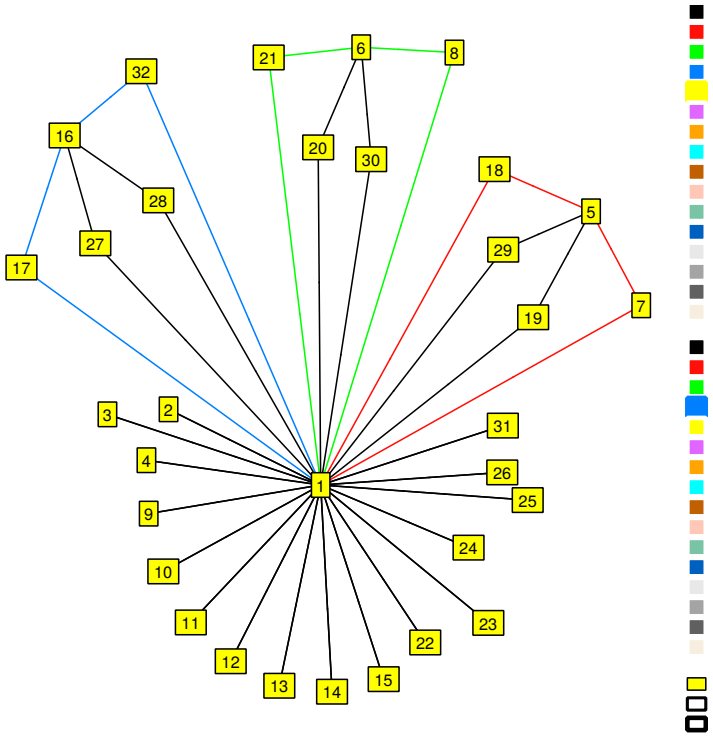

FIGURE 93. $G_{32}^{27} \cong (C_2 \times C_2 \times C_2 \times C_2) \rtimes C_2$. $(2^{16}, 4^6)$. $(x_1^{32} + 19x_2^{16} + 12x_4^8)/32$.

	7	8	17	19	20	27	2	3	4	9	10	11	12	13	14	15	22	23	24	25	26	31	1
7	5	23	14	16	11	4	12	2	17	3	25	8	9	10	27	28	13	20	21	22	32	30	7
8	24	6	13	11	16	25	15	17	2	26	4	7	27	28	9	10	14	18	19	32	22	29	8
17	26	25	16	14	13	6	31	8	7	24	23	2	20	21	18	19	11	9	10	30	29	22	17
19	16	31	26	5	24	15	25	10	28	13	12	21	22	2	32	17	3	30	8	9	27	20	19
20	31	16	25	23	6	13	26	27	9	15	14	18	17	32	2	22	4	7	29	28	10	19	20
27	15	13	6	4	25	16	24	20	18	31	11	9	8	30	7	29	23	2	22	21	19	10	27
2	3	4	11	13	14	23	1	7	8	5	6	17	18	19	20	21	16	27	28	29	30	32	2
3	9	27	20	22	17	8	18	1	11	7	29	4	5	6	23	24	19	14	15	16	31	26	3
4	28	10	19	17	22	29	21	11	1	30	8	3	23	24	5	6	20	12	13	31	16	25	4
9	12	14	23	25	4	11	5	18	20	1	16	27	7	29	8	30	6	17	32	19	21	28	9
10	13	15	24	3	26	31	6	19	21	16	1	28	29	7	30	8	5	32	17	18	20	27	10
11	30	29	22	20	19	10	32	4	3	28	27	1	14	15	12	13	17	5	6	26	25	16	11
12	2	17	8	10	27	20	7	5	23	18	19	14	1	16	11	31	29	4	26	6	24	15	12
13	22	32	30	9	28	21	29	6	24	19	18	15	16	1	31	11	7	26	4	5	23	14	13
14	32	22	29	27	10	19	30	23	5	21	20	12	11	31	1	16	8	3	25	24	6	13	14
15	17	2	7	28	9	18	8	24	6	20	21	13	31	11	16	1	30	25	3	23	5	12	15
22	25	26	31	12	15	24	16	29	30	6	5	32	19	18	21	20	1	28	27	7	8	17	22
23	21	19	10	8	29	22	28	14	12	32	17	5	4	26	3	25	27	1	16	15	13	6	23
24	20	18	9	30	7	2	27	15	13	17	32	6	26	4	25	3	28	16	1	14	12	5	24
25	10	28	21	2	32	30	19	16	31	29	7	26	6	5	24	23	18	15	14	1	11	4	25
26	27	9	18	32	2	7	20	31	16	8	30	25	24	23	6	5	21	13	12	11	1	3	26
31	8	7	2	21	18	9	17	26	25	27	28	16	15	14	13	12	32	6	5	4	3	1	31
1	7	8	17	19	20	27	2	3	4	9	10	11	12	13	14	15	22	23	24	25	26	31	1

TABLE 91. Cayley subtable for G_{32}^{27}.

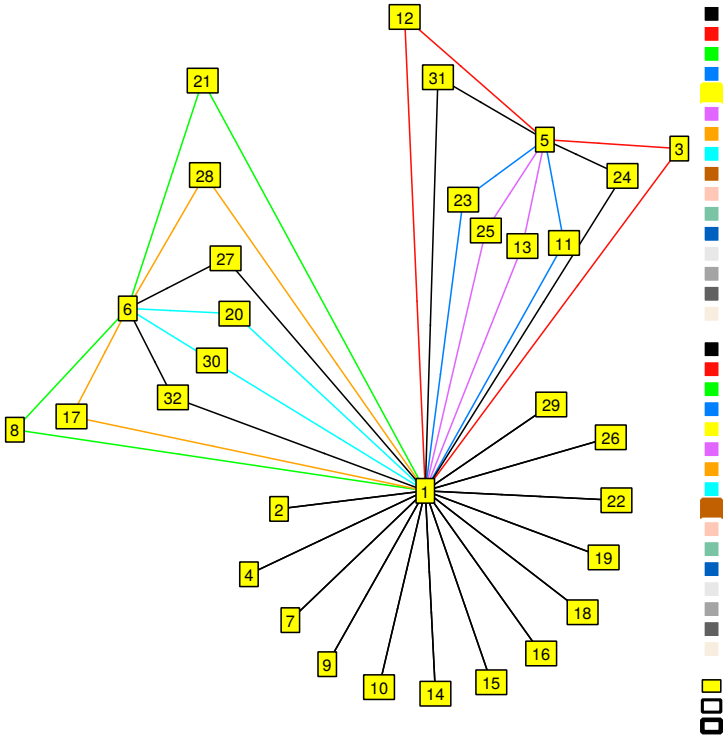

FIGURE 94. $G_{32}^{28} \cong (C_4 \times C_2 \times C_2) \rtimes C_2$. $(2^{13}, 4^8)$. $(x_1^{32} + 15x_2^{16} + 16x_4^8)/32$.

	3	8	11	13	17	20	24	27	2	4	7	9	10	14	15	16	18	19	22	26	29	1
3	5	27	14	16	8	17	26	20	18	11	2	7	29	23	24	25	9	10	19	31	22	3
8	17	6	7	28	13	16	19	25	15	2	24	26	4	9	10	30	31	11	14	22	23	8
11	14	29	5	26	10	19	16	22	32	3	21	28	27	12	13	31	30	8	17	25	20	11
13	16	32	26	5	21	28	14	30	29	24	10	19	18	31	11	12	22	2	7	23	9	13
17	20	25	9	30	6	13	22	16	31	7	15	24	23	18	19	32	26	4	11	29	14	17
20	27	16	18	32	25	6	29	13	26	9	31	15	14	2	22	21	24	23	4	10	11	20
24	26	18	16	14	2	7	5	9	27	13	8	17	32	25	3	23	20	21	28	12	30	24
27	8	13	2	21	16	25	10	6	24	18	26	31	11	7	29	28	15	14	23	19	4	27
2	7	4	17	19	11	14	28	23	1	8	3	5	6	20	21	22	12	13	16	30	25	2
4	11	10	3	24	19	22	13	29	21	1	28	30	8	5	6	26	32	17	20	16	27	4
7	9	23	20	22	4	11	30	14	12	17	1	3	25	27	28	29	5	6	13	32	16	7
9	18	14	27	29	23	4	32	11	5	20	12	1	16	8	30	10	3	25	6	21	13	9
10	19	15	28	7	24	26	17	31	6	21	13	16	1	30	8	9	25	3	5	20	12	10
14	23	22	12	31	29	10	25	19	30	5	32	21	20	1	16	15	28	27	8	6	17	14
15	24	2	13	11	7	9	3	18	8	6	17	20	21	16	1	14	27	28	30	5	32	15
16	25	30	31	12	32	21	23	28	22	26	29	10	9	15	14	1	19	18	2	4	7	16
18	2	11	8	10	14	23	21	4	3	27	5	12	13	17	32	19	1	16	25	28	6	18
19	22	31	30	9	15	24	20	26	25	28	6	13	12	32	17	18	16	1	3	27	5	19
22	29	26	32	18	31	15	27	24	16	30	25	6	5	21	20	2	13	12	1	8	3	22
26	31	9	25	23	18	2	12	7	20	16	27	8	30	6	5	4	17	32	21	1	28	26
29	10	24	21	2	26	31	8	15	13	32	16	25	3	28	27	7	6	5	12	17	1	29
1	3	8	11	13	17	20	24	27	2	4	7	9	10	14	15	16	18	19	22	26	29	1

TABLE 92. Cayley subtable for G_{32}^{28}.

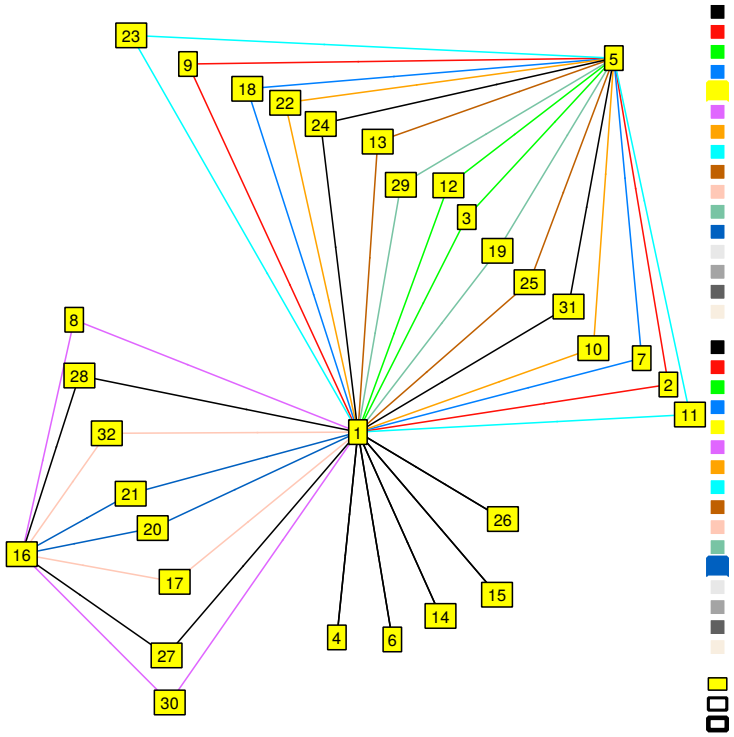

FIGURE 95. $G_{32}^{29} \cong (C_4 \times Q_8) \rtimes C_2$. $(2^5, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

	2	3	7	8	10	11	13	17	19	20	24	27	4	6	14	15	26	1
2	5	7	12	14	16	17	19	23	25	4	28	11	8	10	20	21	30	2
3	18	5	2	27	29	14	16	8	10	17	26	20	11	13	23	24	31	3
7	3	9	5	11	13	20	22	14	16	23	30	4	17	19	27	28	32	7
8	26	17	31	16	14	7	28	25	23	6	19	13	2	21	9	10	22	8
10	16	19	25	26	5	28	7	31	12	15	17	24	21	2	30	8	20	10
11	32	14	21	29	27	5	26	10	8	19	16	22	3	24	12	13	25	11
13	29	16	10	32	18	26	5	21	2	28	14	30	24	3	31	11	23	13
17	24	20	26	13	11	9	30	16	14	25	22	6	7	28	18	19	29	17
19	13	22	16	24	3	30	9	26	5	31	20	15	28	7	32	17	27	19
20	15	27	24	6	4	18	32	13	11	16	29	25	9	30	2	22	10	20
24	27	26	8	18	32	16	14	2	21	7	5	9	13	11	25	3	12	24
27	31	8	15	25	23	2	21	6	4	13	10	16	18	32	7	29	19	27
4	21	11	28	10	8	3	24	19	17	22	13	29	1	15	5	6	16	4
6	10	13	19	21	2	24	3	28	7	30	11	32	15	1	26	4	14	6
14	30	23	32	22	20	12	31	29	27	10	25	19	5	26	1	16	6	14
15	8	24	17	2	21	13	11	7	28	9	3	18	6	4	16	1	5	15
26	20	31	27	9	30	25	23	18	32	2	12	7	16	14	6	5	1	26
1	2	3	7	8	10	11	13	17	19	20	24	27	4	6	14	15	26	1

TABLE 93. Cayley subtable for G_{32}^{29}.

	4	7	8	11	15	17	19	20	24	27	2	3	9	10	12	13	22	25	1
4	5	28	22	12	16	29	17	10	25	19	21	11	30	8	23	24	20	31	4
7	17	5	23	8	28	14	16	11	21	4	12	2	3	25	9	10	13	22	7
8	9	24	16	18	22	25	11	6	29	13	15	17	26	4	27	28	14	32	8
11	12	30	19	5	25	10	20	29	16	22	32	4	28	27	14	15	17	26	11
15	16	17	9	25	5	18	28	2	12	7	8	24	20	21	31	11	30	23	15
17	18	26	13	9	29	6	14	25	22	16	31	8	24	23	20	21	11	30	17
19	28	16	31	21	17	26	5	24	8	15	25	10	13	12	22	2	3	9	19
20	2	31	6	7	10	13	23	16	19	25	26	27	15	14	17	32	4	28	20
24	25	20	7	16	12	2	30	18	5	9	27	15	17	32	26	4	28	14	24
27	7	15	25	2	19	16	4	13	10	6	24	20	31	11	8	30	23	21	27
2	8	3	4	17	21	11	13	14	28	23	1	7	5	6	18	19	16	29	2
3	11	9	27	4	24	20	22	17	15	8	18	1	7	29	5	6	19	16	3
9	20	12	14	27	30	23	25	4	32	11	5	18	1	16	7	29	6	19	9
10	21	13	15	28	8	24	3	26	17	31	6	19	16	1	29	7	5	18	10
12	23	2	17	14	31	8	10	27	26	20	7	5	18	19	1	16	29	6	12
13	24	22	32	15	11	30	9	28	4	21	29	6	19	18	16	1	7	5	13
22	30	25	26	32	20	31	12	15	27	24	16	29	6	5	19	18	1	7	22
25	31	10	28	26	23	21	2	32	14	30	19	16	29	7	6	5	18	1	25
1	4	7	8	11	15	17	19	20	24	27	2	3	9	10	12	13	22	25	1

TABLE 94. Cayley subtable for G_{32}^{30}.

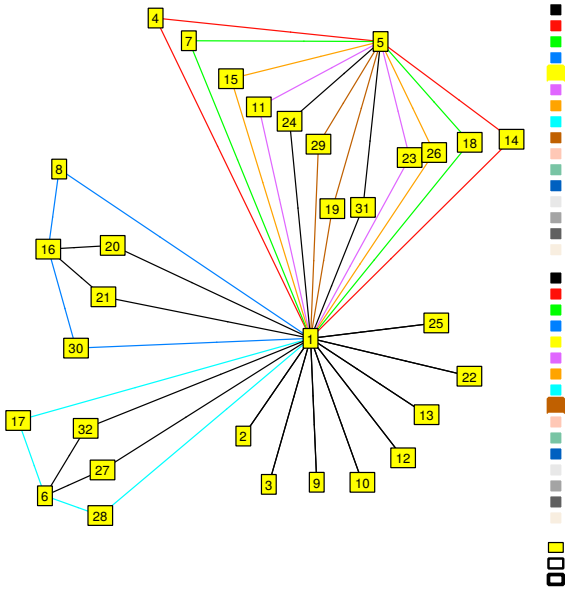

FIGURE 96. $G_{32}^{30} \cong (C_4 \times C_2 \times C_2) \rtimes C_2$. $(2^8, 4^{10})$. $(x_1^{32} + 11x_2^{16} + 20x_4^8)/32$.

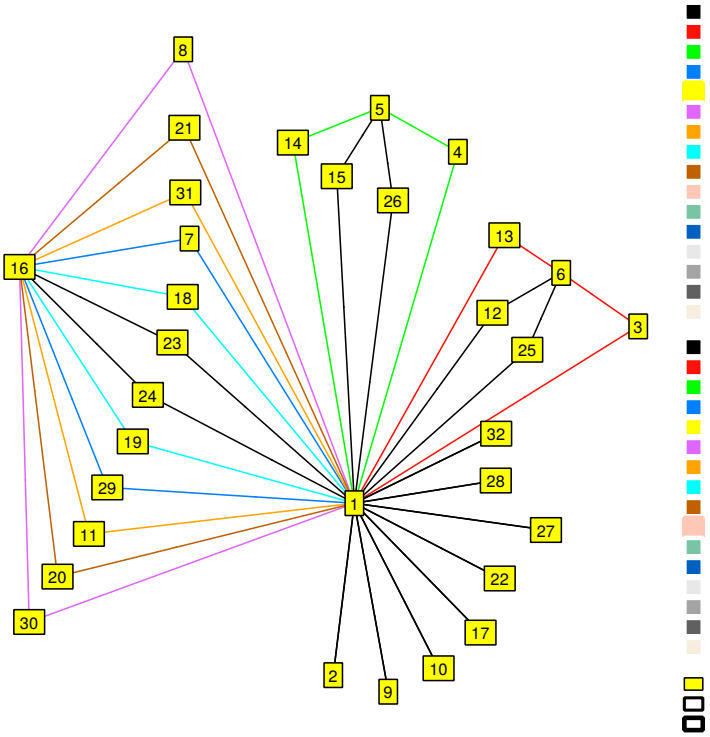

FIGURE 97. $G_{32}^{31} \cong (C_4 \times C_4) \rtimes C_2$. $(2^8, 4^{10})$. $(x_1^{32} + 11x_2^{16} + 20x_4^8)/32$.

	3	4	7	8	11	12	15	18	20	23	2	9	10	17	22	27	28	32	1
3	6	11	22	27	15	16	24	10	17	26	18	7	29	30	19	21	20	8	3
4	11	5	28	22	12	23	16	32	10	3	21	30	8	29	20	19	18	7	4
7	10	17	16	23	21	22	28	6	11	30	12	3	25	26	13	15	14	4	7
8	17	9	24	16	18	27	22	31	6	7	15	26	4	25	14	13	12	3	8
11	15	12	20	19	16	26	25	8	29	6	32	28	27	2	17	9	10	22	11
12	16	23	10	17	26	6	31	22	27	15	7	18	19	21	29	30	8	20	12
15	24	16	17	9	25	31	5	27	2	13	8	20	21	18	30	7	29	19	15
18	22	27	6	11	30	10	32	16	23	21	3	12	13	15	25	26	4	14	18
20	27	2	31	6	7	17	10	24	16	18	26	15	14	13	4	25	3	12	20
23	26	3	8	29	6	15	13	20	19	16	28	32	17	9	27	2	22	10	23
2	7	8	3	4	17	18	21	12	14	27	1	5	6	11	16	23	24	31	2
9	18	20	12	14	27	7	30	3	4	17	5	1	16	23	6	11	31	24	9
10	19	21	13	15	28	29	8	25	26	32	6	16	1	24	5	31	11	23	10
17	21	18	14	13	22	30	29	4	25	10	31	24	23	1	11	5	6	16	17
22	29	30	25	26	32	19	20	13	15	28	16	6	5	31	1	24	23	11	22
27	30	7	4	25	10	21	19	14	13	22	24	31	11	5	23	1	16	6	27
28	8	29	26	3	9	20	18	15	12	2	23	11	31	6	24	16	1	5	28
32	20	19	15	12	2	8	7	26	3	9	11	23	24	16	31	6	5	1	32
1	3	4	7	8	11	12	15	18	20	23	2	9	10	17	22	27	28	32	1

TABLE 95. Cayley subtable for G_{32}^{31}.

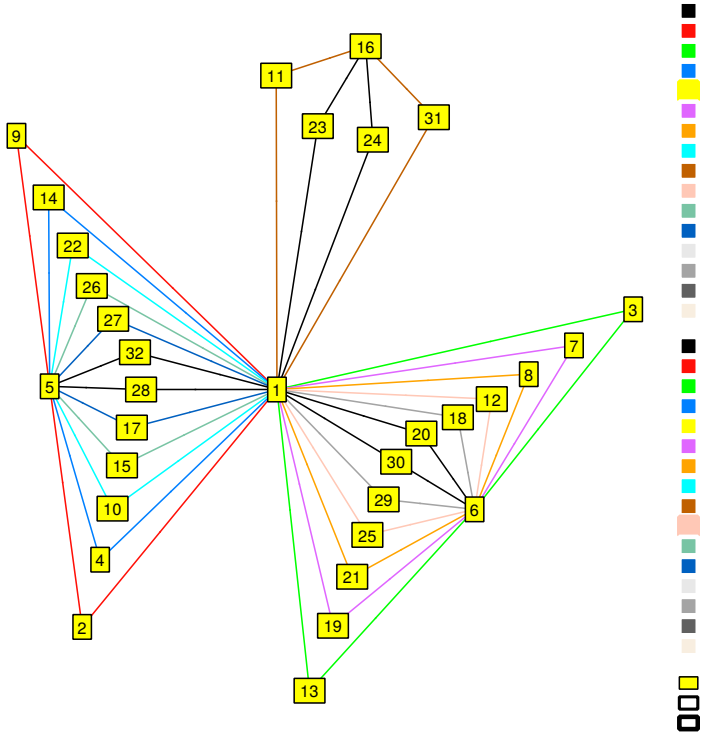

FIGURE 98. $G_{32}^{32} \cong (C_2 \times C_2) \to (C_2 \times C_2 \times C_2)$. (4^{14}). $(x_1^{32} + 3x_2^{16} + 28x_4^8)/32$.

	2	3	4	7	8	10	11	12	15	17	18	20	23	28	1
2	5	7	8	12	14	16	17	18	21	23	3	4	27	31	2
3	18	6	11	22	27	29	15	16	24	30	10	17	26	20	3
4	21	11	5	28	22	8	12	23	16	29	32	10	3	18	4
7	3	10	17	6	11	13	21	22	28	15	16	23	30	4	7
8	26	17	9	31	6	14	18	27	22	13	24	16	7	3	8
10	16	19	21	25	26	5	28	29	8	31	13	15	32	23	10
11	32	15	12	20	19	27	16	26	25	2	8	29	6	10	11
12	7	16	23	10	17	19	26	6	31	21	22	27	15	8	12
15	8	24	16	17	9	21	25	31	5	18	27	2	13	29	15
17	24	21	18	4	25	11	22	30	29	5	14	13	10	16	17
18	12	22	27	16	23	25	30	10	32	26	6	11	21	14	18
20	15	27	2	24	16	4	7	17	10	25	31	6	18	12	20
23	28	26	3	8	29	17	6	15	13	9	20	19	16	22	23
28	11	8	29	15	12	24	9	20	18	16	26	3	2	5	28
1	2	3	4	7	8	10	11	12	15	17	18	20	23	28	1

TABLE 96. Cayley subtable for G_{32}^{32}.

	3	4	7	8	11	12	15	17	18	20	23	28	2	9	10	22	1
3	16	11	10	27	26	6	24	21	22	17	15	8	18	7	29	19	3
4	11	5	28	22	12	23	16	29	32	10	3	18	21	30	8	20	4
7	22	17	6	23	30	10	28	15	16	11	21	4	12	3	25	13	7
8	17	9	24	16	18	27	22	25	31	6	7	12	15	26	4	14	8
11	26	12	8	19	6	15	25	9	20	29	16	22	32	28	27	17	11
12	6	23	22	17	15	16	31	30	10	27	26	20	7	18	19	29	12
15	24	16	17	9	25	31	5	18	27	2	13	29	8	20	21	30	15
17	30	18	4	13	10	21	29	5	14	25	22	16	31	24	23	11	17
18	10	27	16	11	21	22	32	26	6	23	30	14	3	12	13	25	18
20	27	2	31	6	7	17	10	13	24	16	18	3	26	15	14	4	20
23	15	3	20	29	16	26	13	2	8	19	6	10	28	32	17	27	23
28	20	29	15	3	2	8	18	16	26	12	9	5	23	11	31	24	28
2	7	8	3	4	17	18	21	11	12	14	27	24	1	5	6	16	2
9	18	20	12	14	27	7	30	23	3	4	17	31	5	1	16	6	9
10	19	21	13	15	28	29	8	24	25	26	32	11	6	16	1	5	10
22	29	30	25	26	32	19	20	31	13	15	28	23	16	6	5	1	22
1	3	4	7	8	11	12	15	17	18	20	23	28	2	9	10	22	1

TABLE 97. Cayley subtable for G_{32}^{33}.

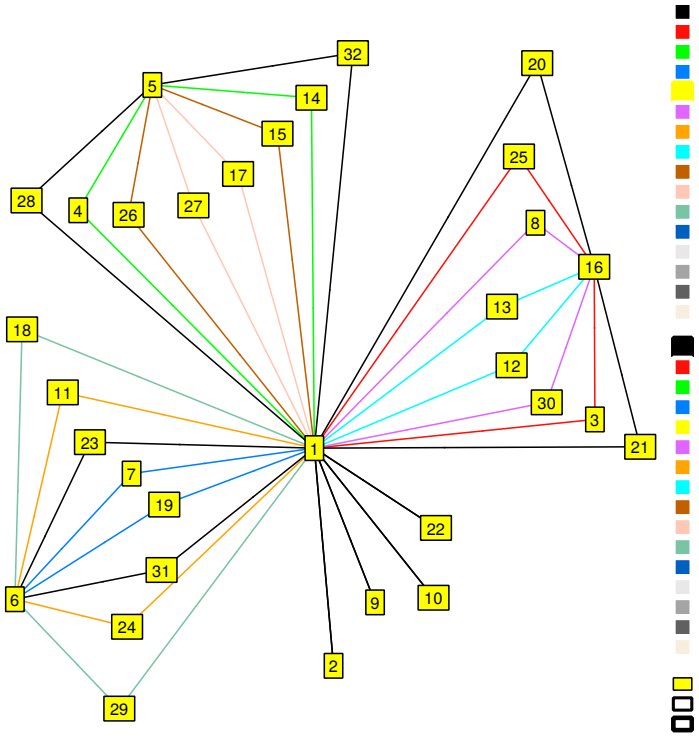

FIGURE 99. $G_{32}^{33} \cong (C_4 \times C_4) \rtimes C_2)$. $(2^4, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

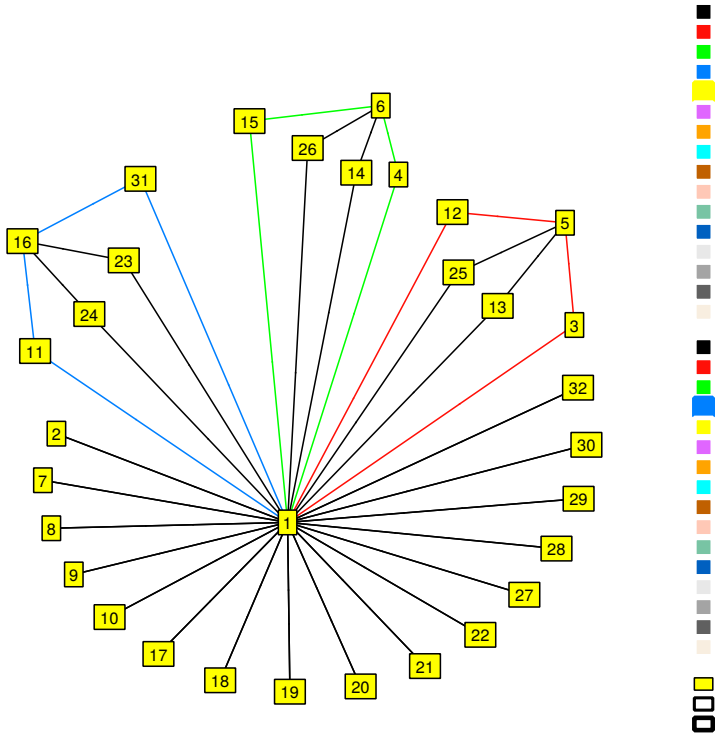

FIGURE 100. $G_{32}^{34} \cong (C_4 \times C_4) \rtimes C_2)$. $(2^{16}, 4^6)$. $(x_1^{32} + 19x_2^{16} + 12x_4^8)/32$.

	3	4	11	13	14	23	2	7	8	9	10	17	18	19	20	21	22	27	28	29	30	32	1
3	5	11	14	16	23	4	18	2	27	7	29	8	9	10	17	32	19	20	21	22	28	30	3
4	11	6	13	24	16	25	21	28	2	30	8	7	32	17	9	10	20	18	19	27	22	29	4
11	14	13	16	26	25	6	32	21	18	28	27	2	30	8	7	29	17	9	10	20	19	22	11
13	16	24	26	5	31	15	29	10	32	19	18	21	22	2	28	27	7	30	8	9	17	20	13
14	23	16	25	31	6	13	30	32	9	21	20	18	28	27	2	22	8	7	29	17	10	19	14
23	4	25	6	15	13	16	28	30	7	32	17	9	21	20	18	19	27	2	22	8	29	10	23
2	7	8	17	19	20	27	1	3	4	5	6	11	12	13	14	15	16	23	24	25	26	31	2
7	9	17	20	22	27	8	12	1	23	3	25	4	5	6	11	31	13	14	15	16	24	26	7
8	17	10	19	28	22	29	15	24	1	26	4	3	31	11	5	6	14	12	13	23	16	25	8
9	18	20	27	29	8	17	5	12	14	1	16	23	3	25	4	26	6	11	31	13	15	24	9
10	19	21	28	7	30	32	6	13	15	16	1	24	25	3	26	4	5	31	11	12	14	23	10
17	20	19	22	30	29	10	31	15	12	24	23	1	26	4	3	25	11	5	6	14	13	16	17
18	2	27	8	10	17	20	3	5	11	12	13	14	1	16	23	24	25	4	26	6	31	15	18
19	22	28	30	9	32	21	25	6	31	13	12	15	16	1	24	23	3	26	4	5	11	14	19
20	27	22	29	32	10	19	26	31	5	15	14	12	24	23	1	16	4	3	25	11	6	13	20
21	28	2	7	17	9	18	4	11	6	14	15	13	23	24	16	1	26	25	3	31	5	12	21
22	29	30	32	18	21	28	16	25	26	6	5	31	13	12	15	14	1	24	23	3	4	11	22
27	8	29	10	21	19	22	24	26	3	31	11	5	15	14	12	13	23	1	16	4	25	6	27
28	30	7	9	20	18	2	23	4	25	11	31	6	14	15	13	12	24	16	1	26	3	5	28
29	10	32	21	2	28	30	13	16	24	25	3	26	6	5	31	11	12	15	14	1	23	4	29
30	32	9	18	27	2	7	14	23	16	4	26	25	11	31	6	5	15	13	12	24	1	3	30
32	21	18	2	8	7	9	11	14	13	23	24	16	4	26	25	3	31	6	5	15	12	1	32
1	3	4	11	13	14	23	2	7	8	9	10	17	18	19	20	21	22	27	28	29	30	32	1

TABLE 98. Cayley subtable for G_{32}^{34}.

	2	3	4	7	8	10	11	13	14	17	19	21	23	28	1
2	5	7	8	12	14	16	17	19	20	23	25	26	27	31	2
3	18	5	11	2	27	29	14	16	23	8	10	32	4	21	3
4	21	11	6	28	2	8	13	24	16	7	17	10	25	19	4
7	3	9	17	5	11	13	20	22	27	14	16	24	8	26	7
8	26	17	10	31	5	14	19	28	22	12	23	16	29	25	8
10	16	19	21	25	26	5	28	7	30	31	12	14	32	23	10
11	32	14	13	21	18	27	16	26	25	2	8	29	6	10	11
13	29	16	24	10	32	18	26	5	31	21	2	27	15	8	13
14	30	23	16	32	9	20	25	31	6	18	27	22	13	29	14
17	24	20	19	26	3	11	22	30	29	5	14	13	10	16	17
19	13	22	28	16	24	3	30	9	32	26	5	11	21	14	19
21	14	28	2	23	16	26	7	17	9	25	31	5	18	12	21
23	28	4	25	30	7	17	6	15	13	9	20	19	16	22	23
28	11	30	7	14	13	24	9	20	18	16	26	3	2	5	28
1	2	3	4	7	8	10	11	13	14	17	19	21	23	28	1

TABLE 99. Cayley subtable for G_{32}^{35}.

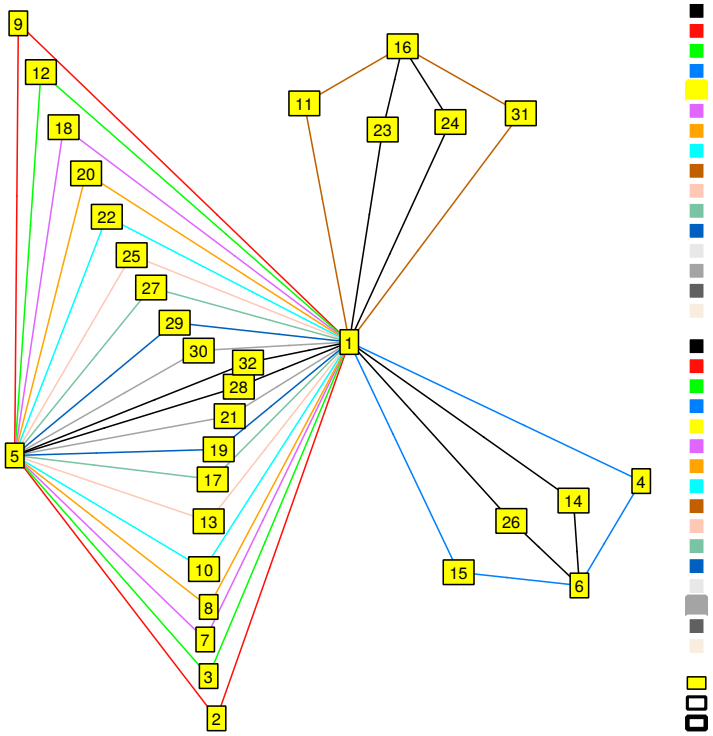

FIGURE 101. $G_{32}^{35} \cong C_4 \rtimes Q_8$. (4^{14}). $(x_1^{32} + 3x_2^{16} + 28x_4^8)/32$.

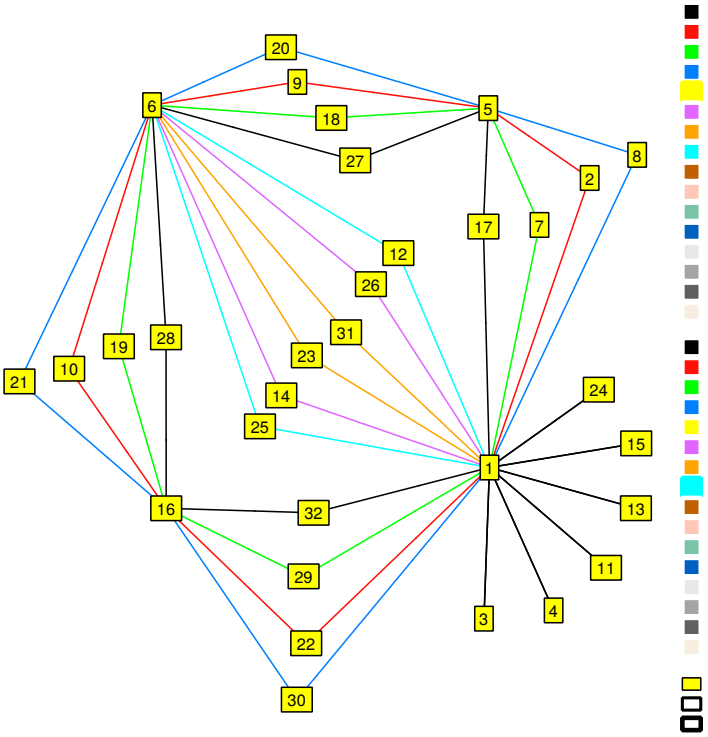

FIGURE 102. $G_{32}^{36} \cong C_8 \times C_2 \times C_2$. $(2^6, 4^3, 8^4)$. $(x_1^{12} + 2x_1^{10}x_2 + x_1^8x_2^2 + x_1^4x_2^4 + 2x_1^2x_2^5 + 2x_1^4x_4^2 + x_2^6 + 4x_1^4x_8 + 4x_1^2x_2x_4^2 + 8x_1^2x_2x_8 + 2x_2^2x_4^2 + 4x_2^2x_8)/32$.

	2	7	8	17	12	14	23	3	4	11	13	15	24	1
2	5	12	14	23	18	20	27	7	8	17	19	21	28	2
7	12	5	23	14	9	27	20	2	17	8	10	28	21	7
8	14	23	5	12	27	9	18	17	2	7	28	10	19	8
17	23	14	12	5	20	18	9	8	7	2	21	19	10	17
12	18	9	27	20	6	24	15	5	23	14	16	31	26	12
14	20	27	9	18	24	6	13	23	5	12	31	16	25	14
23	27	20	18	9	15	13	6	14	12	5	26	25	16	23
3	7	2	17	8	5	23	14	1	11	4	6	24	15	3
4	8	17	2	7	23	5	12	11	1	3	24	6	13	4
11	17	8	7	2	14	12	5	4	3	1	15	13	6	11
13	19	10	28	21	16	31	26	6	24	15	1	11	4	13
15	21	28	10	19	31	16	25	24	6	13	11	1	3	15
24	28	21	19	10	26	25	16	15	13	6	4	3	1	24
1	2	7	8	17	12	14	23	3	4	11	13	15	24	1

TABLE 100. Cayley subtable for G_{32}^{36}.

	2	7	8	17	12	14	23	3	4	11	13	15	24	1
2	5	12	14	23	18	20	27	7	8	17	19	21	28	2
7	25	16	31	26	9	27	20	2	17	8	10	28	21	7
8	14	23	5	12	27	9	18	17	2	7	28	10	19	8
17	31	26	25	16	20	18	9	8	7	2	21	19	10	17
12	29	22	32	30	6	24	15	5	23	14	16	31	26	12
14	20	27	9	18	24	6	13	23	5	12	31	16	25	14
23	32	30	29	22	15	13	6	14	12	5	26	25	16	23
3	19	10	28	21	5	23	14	1	11	4	6	24	15	3
4	8	17	2	7	23	5	12	11	1	3	24	6	13	4
11	28	21	19	10	14	12	5	4	3	1	15	13	6	11
13	7	2	17	8	16	31	26	6	24	15	1	11	4	13
15	21	28	10	19	31	16	25	24	6	13	11	1	3	15
24	17	8	7	2	26	25	16	15	13	6	4	3	1	24
1	2	7	8	17	12	14	23	3	4	11	13	15	24	1

TABLE 101. Cayley subtable for G_{32}^{37}.

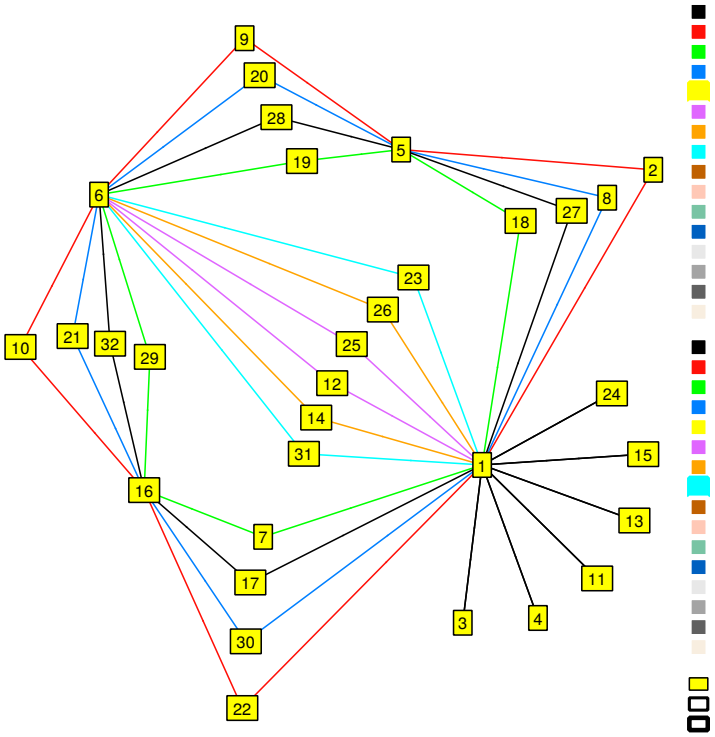

FIGURE 103. $G_{32}^{37} \cong C_2 \times (C_8 \rtimes C_2)$. $(2^6, 4^3, 8^4)$. $(x_1^{32} + 7x_2^{16} + 8x_4^8 + 16x_8^4)/32$.

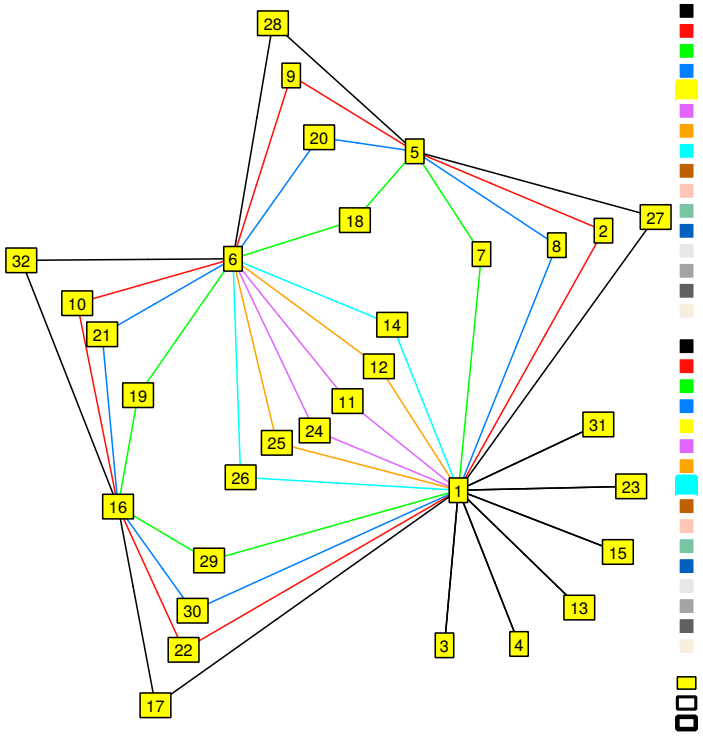

FIGURE 104. $G_{32}^{38} \cong (C_8 \times C_2) \rtimes C_2)$. $(2^6, 4^3, 8^4)$. $(x_1^{32} + 7x_2^{16} + 8x_4^8 + 16x_8^4)/32$.

	2	7	8	17	11	12	14	3	4	13	15	23	31	1
2	5	12	14	23	17	18	20	7	8	19	21	27	32	2
7	12	5	23	14	8	9	27	2	17	10	28	20	30	7
8	14	31	5	25	19	32	9	28	2	17	10	29	18	8
17	23	26	12	16	10	30	18	21	7	8	19	22	9	17
11	17	21	7	10	6	26	12	15	3	4	13	16	5	11
12	18	9	27	20	14	6	24	5	23	16	31	15	4	12
14	20	32	9	29	25	11	6	31	5	23	16	3	13	14
3	7	2	17	8	4	5	23	1	11	6	24	14	26	3
4	8	28	2	19	13	31	5	24	1	11	6	25	12	4
13	19	10	28	21	15	16	31	6	24	1	11	26	14	13
15	21	17	10	7	3	23	16	11	6	24	1	12	25	15
23	27	30	18	22	16	4	13	26	12	14	25	1	6	23
31	32	20	29	9	5	15	3	14	25	26	12	6	1	31
1	2	7	8	17	11	12	14	3	4	13	15	23	31	1

TABLE 102. Cayley subtable for G_{32}^{38}.

	7	17	14	2	3	4	8	9	10	11	12	13	15	20	21	22	23	24	25	30	31	1
7	16	26	27	12	2	17	23	13	25	8	9	10	28	24	31	3	20	21	22	11	30	7
17	26	16	18	23	8	7	12	24	31	2	20	21	19	13	25	11	9	10	30	3	22	17
14	27	18	6	30	31	5	22	8	20	25	11	23	16	2	9	21	3	12	24	10	13	14
2	3	11	20	1	7	8	4	5	6	17	18	19	21	14	15	16	27	28	29	26	32	2
3	22	30	23	18	1	11	27	19	29	4	5	6	24	28	32	7	14	15	16	17	26	3
4	17	7	5	8	11	1	2	20	21	3	23	24	6	9	10	30	12	13	31	22	25	4
8	11	3	9	4	17	2	1	14	15	7	27	28	10	5	6	26	18	19	32	16	29	8
9	12	23	21	16	29	20	26	1	5	32	7	18	30	4	14	6	17	27	19	15	28	9
10	13	24	30	6	19	21	15	16	1	28	29	7	8	26	4	5	32	17	18	14	27	10
11	30	22	12	27	4	3	18	28	32	1	14	15	13	19	29	17	5	6	26	7	16	11
12	2	8	24	7	16	23	17	18	19	26	1	5	31	27	28	29	4	14	6	32	15	12
13	9	20	31	29	6	24	32	7	18	15	16	1	11	17	27	19	26	4	5	28	14	13
15	28	19	16	21	24	6	10	30	8	13	31	11	1	22	2	20	25	3	23	9	12	15
20	23	12	10	26	32	9	16	4	14	29	17	27	22	1	5	15	7	18	28	6	19	20
21	24	13	22	15	28	10	6	26	4	19	32	17	2	16	1	14	29	7	27	5	18	21
22	25	31	8	5	18	30	14	6	16	27	19	29	20	15	26	1	28	32	7	4	17	22
23	8	2	13	17	26	12	7	27	28	16	4	14	25	18	19	32	1	5	15	29	6	23
24	20	9	25	32	15	13	29	17	27	6	26	4	3	7	18	28	16	1	14	19	5	24
25	10	21	11	19	5	31	28	29	7	14	6	16	23	32	17	18	15	26	1	27	4	25
30	31	25	2	14	27	22	5	15	26	18	28	32	9	6	16	4	19	29	17	1	7	30
31	21	10	3	28	14	25	19	32	17	5	15	26	12	29	7	27	6	16	4	18	1	31
1	7	17	14	2	3	4	8	9	10	11	12	13	15	20	21	22	23	24	25	30	31	1

TABLE 103. Cayley subtable for G_{32}^{39}.

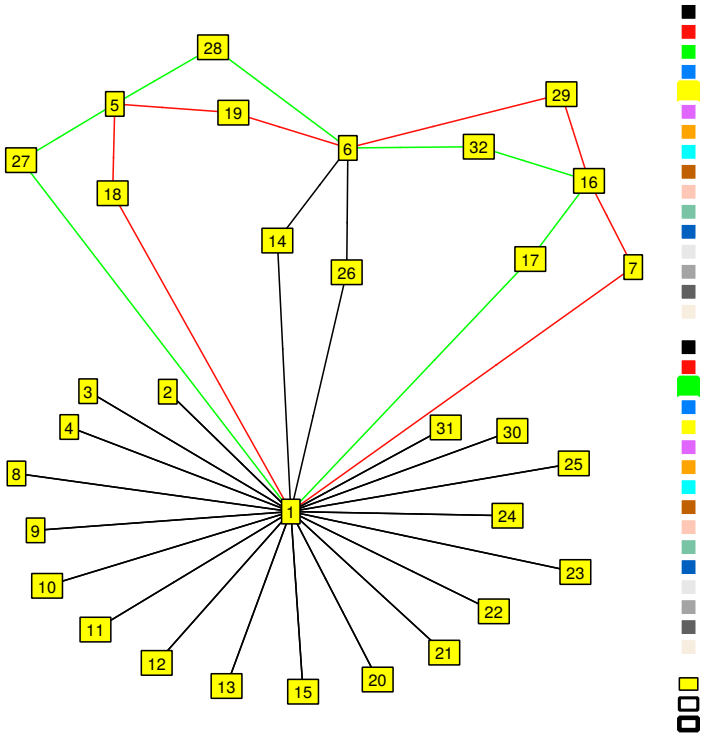

FIGURE 105. $G_{32}^{39} \cong C_2 \times D_{16}$. $(2^{18}, 4^1, 8^2)$. $(x_1^{32} + 19x_2^{16} + 4x_4^8 + 8x_8^4)/32$.

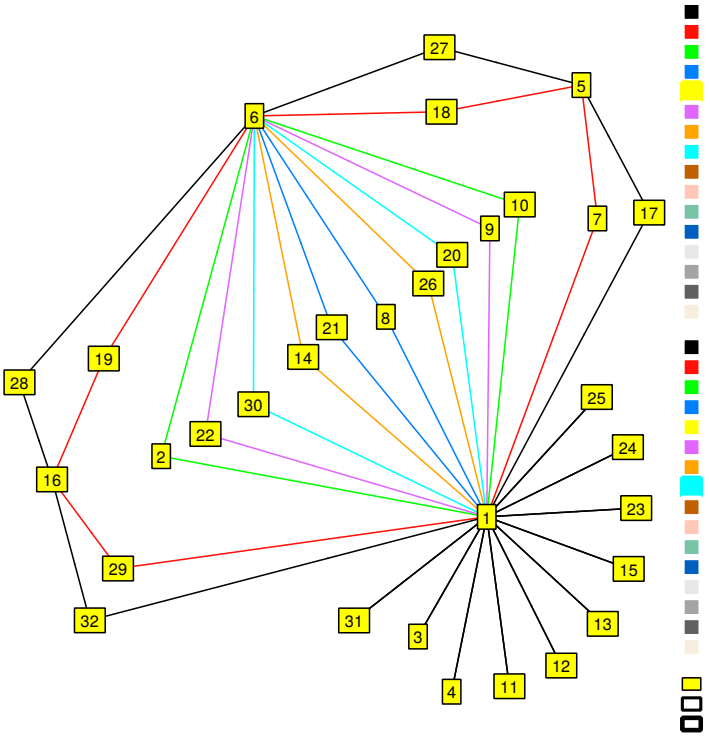

FIGURE 106. $G_{32}^{40} \cong C_2 \times QD_{16}$. $(2^{10}, 4^5, 8^2)$. $(x_1^{32} + 11x_2^{16} + 12x_4^8 + 8x_8^4)/32$.

	7	17	2	8	9	14	20	3	4	11	12	13	15	23	24	25	31	1
7	5	14	25	31	3	27	11	2	17	8	9	10	28	20	21	22	30	7
17	14	5	31	25	11	18	3	8	7	2	20	21	19	9	10	30	22	17
2	13	24	6	15	16	20	26	7	8	17	18	19	21	27	28	29	32	2
8	24	13	15	6	26	9	16	17	2	7	27	28	10	18	19	32	29	8
9	25	31	5	14	6	21	15	29	20	32	7	18	30	17	27	19	28	9
14	27	18	30	22	8	6	2	31	5	25	11	23	16	3	12	24	13	14
20	31	25	14	5	15	10	6	32	9	29	17	27	22	7	18	28	19	20
3	22	30	18	27	19	23	28	1	11	4	5	6	24	14	15	16	26	3
4	17	7	8	2	20	5	9	11	1	3	23	24	6	12	13	31	25	4
11	30	22	27	18	28	12	19	4	3	1	14	15	13	5	6	26	16	11
12	2	8	7	17	18	24	27	16	23	26	1	5	31	4	14	6	15	12
13	9	20	29	32	7	31	17	6	24	15	16	1	11	26	4	5	14	13
15	28	19	21	10	30	16	22	24	6	13	31	11	1	25	3	23	12	15
23	8	2	17	7	27	13	18	26	12	16	4	14	25	1	5	15	6	23
24	20	9	32	29	17	25	7	15	13	6	26	4	3	16	1	14	5	24
25	10	21	19	28	29	11	32	5	31	14	6	16	23	15	26	1	4	25
31	21	10	28	19	32	3	29	14	25	5	15	26	12	6	16	4	1	31
1	7	17	2	8	9	14	20	3	4	11	12	13	15	23	24	25	31	1

TABLE 104. Cayley subtable for G_{32}^{40}.

	7	17	2	3	8	9	11	12	14	20	23	4	15	1
7	16	26	25	10	31	3	21	22	27	11	30	17	28	7
17	26	16	31	21	25	11	10	30	18	3	22	7	19	17
2	13	24	6	7	15	16	17	18	20	26	27	8	21	2
3	9	20	18	6	27	19	15	16	23	28	26	11	24	3
8	24	13	15	17	6	26	7	27	9	16	18	2	10	8
9	25	31	5	29	14	6	32	7	21	15	17	20	30	9
11	20	9	27	15	18	28	6	26	12	19	16	3	13	11
12	10	21	7	5	17	18	14	6	24	27	15	23	31	12
14	27	18	30	31	22	8	25	11	6	2	3	5	16	14
20	31	25	14	32	5	15	29	17	10	6	7	9	22	20
23	21	10	17	14	7	27	5	15	13	18	6	12	25	23
4	17	7	8	11	2	20	3	23	5	9	12	1	6	4
15	28	19	21	24	10	30	13	31	16	22	25	6	1	15
1	7	17	2	3	8	9	11	12	14	20	23	4	15	1

TABLE 105. Cayley subtable for G_{32}^{41}.

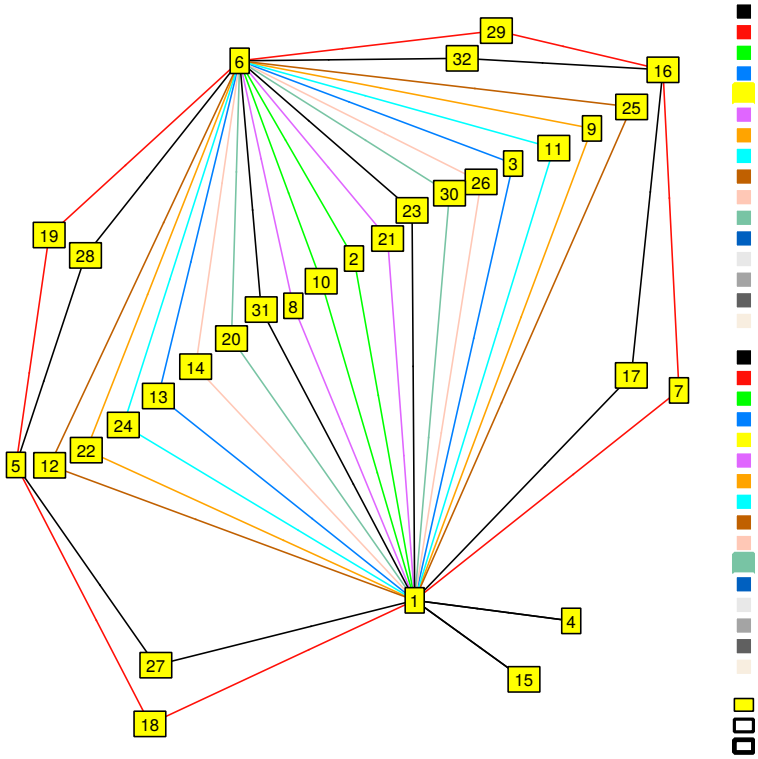

FIGURE 107. $G_{32}^{41} \cong C_2 \times Q_{16}$. $(2^2, 4^9, 8^2)$. $(x_1^{32} + 3x_2^{16} + 20x_4^8 + 8x_8^4)/32$.

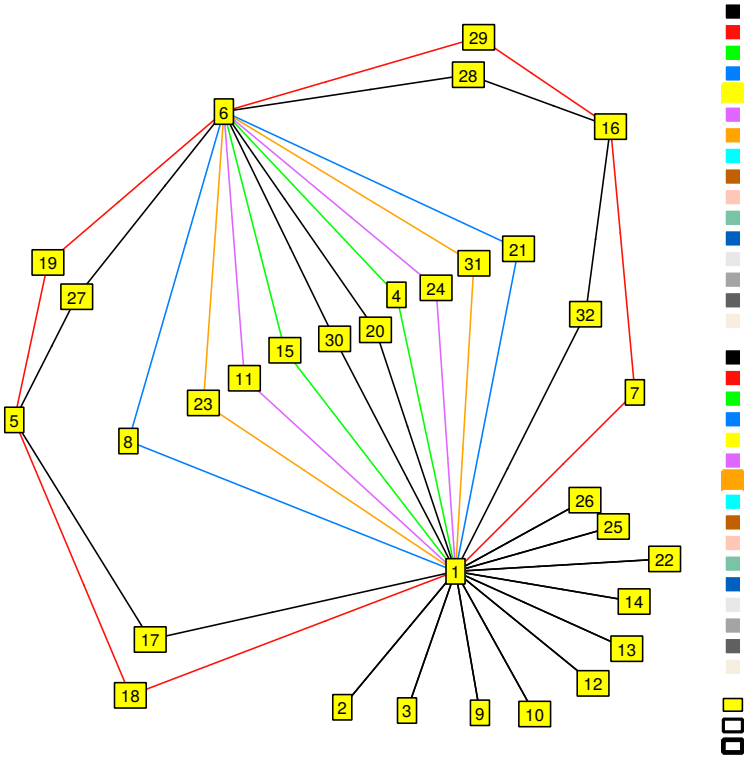

FIGURE 108. $G_{32}^{42} \cong (C_8 \times C_2) \rtimes C_2$. $(2^{10}, 4^5, 8^2)$. $(x_1^{32} + 11x_2^{16} + 12x_4^8 + 8x_8^4)/32$.

	7	17	4	8	11	20	23	2	3	9	10	12	13	14	22	25	26	1
7	16	26	17	23	8	24	20	12	2	13	25	9	10	27	3	22	32	7
17	26	5	19	25	10	3	22	23	8	24	31	20	21	29	11	30	18	17
4	17	19	6	10	13	22	25	8	11	20	21	23	24	16	30	31	5	4
8	11	13	10	6	19	16	29	4	17	14	15	27	28	22	26	32	9	8
11	30	9	13	29	6	7	16	27	4	28	32	14	15	25	17	26	12	11
20	23	25	22	5	18	6	19	26	32	4	14	17	27	2	15	28	10	20
23	8	10	25	19	5	29	6	17	26	27	28	4	14	3	32	15	13	23
2	3	11	8	4	17	14	27	1	7	5	6	18	19	20	16	29	30	2
3	22	30	11	27	4	28	14	18	1	19	29	5	6	23	7	16	31	3
9	12	23	20	26	32	4	17	16	29	1	5	7	18	21	6	19	8	9
10	13	24	21	15	28	26	32	6	19	16	1	29	7	30	5	18	20	10
12	2	8	23	17	26	27	4	7	16	18	19	1	5	24	29	6	11	12
13	9	20	24	32	15	17	26	29	6	7	18	16	1	31	19	5	23	13
14	27	29	16	9	12	10	13	30	31	8	20	11	23	1	21	24	6	14
22	25	31	30	14	27	15	28	5	18	6	16	19	29	8	1	7	21	22
25	10	21	31	28	14	32	15	19	5	29	7	6	16	11	18	1	24	25
26	32	18	5	22	25	2	3	20	23	21	30	24	31	6	8	11	1	26
1	7	17	4	8	11	20	23	2	3	9	10	12	13	14	22	25	26	1

TABLE 106. Cayley subtable for G_{32}^{42}.

	7	17	8	14	20	2	3	4	9	10	11	12	13	15	22	23	24	25	31	1
7	16	26	23	27	24	12	2	17	13	25	8	9	10	28	3	20	21	22	30	7
17	14	5	25	18	3	31	8	7	11	23	2	20	21	19	24	9	10	30	22	17
8	24	13	6	9	16	15	17	2	26	4	7	27	28	10	14	18	19	32	29	8
14	32	29	9	6	10	20	31	5	21	30	25	11	23	16	8	3	12	24	13	14
20	31	25	5	10	6	14	32	9	15	26	29	17	27	22	4	7	18	28	19	20
2	3	11	4	20	14	1	7	8	5	6	17	18	19	21	16	27	28	29	32	2
3	22	30	27	23	28	18	1	11	19	29	4	5	6	24	7	14	15	16	26	3
4	28	19	10	5	22	21	11	1	30	8	3	23	24	6	20	12	13	31	25	4
9	12	23	26	21	4	16	29	20	1	5	32	7	18	30	6	17	27	19	28	9
10	13	24	15	30	26	6	19	21	16	1	28	29	7	8	5	32	17	18	27	10
11	20	9	29	12	7	32	4	3	17	27	1	14	15	13	28	5	6	26	16	11
12	2	8	17	24	27	7	16	23	18	19	26	1	5	31	29	4	14	6	15	12
13	9	20	32	31	17	29	6	24	7	18	15	16	1	11	19	26	4	5	14	13
15	17	7	2	16	9	8	24	6	20	21	13	31	11	1	30	25	3	23	12	15
22	25	31	14	8	15	5	18	30	6	16	27	19	29	20	1	28	32	7	17	22
23	21	10	19	13	29	28	26	12	32	17	16	4	14	25	27	1	5	15	6	23
24	30	22	18	25	19	27	15	13	28	32	6	26	4	3	17	16	1	14	5	24
25	10	21	28	11	32	19	5	31	29	7	14	6	16	23	18	15	26	1	4	25
31	8	2	7	3	18	17	14	25	27	28	5	15	26	12	32	6	16	4	1	31
1	7	17	8	14	20	2	3	4	9	10	11	12	13	15	22	23	24	25	31	1

TABLE 107. Cayley subtable for G_{32}^{43}.

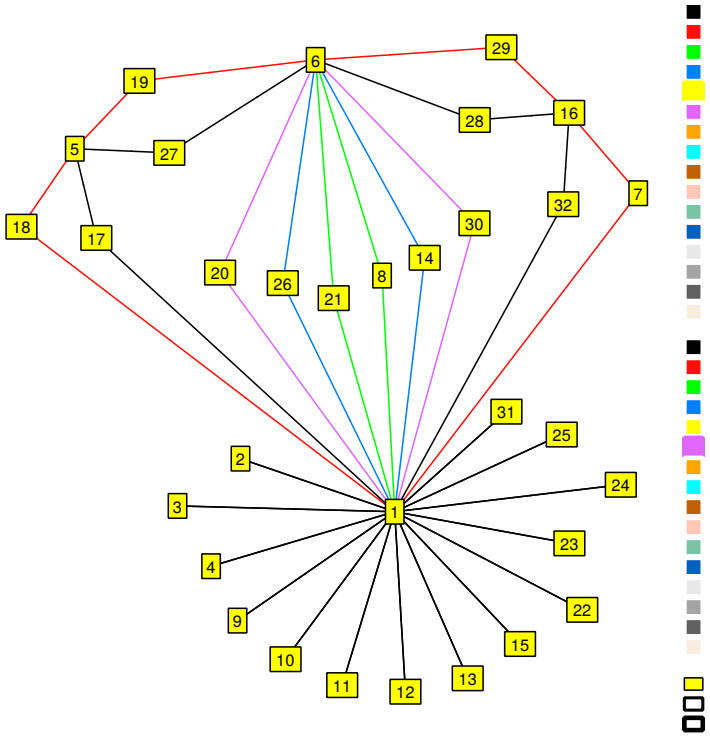

FIGURE 109. $G_{32}^{43} \cong (C_2 \times D_8) \rtimes C_2$. $(2^{14}, 4^3, 8^2)$. $(x_1^{32} + 15x_2^{16} + 8x_4^8 + 8x_8^4)/32$.

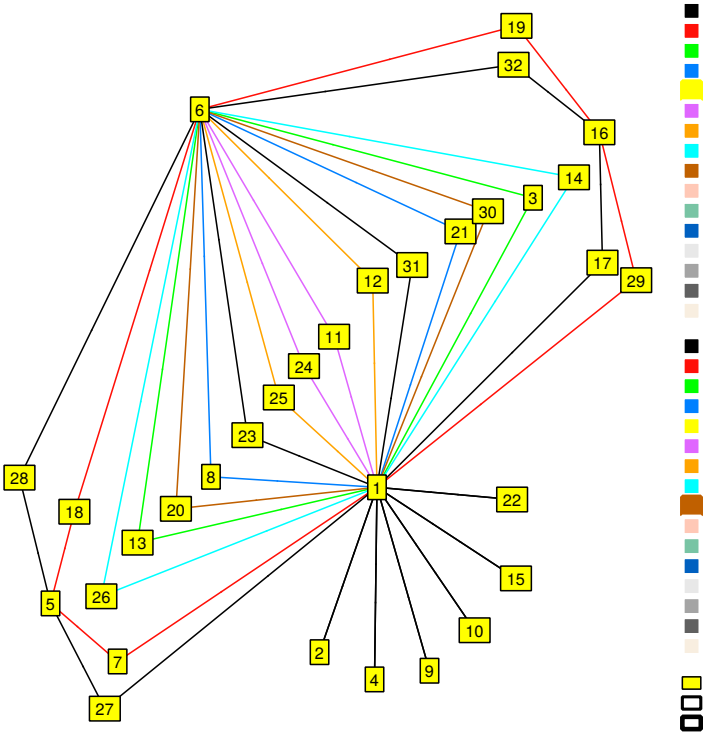

FIGURE 110. $G_{32}^{44} \cong (C_2 \times Q_8) \rtimes C_2$. $(2^6, 4^7, 8^2)$. $(x_1^{32} + 7x_2^{16} + 16x_4^8 + 8x_8^4)/32$.

	7	17	3	8	11	12	14	20	23	2	4	9	10	15	22	1
7	5	14	10	23	21	22	27	24	30	12	17	13	25	28	3	7
17	26	16	21	25	10	30	18	3	22	31	7	11	23	19	24	17
3	9	20	6	27	15	16	23	28	26	18	11	19	29	24	7	3
8	24	13	17	6	7	27	9	16	18	15	2	26	4	10	14	8
11	30	22	15	29	6	26	12	7	16	32	3	17	27	13	28	11
12	10	21	5	17	14	6	24	27	15	7	23	18	19	31	29	12
14	32	29	31	9	25	11	6	10	3	20	5	21	30	16	8	14
20	31	25	32	5	29	17	10	6	7	14	9	15	26	22	4	20
23	8	2	14	19	5	15	13	29	6	28	12	32	17	25	27	23
2	3	11	7	4	17	18	20	14	27	1	8	5	6	21	16	2
4	28	19	11	10	3	23	5	22	12	21	1	30	8	6	20	4
9	12	23	29	26	32	7	21	4	17	16	20	1	5	30	6	9
10	13	24	19	15	28	29	30	26	32	6	21	16	1	8	5	10
15	17	7	24	2	13	31	16	9	25	8	6	20	21	1	30	15
22	25	31	18	14	27	19	8	15	28	5	30	6	16	20	1	22
1	7	17	3	8	11	12	14	20	23	2	4	9	10	15	22	1

TABLE 108. Cayley subtable for G_{32}^{44}.

	2	7	8	9	17	18	20	27	3	4	5	11	12	13	14	15	16	23	24	25	26	31	1
2	6	13	15	16	24	25	26	31	7	8	9	17	18	19	20	21	22	27	28	29	30	32	2
7	13	6	24	25	15	16	31	26	2	17	18	8	9	10	27	28	29	20	21	22	32	30	7
8	15	24	6	26	13	31	16	25	17	2	20	7	27	28	9	10	30	18	19	32	22	29	8
9	16	25	26	6	31	13	15	24	18	20	2	27	7	29	8	30	10	17	32	19	21	28	9
17	24	15	13	31	6	26	25	16	8	7	27	2	20	21	18	19	32	9	10	30	29	22	17
18	25	16	31	13	26	6	24	15	9	27	7	20	2	22	17	32	19	8	30	10	28	21	18
20	26	31	16	15	25	24	6	13	27	9	8	18	17	32	2	22	21	7	29	28	10	19	20
27	31	26	25	24	16	15	13	6	20	18	17	9	8	30	7	29	28	2	22	21	19	10	27
3	7	2	17	18	8	9	27	20	1	11	12	4	5	6	23	24	25	14	15	16	31	26	3
4	8	17	2	20	7	27	9	18	11	1	14	3	23	24	5	6	26	12	13	31	16	25	4
5	9	18	20	2	27	7	8	17	12	14	1	23	3	25	4	26	6	11	31	13	15	24	5
11	17	8	7	27	2	20	18	9	4	3	23	1	14	15	12	13	31	5	6	26	25	16	11
12	18	9	27	7	20	2	17	8	5	23	3	14	1	16	11	31	13	4	26	6	24	15	12
13	19	10	28	29	21	22	32	30	6	24	25	15	16	1	31	11	12	26	4	5	23	14	13
14	20	27	9	8	18	17	2	7	23	5	4	12	11	31	1	16	15	3	25	24	6	13	14
15	21	28	10	30	19	32	22	29	24	6	26	13	31	11	16	1	14	25	3	23	5	12	15
16	22	29	30	10	32	19	21	28	25	26	6	31	13	12	15	14	1	24	23	3	4	11	16
23	27	20	18	17	9	8	7	2	14	12	11	5	4	26	3	25	24	1	16	15	13	6	23
24	28	21	19	32	10	30	29	22	15	13	31	6	26	4	25	3	23	16	1	14	12	5	24
25	29	22	32	19	30	10	28	21	16	31	13	26	6	5	24	23	3	15	14	1	11	4	25
26	30	32	22	21	29	28	10	19	31	16	15	25	24	23	6	5	4	13	12	11	1	3	26
31	32	30	29	28	22	21	19	10	26	25	24	16	15	14	13	12	11	6	5	4	3	1	31
1	2	7	8	9	17	18	20	27	3	4	5	11	12	13	14	15	16	23	24	25	26	31	1

TABLE 109. Cayley subtable for G_{32}^{45}.

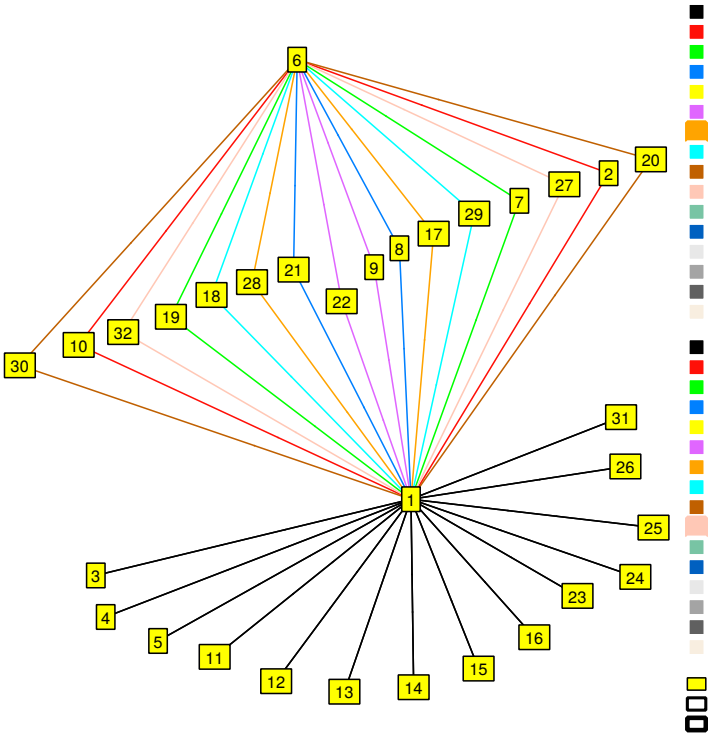

FIGURE 111. $G_{32}^{45} \cong C_4 \times C_2 \times C_2 \times C_2$. $(2^{14}, 4^8)$. $(x_1^{10} + 3x_1^8x_2 + 4x_1^6x_2^2 + 2x_1^6x_4 + 4x_1^4x_2^3 + 6x_1^4x_2x_4 + 3x_1^2x_2^4 + 6x_1^2x_2^2x_4 + x_2^5 + 2x_2^3x_4)/32$.

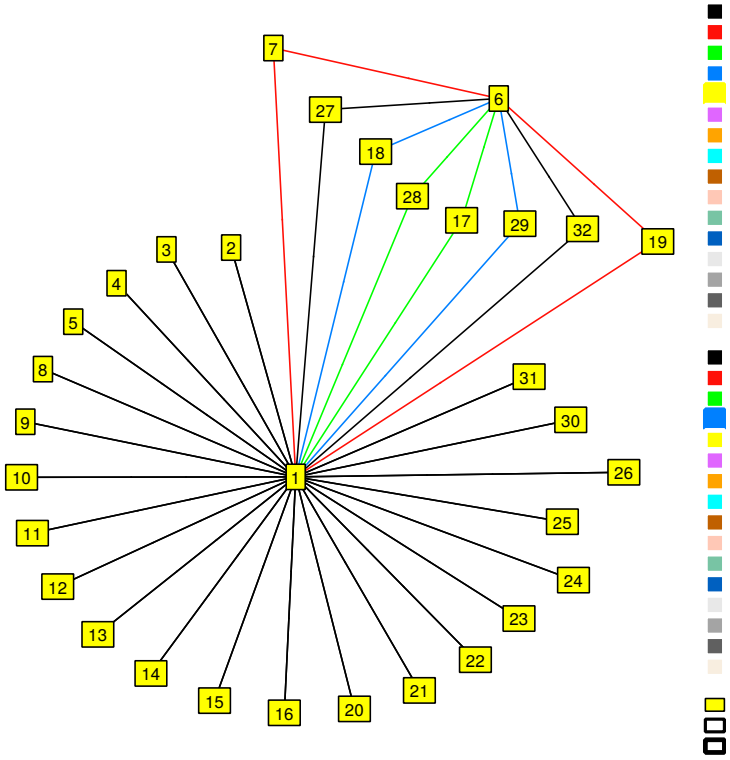

FIGURE 112. $G_{32}^{46} \cong C_2 \times C_2 \times D_8$. $(2^{22}, 4^4)$. $(x_1^{32} + 23x_2^{16} + 8x_4^8)/32$.

	7	17	18	27	2	3	4	5	8	9	10	11	12	13	14	15	16	20	21	22	23	24	25	2
7	6	15	16	26	13	2	17	18	24	25	3	8	9	10	27	28	29	31	11	12	20	21	22	3
17	15	6	26	16	24	8	7	27	13	31	11	2	20	21	18	19	32	25	3	23	9	10	30	2
18	16	26	6	15	25	9	27	7	31	13	12	20	2	22	17	32	19	24	23	3	8	30	10	2
27	26	16	15	6	31	20	18	17	25	24	23	9	8	30	7	29	28	13	12	11	2	22	21	1
2	3	11	12	23	1	7	8	9	4	5	6	17	18	19	20	21	22	14	15	16	27	28	29	3
3	10	21	22	30	19	1	11	12	28	29	7	4	5	6	23	24	25	32	17	18	14	15	16	3
4	17	7	27	18	8	11	1	14	2	20	21	3	23	24	5	6	26	9	10	30	12	13	31	1
5	18	27	7	17	9	12	14	1	20	2	22	23	3	25	4	26	6	8	30	10	11	31	13	1
8	11	3	23	12	4	17	2	20	1	14	15	7	27	28	9	10	30	5	6	26	18	19	32	2
9	12	23	3	11	5	18	20	2	14	1	16	27	7	29	8	30	10	4	26	6	17	32	19	2
10	13	24	25	31	6	19	21	22	15	16	1	28	29	7	30	8	9	26	4	5	32	17	18	2
11	21	10	30	22	28	4	3	23	19	32	17	1	14	15	12	13	31	29	7	27	5	6	26	2
12	22	30	10	21	29	5	23	3	32	19	18	14	1	16	11	31	13	28	27	7	4	26	6	2
13	2	8	9	20	7	6	24	25	17	18	19	15	16	1	31	11	12	27	28	29	26	4	5	2
14	27	18	17	7	20	23	5	4	9	8	30	12	11	31	1	16	15	2	22	21	3	25	24	
15	28	19	32	29	21	24	6	26	10	30	8	13	31	11	16	1	14	22	2	20	25	3	23	
16	29	32	19	28	22	25	26	6	30	10	9	31	13	12	15	14	1	21	20	2	24	23	3	
20	23	12	11	3	14	27	9	8	5	4	26	18	17	32	2	22	21	1	16	15	7	29	28	1
21	24	13	31	25	15	28	10	30	6	26	4	19	32	17	22	2	20	16	1	14	29	7	27	
22	25	31	13	24	16	29	30	10	26	6	5	32	19	18	21	20	2	15	14	1	28	27	7	
23	30	22	21	10	32	14	12	11	29	28	27	5	4	26	3	25	24	19	18	17	1	16	15	1
24	8	2	20	9	17	15	13	31	7	27	28	6	26	4	25	3	23	18	19	32	16	1	14	1
25	9	20	2	8	18	16	31	13	27	7	29	26	6	5	24	23	3	17	32	19	15	14	1	1
26	32	29	28	19	30	31	16	15	22	21	20	25	24	23	6	5	4	10	9	8	13	12	11	
30	31	25	24	13	26	32	22	21	16	15	14	29	28	27	10	9	8	6	5	4	19	18	17	
31	20	9	8	2	27	26	25	24	18	17	32	16	15	14	13	12	11	7	29	28	6	5	4	
1	7	17	18	27	2	3	4	5	8	9	10	11	12	13	14	15	16	20	21	22	23	24	25	2

TABLE 110. Cayley subtable for G_{32}^{46}.

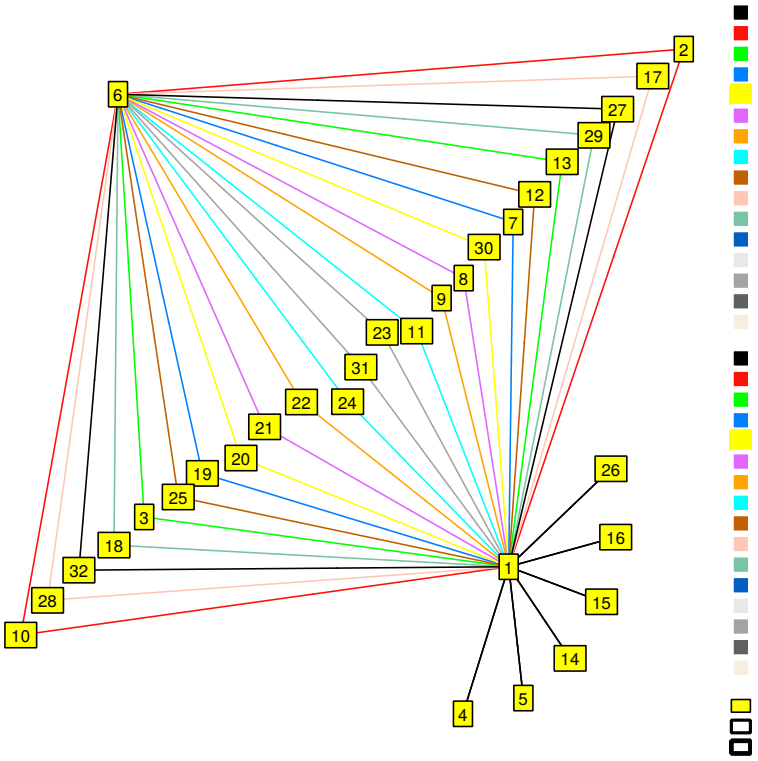

FIGURE 113. $G_{32}^{47} \cong C_2 \times C_2 \times Q_8$. $(2^6, 4^{12})$. $(x_1^{32} + 7x_2^{16} + 24x_4^8)/32$.

	2	3	7	8	9	11	12	17	18	20	23	27	4	5	14	15	16	26	1
2	6	7	13	15	16	17	18	24	25	26	27	31	8	9	20	21	22	30	2
3	19	6	2	28	29	15	16	8	9	32	26	20	11	12	23	24	25	31	3
7	3	10	6	11	12	21	22	15	16	23	30	26	17	18	27	28	29	32	7
8	15	17	24	6	26	7	27	13	31	16	18	25	2	20	9	10	30	22	8
9	16	18	25	26	6	27	7	31	13	15	17	24	20	2	8	30	10	21	9
11	28	15	8	19	32	6	26	2	20	29	16	9	3	23	12	13	31	25	11
12	29	16	9	32	19	26	6	20	2	28	15	8	23	3	11	31	13	24	12
17	11	21	15	3	23	10	30	6	26	12	22	16	7	27	18	19	32	29	17
18	12	22	16	23	3	30	10	26	6	11	21	15	27	7	17	32	19	28	18
20	26	27	31	16	15	18	17	25	24	6	7	13	9	8	2	22	21	10	20
23	32	26	20	29	28	16	15	9	8	19	6	2	12	11	3	25	24	13	23
27	23	30	26	12	11	22	21	16	15	3	10	6	18	17	7	29	28	19	27
4	8	11	17	2	20	3	23	7	27	9	12	18	1	14	5	6	26	16	4
5	9	12	18	20	2	23	3	27	7	8	11	17	14	1	4	26	6	15	5
14	20	23	27	9	8	12	11	18	17	2	3	7	5	4	1	16	15	6	14
15	21	24	28	10	30	13	31	19	32	22	25	29	6	26	16	1	14	5	15
16	22	25	29	30	10	31	13	32	19	21	24	28	26	6	15	14	1	4	16
26	30	31	32	22	21	25	24	29	28	10	13	19	16	15	6	5	4	1	26
1	2	3	7	8	9	11	12	17	18	20	23	27	4	5	14	15	16	26	1

TABLE 111. Cayley subtable for G_{32}^{47}.

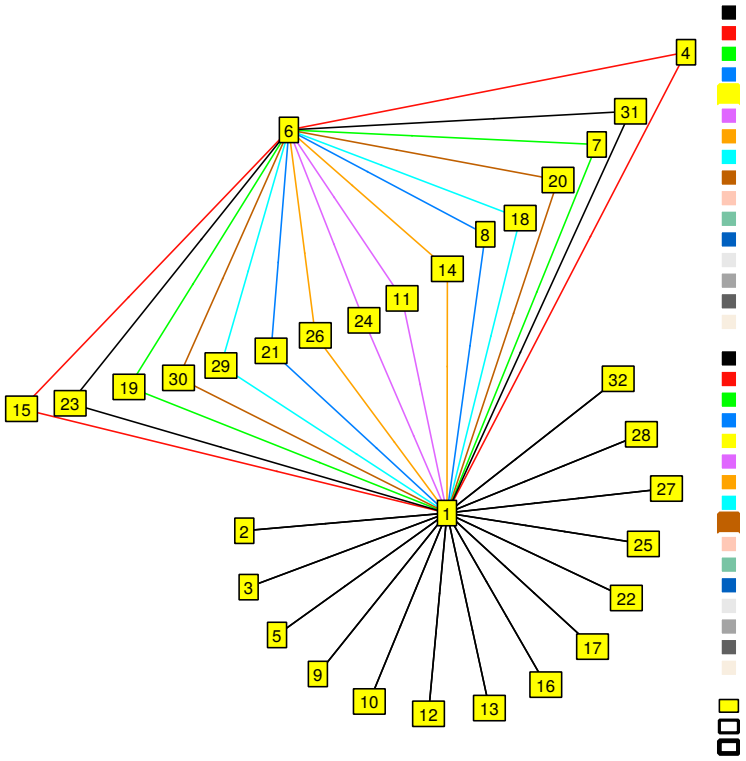

FIGURE 114. $G_{32}^{48} \cong C_2 \times ((C_4 \times C_2) \rtimes C_2)$. $(2^{14}, 4^8)$. $(x_1^{32} + 15x_2^{16} + 16x_4^8)/32$.

	4	7	8	11	14	18	20	23	2	3	5	9	10	12	13	16	17	22	25	27	28	32	1
4	6	17	10	13	16	27	22	25	8	11	14	20	21	23	24	26	19	30	31	29	7	18	4
7	17	6	24	8	27	16	31	20	13	2	18	25	3	9	10	29	15	12	22	26	4	14	7
8	10	11	6	19	22	23	16	29	4	17	20	14	15	27	28	30	13	26	32	25	3	12	8
11	13	21	7	6	25	30	18	16	28	4	23	32	17	14	15	31	2	27	26	9	10	22	11
14	16	27	22	25	6	17	10	13	20	23	4	8	30	11	31	15	29	21	24	19	18	7	14
18	27	16	31	20	17	6	24	8	25	9	7	13	12	2	22	19	26	3	10	15	14	4	18
20	22	23	16	29	10	11	6	19	14	27	8	4	26	17	32	21	25	15	28	13	12	3	20
23	25	30	18	16	13	21	7	6	32	14	11	28	27	4	26	24	9	17	15	2	22	10	23
2	8	3	4	17	20	12	14	27	1	7	9	5	6	18	19	22	11	16	29	23	24	31	2
3	11	10	28	4	23	22	32	14	19	1	12	29	7	5	6	25	21	18	16	30	8	20	3
5	14	18	20	23	4	7	8	11	9	12	1	2	22	3	25	6	27	10	13	17	32	28	5
9	20	12	14	27	8	3	4	17	5	18	2	1	16	7	29	10	23	6	19	11	31	24	9
10	21	13	15	28	30	25	26	32	6	19	22	16	1	29	7	9	24	5	18	31	11	23	10
12	23	22	32	14	11	10	28	4	29	5	3	19	18	1	16	13	30	7	6	21	20	8	12
13	24	2	17	15	31	9	27	26	7	6	25	18	19	16	1	12	8	29	5	20	21	30	13
16	26	29	30	31	15	19	21	24	22	25	6	10	9	13	12	1	32	2	3	28	27	17	16
17	19	15	3	10	29	26	12	22	24	8	27	31	11	20	21	32	1	23	30	5	6	16	17
22	30	25	26	32	21	13	15	28	16	29	10	6	5	19	18	2	31	1	7	24	23	11	22
25	31	9	27	26	24	2	17	15	18	16	13	7	29	6	5	3	20	19	1	8	30	21	25
27	29	26	12	22	19	15	3	10	31	20	17	24	23	8	30	28	5	11	21	1	16	6	27
28	7	4	13	2	18	14	25	9	11	21	32	23	24	30	8	27	6	31	20	16	1	5	28
32	18	14	25	9	7	4	13	2	23	30	28	11	31	21	20	17	16	24	8	6	5	1	32
1	4	7	8	11	14	18	20	23	2	3	5	9	10	12	13	16	17	22	25	27	28	32	1

TABLE 112. Cayley subtable for G_{32}^{48}.

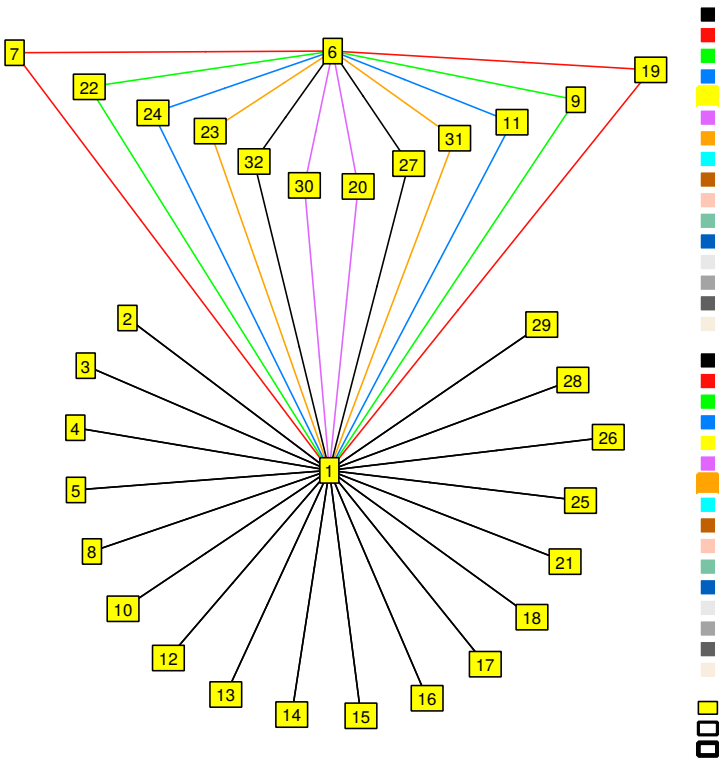

FIGURE 115. $G_{32}^{49} \cong (C_2 \times D_8) \rtimes C_2$. $(2^{18}, 4^6)$. $(x_1^{32} + 19x_2^{16} + 12x_4^8)/32$.

	7	9	11	20	23	27	2	3	4	5	8	10	12	13	14	15	16	17	18	21	25	26	28	2
7	6	25	8	31	20	26	13	2	17	18	24	3	9	10	27	28	29	15	16	11	22	32	4	
9	25	6	27	15	17	24	16	18	20	2	26	5	7	29	8	30	10	31	13	14	19	21	23	
11	8	32	6	29	16	9	28	15	3	23	19	17	26	4	12	13	31	2	20	7	14	25	10	3
20	23	15	29	6	19	3	26	32	9	8	16	14	28	27	2	22	21	12	11	5	17	10	25	2
23	30	17	16	7	6	10	27	26	12	11	18	32	15	14	3	25	24	22	21	29	4	13	9	
27	26	11	22	3	10	6	23	30	18	17	12	31	21	20	7	29	28	16	15	25	8	19	5	
2	3	5	17	14	27	23	1	7	8	9	4	6	18	19	20	21	22	11	12	15	29	30	24	2
3	10	29	4	32	14	30	19	1	11	12	28	7	5	6	23	24	25	21	22	17	16	31	8	
4	28	20	13	9	25	29	8	24	1	14	2	21	31	11	5	6	26	19	32	10	23	16	7	2
5	29	10	23	21	11	28	22	12	14	1	30	9	3	25	4	26	6	32	19	20	13	15	27	
8	24	14	19	5	29	25	4	28	2	20	1	15	32	17	9	10	30	13	31	6	27	22	3	2
10	13	16	28	26	32	31	6	19	21	22	15	1	29	7	30	8	9	24	25	4	18	20	11	1
12	9	7	14	17	4	8	18	5	23	3	27	29	1	16	11	31	13	20	2	32	6	24	30	1
13	2	18	15	27	26	20	7	6	24	25	17	19	16	1	31	11	12	8	9	28	5	23	21	2
14	27	21	25	10	13	7	30	31	5	4	22	20	24	23	1	16	15	18	17	9	11	6	29	2
15	17	30	3	22	12	18	21	11	6	26	10	8	23	24	16	1	14	7	27	2	31	5	19	3
16	18	2	31	8	24	17	9	25	26	6	20	22	13	12	15	14	1	27	7	30	3	4	32	1
17	4	31	10	25	22	5	24	21	7	27	13	11	30	8	18	19	32	1	14	3	20	29	6	2
18	5	3	20	11	8	4	12	9	27	7	23	25	2	22	17	32	19	14	1	31	10	28	26	
21	11	26	7	16	18	12	15	17	10	30	6	4	27	28	22	2	20	3	23	1	32	9	13	3
25	22	19	26	28	15	21	29	16	31	13	32	18	6	5	24	23	3	30	10	27	1	11	20	
26	32	8	12	2	3	19	20	23	16	15	9	30	11	31	6	5	4	29	28	22	24	1	18	1
28	15	23	2	12	9	16	11	8	19	32	3	24	20	21	29	7	27	6	26	13	30	18	1	1
29	16	13	30	24	21	15	25	22	32	19	31	12	10	9	28	27	7	26	6	23	2	17	14	
1	7	9	11	20	23	27	2	3	4	5	8	10	12	13	14	15	16	17	18	21	25	26	28	2

TABLE 113. Cayley subtable for G_{32}^{49}.

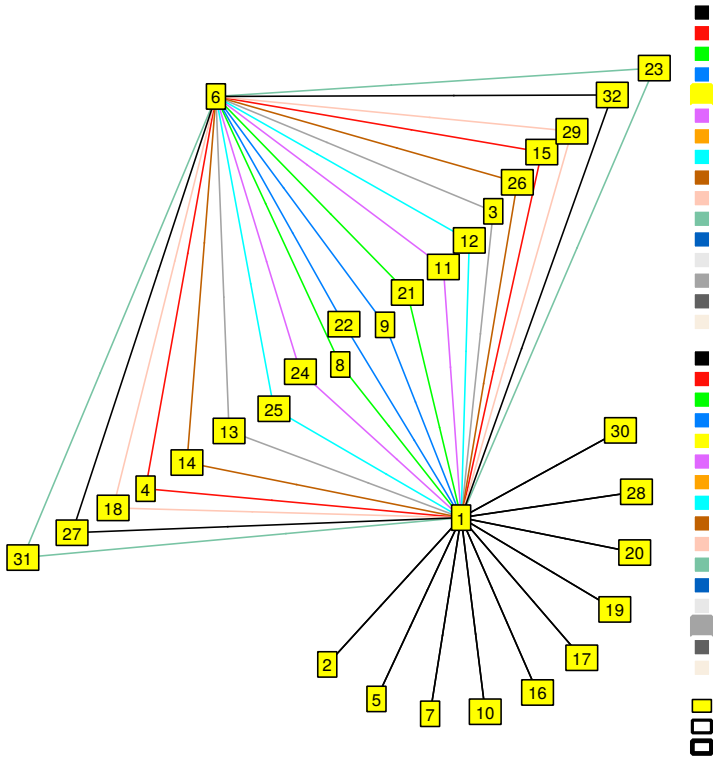

FIGURE 116. $G_{32}^{50} \cong (C_2 \times Q_8) \rtimes C_2$. $(2^{10}, 4^{10})$. $(x_1^{32} + 11x_2^{16} + 20x_4^8)/32$.

	3	4	8	9	11	12	14	18	23	27	2	5	7	10	16	17	19	20	28	30	1
3	6	11	28	29	15	16	23	9	26	20	19	12	2	7	25	8	10	32	21	27	3
4	24	6	10	20	3	31	16	32	12	18	8	14	28	21	26	7	17	22	19	9	4
8	28	10	6	14	7	32	22	31	18	12	4	20	24	15	30	3	11	16	13	5	8
9	18	20	26	6	27	7	8	13	17	24	16	2	25	5	10	31	12	15	23	4	9
11	4	13	7	32	6	14	25	30	16	9	28	23	21	17	31	2	8	18	10	29	11
12	16	23	27	7	26	6	11	10	15	21	18	3	22	29	13	30	9	17	20	28	12
14	31	16	9	21	12	24	6	17	3	19	30	4	27	20	15	29	32	2	18	10	14
18	22	27	23	3	30	10	17	6	21	15	12	7	16	25	19	26	5	11	14	24	18
23	14	25	29	17	16	4	13	8	6	10	27	11	20	32	24	22	30	19	9	7	23
27	20	29	25	11	22	8	19	4	10	6	23	17	14	31	28	16	26	13	5	3	27
2	7	8	4	5	17	18	20	12	27	23	1	9	3	6	22	11	13	14	24	26	2
5	12	14	30	10	23	3	4	19	11	28	22	1	29	9	6	32	18	21	27	8	5
7	10	17	24	25	21	22	27	5	30	14	13	18	1	3	29	4	6	31	15	23	7
10	19	21	15	16	28	29	30	25	32	31	6	22	13	1	9	24	3	26	11	14	10
16	25	26	20	2	31	13	15	7	24	17	9	6	18	22	1	27	29	8	32	21	16
17	8	19	3	31	10	20	29	26	22	5	24	27	15	11	32	1	4	12	6	25	17
19	2	28	11	12	8	9	32	16	20	26	3	29	6	13	18	15	1	23	4	31	19
20	32	22	5	15	18	28	10	11	7	13	26	8	23	14	21	25	31	1	12	6	20
28	21	7	13	23	2	30	18	14	9	16	11	32	4	24	27	6	15	25	1	12	28
30	27	9	16	4	29	17	2	24	19	3	14	21	31	26	8	12	23	6	25	1	30
1	3	4	8	9	11	12	14	18	23	27	2	5	7	10	16	17	19	20	28	30	1

TABLE 114. Cayley subtable for G_{32}^{50}.

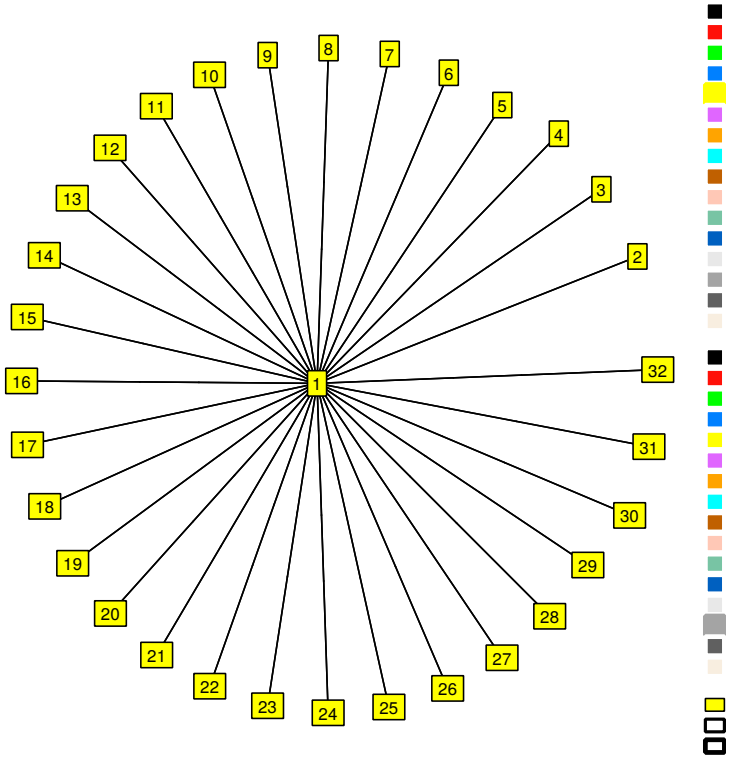

FIGURE 117. $G_{32}^{51} \cong C_2 \times C_2 \times C_2 \times C_2 \times C_2$. (2^{31}). $(x_1^{10} + 5x_1^8x_2 + 10x_1^6x_2^2 + 10x_1^4x_2^3 + 5x_1^2x_2^4 + x_2^5)/32$.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	2
2	1	7	8	9	10	3	4	5	6	17	18	19	20	21	22	11	12	13	14	15	16	27	28	2
3	7	1	11	12	13	2	17	18	19	4	5	6	23	24	25	8	9	10	27	28	29	14	15	1
4	8	11	1	14	15	17	2	20	21	3	23	24	5	6	26	7	27	28	9	10	30	12	13	3
5	9	12	14	1	16	18	20	2	22	23	3	25	4	26	6	27	7	29	8	30	10	11	31	1
6	10	13	15	16	1	19	21	22	2	24	25	3	26	4	5	28	29	7	30	8	9	31	11	1
7	3	2	17	18	19	1	11	12	13	8	9	10	27	28	29	4	5	6	23	24	25	20	21	2
8	4	17	2	20	21	11	1	14	15	7	27	28	9	10	30	3	23	24	5	6	26	18	19	3
9	5	18	20	2	22	12	14	1	16	27	7	29	8	30	10	23	3	25	4	26	6	17	32	1
10	6	19	21	22	2	13	15	16	1	28	29	7	30	8	9	24	25	3	26	4	5	32	17	1
11	17	4	3	23	24	8	7	27	28	1	14	15	12	13	31	2	20	21	18	19	32	5	6	2
12	18	5	23	3	25	9	27	7	29	14	1	16	11	31	13	20	2	22	17	32	19	4	26	
13	19	6	24	25	3	10	28	29	7	15	16	1	31	11	12	21	22	2	32	17	18	26	4	
14	20	23	5	4	26	27	9	8	30	12	11	31	1	16	15	18	17	32	2	22	21	3	25	2
15	21	24	6	26	4	28	10	30	8	13	31	11	16	1	14	19	32	17	22	2	20	25	3	2
16	22	25	26	6	5	29	30	10	9	31	13	12	15	14	1	32	19	18	21	20	2	24	23	
17	11	8	7	27	28	4	3	23	24	2	20	21	18	19	32	1	14	15	12	13	31	9	10	3
18	12	9	27	7	29	5	23	3	25	20	2	22	17	32	19	14	1	16	11	31	13	8	30	1
19	13	10	28	29	7	6	24	25	3	21	22	2	32	17	18	15	16	1	31	11	12	30	8	
20	14	27	9	8	30	23	5	4	26	18	17	32	2	22	21	12	11	31	1	16	15	7	29	2
21	15	28	10	30	8	24	6	26	4	19	32	17	22	2	20	13	31	11	16	1	14	29	7	2
22	16	29	30	10	9	25	26	6	5	32	19	18	21	20	2	31	13	12	15	14	1	28	27	
23	27	14	12	11	31	20	18	17	32	5	4	26	3	25	24	9	8	30	7	29	28	1	16	1
24	28	15	13	31	11	21	19	32	17	6	26	4	25	3	23	10	30	8	29	7	27	16	1	1
25	29	16	31	13	12	22	32	19	18	26	6	5	24	23	3	30	10	9	28	27	7	15	14	
26	30	31	16	15	14	32	22	21	20	25	24	23	6	5	4	29	28	27	10	9	8	13	12	1
27	23	20	18	17	32	14	12	11	31	9	8	30	7	29	28	5	4	26	3	25	24	2	22	2
28	24	21	19	32	17	15	13	31	11	10	30	8	29	7	27	6	26	4	25	3	23	22	2	2
29	25	22	32	19	18	16	31	13	12	30	10	9	28	27	7	26	6	5	24	23	3	21	20	
30	26	32	22	21	20	31	16	15	14	29	28	27	10	9	8	25	24	23	6	5	4	19	18	1
31	32	26	25	24	23	30	29	28	27	16	15	14	13	12	11	22	21	20	19	18	17	6	5	
32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	2

TABLE 115. Cayley subtable for G_{32}^{51}.

20. Order 33

The graph of the group $G_{33}^1 \cong C_{33}$ is the simple cyclic ring with 33 elements and not shown for that reason. The group is a subgroup of S_{14} with cycle index $(x_1^{14} + 2x_1^{11}x_3 + 10x_1^3x_{11} + 20x_3x_{11})/33$.

21. Order 34

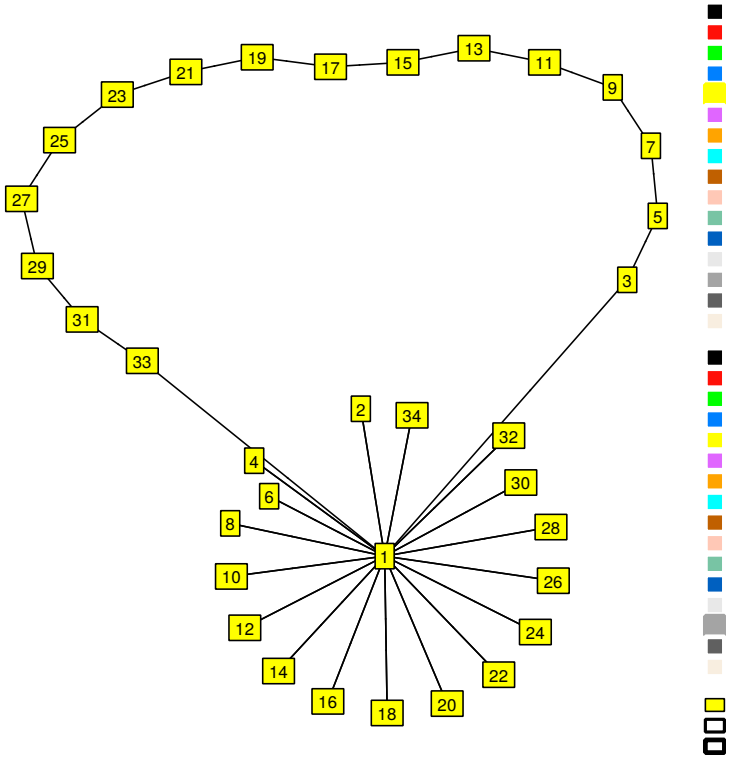

Figure 118. $G_{34}^1 \cong D_{34}$. $(2^{17}, 17^1)$. $(x_1^{34} + 17x_2^{17} + 16x_{17}^2)/34$.

	3	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	1
3	5	34	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	3
2	4	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	2
4	6	33	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	4
6	8	31	33	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	6
8	10	29	31	33	1	3	5	7	9	11	13	15	17	19	21	23	25	27	8
10	12	27	29	31	33	1	3	5	7	9	11	13	15	17	19	21	23	25	10
12	14	25	27	29	31	33	1	3	5	7	9	11	13	15	17	19	21	23	12
14	16	23	25	27	29	31	33	1	3	5	7	9	11	13	15	17	19	21	14
16	18	21	23	25	27	29	31	33	1	3	5	7	9	11	13	15	17	19	16
18	20	19	21	23	25	27	29	31	33	1	3	5	7	9	11	13	15	17	18
20	22	17	19	21	23	25	27	29	31	33	1	3	5	7	9	11	13	15	20
22	24	15	17	19	21	23	25	27	29	31	33	1	3	5	7	9	11	13	22
24	26	13	15	17	19	21	23	25	27	29	31	33	1	3	5	7	9	11	24
26	28	11	13	15	17	19	21	23	25	27	29	31	33	1	3	5	7	9	26
28	30	9	11	13	15	17	19	21	23	25	27	29	31	33	1	3	5	7	28
30	32	7	9	11	13	15	17	19	21	23	25	27	29	31	33	1	3	5	30
32	34	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	1	3	32
34	2	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	1	34
1	3	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	1

TABLE 116. Cayley subtable for G_{34}^{1}.

The graph of the group $G_{34}^{2} \cong C_{34}$ is the simple cyclic ring with 34 elements and not shown for that reason. The group is a subgroup of S_{19} with cycle index $(x_1^{19} + x_1^{17}x_2 + 16x_1^2x_{17} + 16x_2x_{17})/34$.

22. Order 35

The graph of the group $G_{35}^{1} \cong C_{35}$ is the simple cyclic ring with 35 elements and not shown for that reason. The group is a subgroup of S_{12} with cycle index $(x_1^{12} + 4x_1^7x_5 + 6x_1^5x_7 + 24x_5x_7)/35$.

23. Order 36

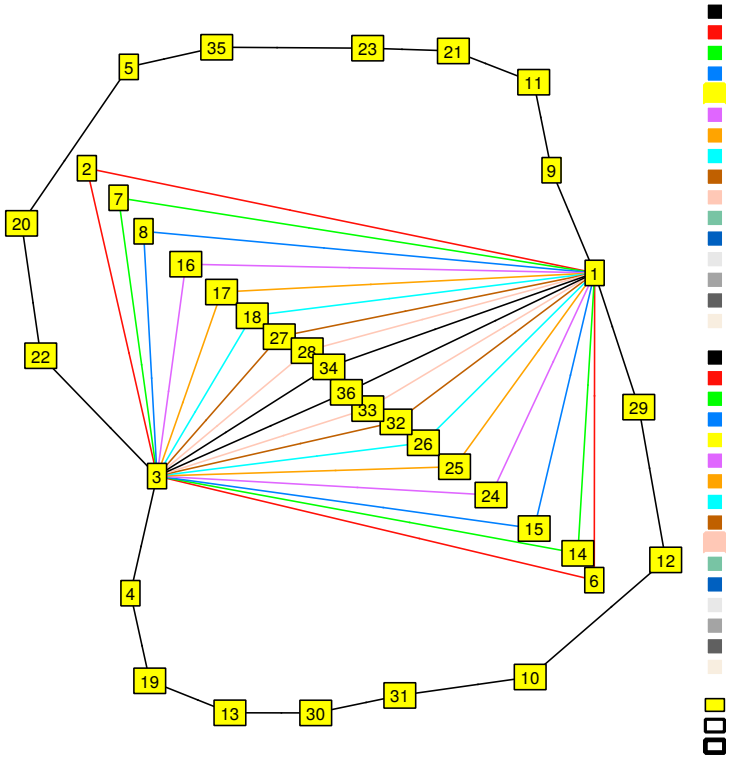

Figure 119. $G_{36}^{1} \cong C_9 \rtimes C_4$. $(4^9, 18^1)$. $(x_1^{36} + x_2^{18} + 2x_3^{12} + 18x_4^9 + 2x_6^6 + 6x_9^4 + 6x_{18}^2)/36$.

	9	2	7	8	16	17	18	27	28	34	1
9	11	32	6	36	14	15	24	25	26	33	9
2	14	3	9	10	19	20	21	29	30	35	2
7	24	29	3	35	9	10	19	20	21	30	7
8	25	21	30	3	35	9	10	19	20	29	8
16	26	20	29	30	3	35	9	10	19	21	16
17	32	19	21	29	30	3	35	9	10	20	17
18	33	10	20	21	29	30	3	35	9	19	18
27	6	9	19	20	21	29	30	3	35	10	27
28	36	35	10	19	20	21	29	30	3	9	28
34	15	30	35	9	10	19	20	21	29	3	34
1	9	2	7	8	16	17	18	27	28	34	1

TABLE 117. Cayley subtable for G_{36}^{1}.

The graph of the group $G_{36}^2 \cong C_{36}$ is the simple cyclic ring with 36 elements and not shown for that reason. The group is a subgroup of S_{13} with cycle index $(x_1^{13} + x_1^9x_2^2 + 2x_1^9x_4 + 2x_1^4x_3^3 + 6x_1^4x_9 + 2x_2^2x_3^3 + 4x_3^3x_4 + 6x_2^2x_9 + 12x_4x_9)/36$.

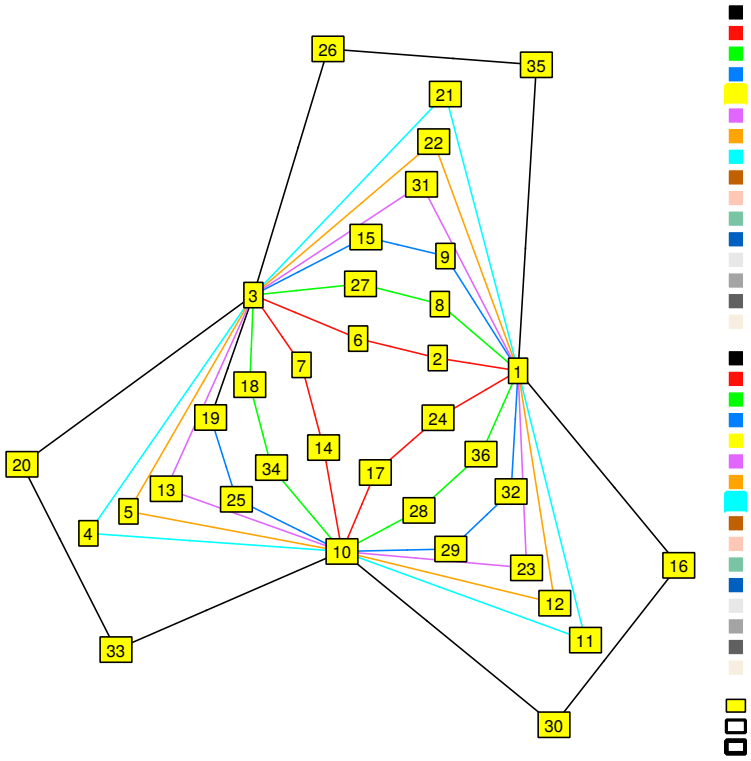

FIGURE 120. $G_{36}^3 \cong (C_2 \times C_2) \rtimes C_9$. $(6^3, 9^4)$. $(x_1^{36} + 3x_2^{18} + 2x_3^{12} + 6x_6^6 + 24x_9^4)/36$.

	2	8	9	16	11	12	23	1
2	6	15	16	12	18	19	30	2
8	16	27	6	11	7	30	19	8
9	27	16	15	23	30	7	18	9
16	23	12	11	30	34	14	25	16
11	19	30	7	25	10	31	22	11
12	30	19	18	34	31	10	21	12
23	18	7	30	14	22	21	10	23
1	2	8	9	16	11	12	23	1

TABLE 118. Cayley subtable for G_{36}^{3}.

	9	2	6	7	8	14	15	16	17	18	24	25	26	27	28	32	33	34	36	1
9	11	32	27	6	36	2	34	14	15	24	7	8	16	25	26	17	18	33	28	9
2	14	1	3	4	5	9	10	11	12	13	19	20	21	22	23	29	30	31	35	2
6	7	3	1	9	10	4	5	19	20	21	11	12	13	29	30	22	23	35	31	6
7	24	22	29	1	31	3	35	4	5	11	9	10	19	12	13	20	21	23	30	7
8	25	13	21	23	1	30	3	31	4	5	35	9	10	11	12	19	20	22	29	8
14	16	29	22	3	35	1	31	9	10	19	4	5	11	20	21	12	13	30	23	14
15	17	21	13	30	3	23	1	35	9	10	31	4	5	19	20	11	12	29	22	15
16	26	12	20	22	23	29	30	1	31	4	3	35	9	5	11	10	19	13	21	16
17	32	11	19	13	22	21	29	23	1	31	30	3	35	4	5	9	10	12	20	17
18	33	5	10	12	13	20	21	22	23	1	29	30	3	31	4	35	9	11	19	18
24	18	20	12	29	30	22	23	3	35	9	1	31	4	10	19	5	11	21	13	24
25	27	19	11	21	29	13	22	30	3	35	23	1	31	9	10	4	5	20	12	25
26	28	10	5	20	21	12	13	29	30	3	22	23	1	35	9	31	4	19	11	26
27	6	4	9	11	12	19	20	13	22	23	21	29	30	1	31	3	35	5	10	27
28	36	31	35	5	11	10	19	12	13	22	20	21	29	23	1	30	3	4	9	28
32	2	9	4	19	20	11	12	21	29	30	13	22	23	3	35	1	31	10	5	32
33	34	35	31	10	19	5	11	20	21	29	12	13	22	30	3	23	1	9	4	33
34	15	23	30	31	4	35	9	5	11	12	10	19	20	13	22	21	29	1	3	34
36	8	30	23	35	9	31	4	10	19	20	5	11	12	21	29	13	22	3	1	36
1	9	2	6	7	8	14	15	16	17	18	24	25	26	27	28	32	33	34	36	1

TABLE 119. Cayley subtable for G_{36}^{4}.

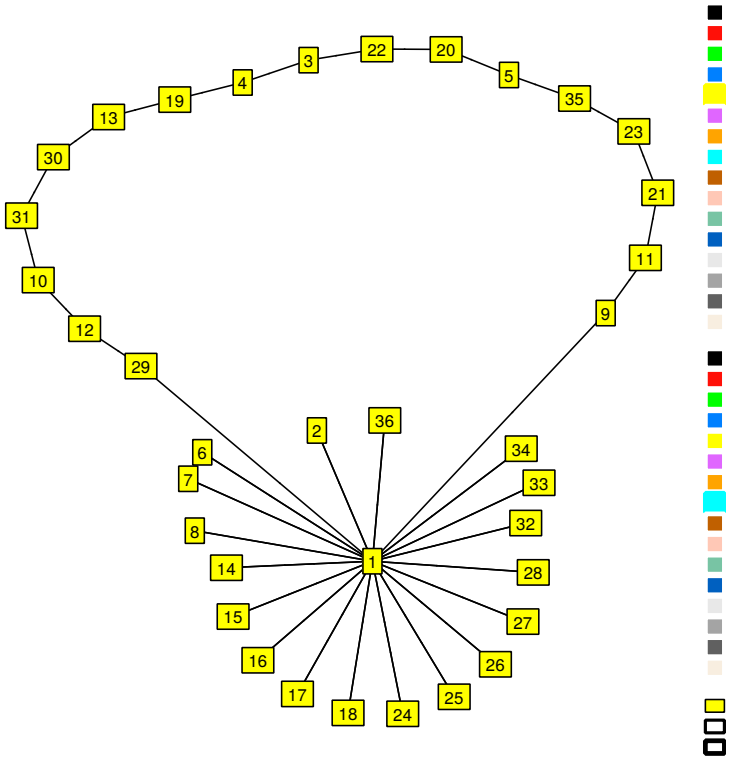

FIGURE 121. $G_{36}^4 \cong D_{36}$. $(2^{18}, 18^1)$. $(x_1^{36} + 19x_2^{18} + 2x_3^{12} + 2x_6^6 + 6x_9^4 + 6x_{18}^2)/36$.

	7	9	14	1
7	11	24	19	7
9	24	11	16	9
14	19	16	11	14
1	7	9	14	1

TABLE 120. Cayley subtable for G_{36}^5.

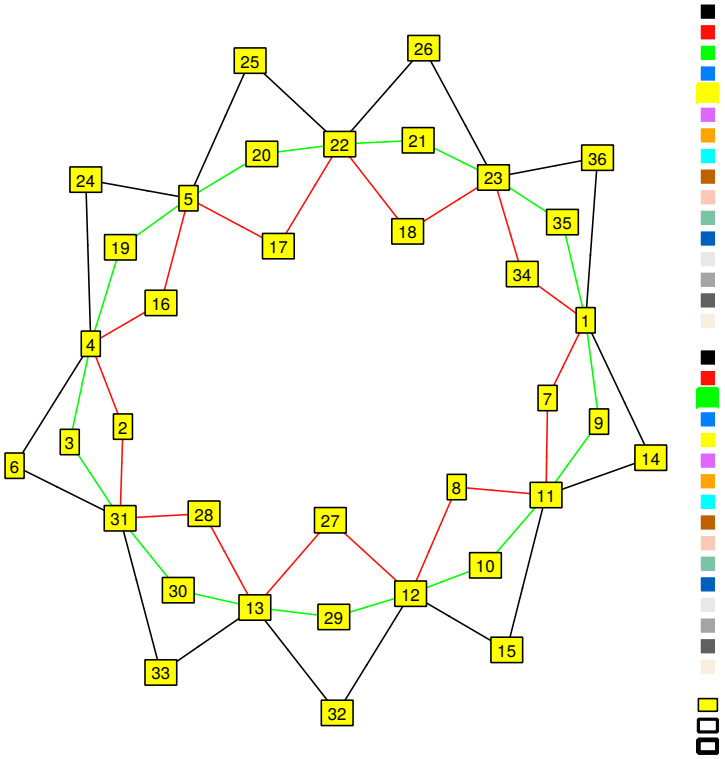

FIGURE 122. $G_{36}^5 \cong C_{18} \times C_2$. (18^3). $(x_1^{13} + 2x_1^{11}x_2 + x_1^9x_2^2 + 2x_1^4x_3^3 + 4x_1^2x_2x_3^3 + 6x_1^4x_9 + 2x_2^2x_3^3 + 12x_1^2x_2x_9 + 6x_2^2x_9)/36$.

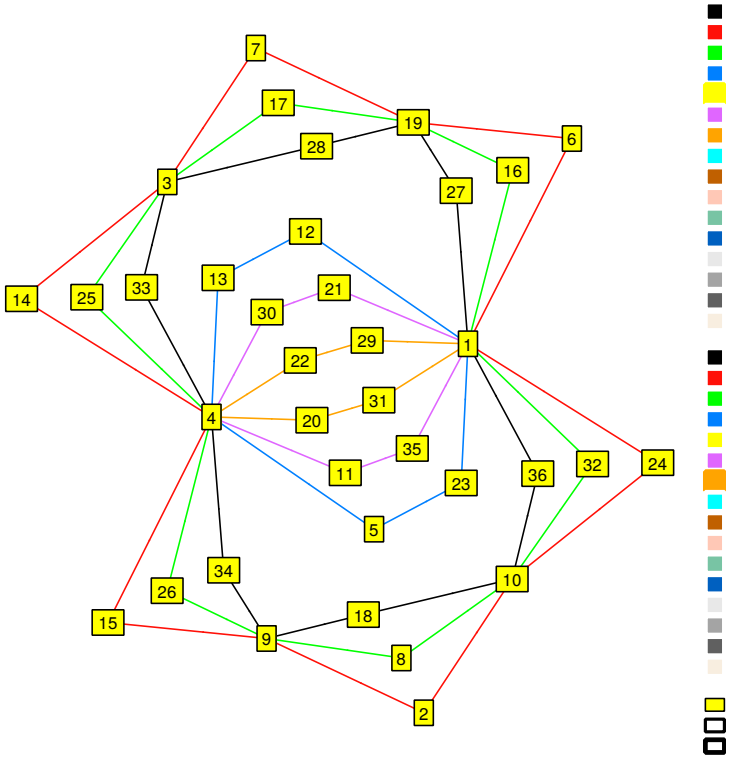

FIGURE 123. $G_{36}^{6} \cong C_3 \times (C_3 \rtimes C_4)$. $(6^3, 12^3)$. $(x_1^{36} + x_2^{18} + 8x_3^{12} + 6x_4^9 + 8x_6^6 + 12x_{12}^3)/36$.

	6	16	27	12	21	29	1
6	19	29	35	26	32	17	6
16	35	19	29	34	36	28	16
27	29	35	19	15	24	7	27
12	34	15	26	13	22	30	12
21	36	24	32	22	30	13	21
29	28	7	17	30	13	22	29
1	6	16	27	12	21	29	1

TABLE 121. Cayley subtable for G_{36}^{6}.

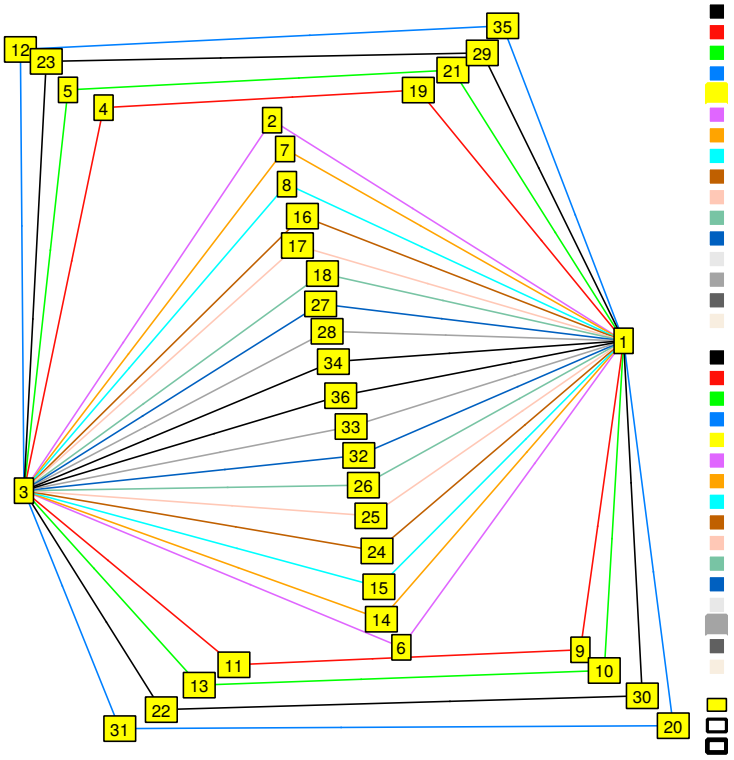

FIGURE 124. $G_{36}^{7} \cong (C_3 \times C_3) \rtimes C_4$). $(4^9, 6^4)$. $(x_1^{36} + x_2^{18} + 8x_3^{12} + 18x_4^9 + 8x_6^6)/36$.

	9	10	20	29	2	7	8	16	17	18	27	28	34	1
9	11	12	22	5	24	6	32	14	15	36	25	26	33	9
10	12	13	23	31	26	33	6	36	14	15	24	25	32	10
20	22	23	31	13	36	26	24	33	6	32	14	15	25	20
29	5	31	13	23	33	36	14	26	24	25	6	32	15	29
2	14	15	25	32	3	9	10	19	20	21	29	30	35	2
7	24	25	32	15	19	3	29	9	10	35	20	21	30	7
8	25	26	33	36	21	30	3	35	9	10	19	20	29	8
16	6	32	15	25	9	19	20	3	29	30	10	35	21	16
17	32	33	36	26	35	21	19	30	3	29	9	10	20	17
18	33	6	14	24	10	20	21	29	30	3	35	9	19	18
27	15	36	26	33	30	35	9	21	19	20	3	29	10	27
28	36	14	24	6	29	10	35	20	21	19	30	3	9	28
34	26	24	6	14	20	29	30	10	35	9	21	19	3	34
1	9	10	20	29	2	7	8	16	17	18	27	28	34	1

TABLE 122. Cayley subtable for G_{36}^{7}.

	6	7	15	24	1
6	20	22	30	13	6
7	22	23	31	35	7
15	30	31	35	23	15
24	13	35	23	31	24
1	6	7	15	24	1

TABLE 123. Cayley subtable for G_{36}^{8}.

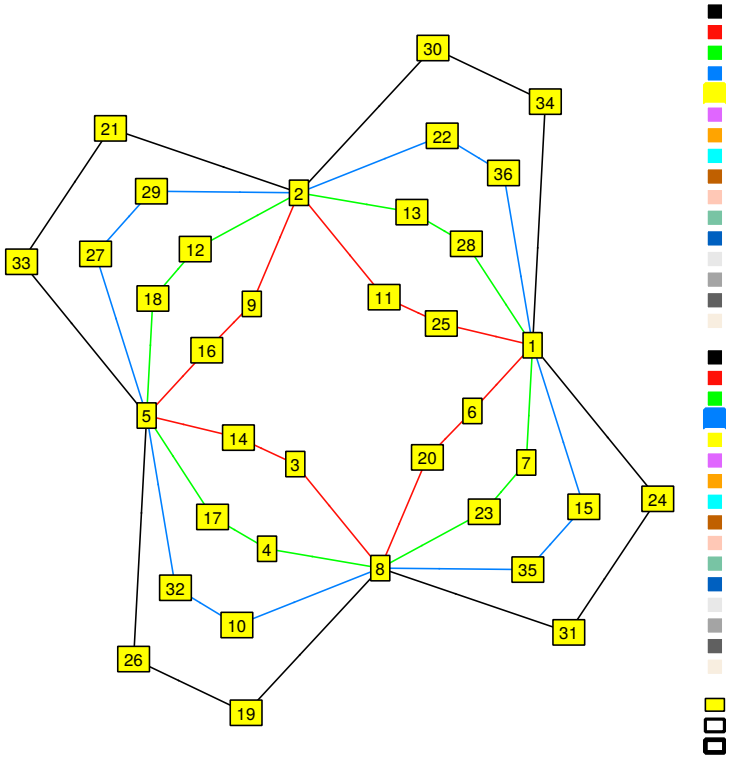

FIGURE 125. $G_{36}^{8} \cong C_{12} \times C_3$. (12^4). $(x_1^{10} + 4x_1^7x_3 + x_1^6x_2^2 + 2x_1^6x_4 + 4x_1^4x_3^2 + 4x_1^3x_2^2x_3 + 8x_1^3x_3x_4 + 4x_2^2x_3^2 + 8x_3^2x_4)/36$.

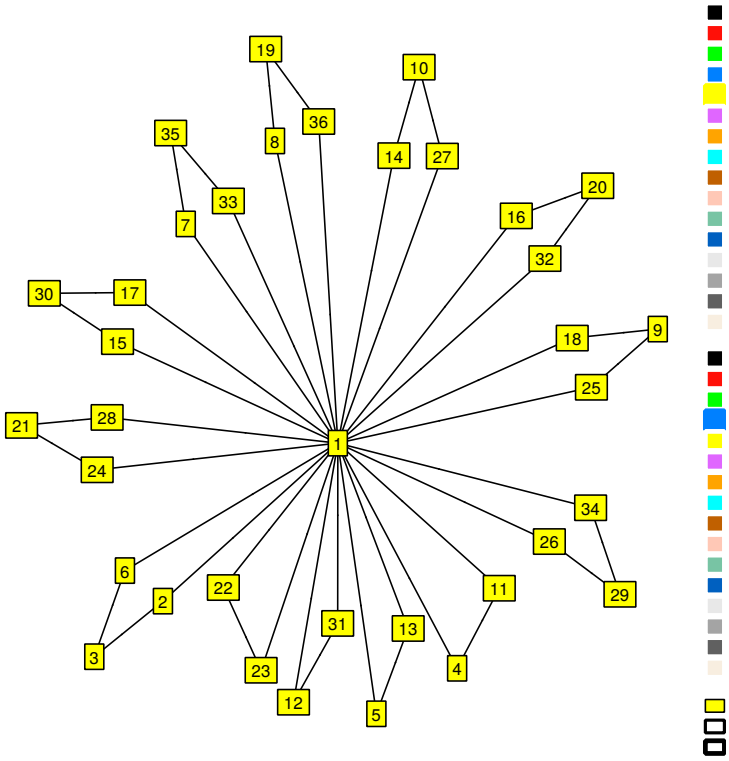

FIGURE 126. $G_{36}^{9} \cong (C_3 \times C_3) \rtimes C_4$. $(3^4, 4^9)$. $(x_1^{36} + 9x_2^{18} + 8x_3^{12} + 18x_4^9)/36$.

	2	7	8	14	15	16	18	24	26	4	5	12	22	1
2	3	9	10	4	5	19	21	11	13	7	8	17	27	2
7	30	35	9	5	31	21	20	12	11	16	17	27	8	7
8	35	21	19	22	23	30	29	5	4	17	18	28	34	8
14	23	31	4	10	35	13	12	20	19	24	25	32	15	14
15	31	13	11	29	30	23	22	10	9	25	26	33	36	15
16	29	10	35	31	4	20	19	13	12	2	27	8	17	16
18	20	29	30	13	11	10	9	23	22	28	2	7	16	18
24	22	5	31	35	9	12	11	21	20	6	32	15	25	24
26	12	22	23	21	19	5	4	30	29	33	6	14	24	26
4	28	34	7	15	36	18	17	25	24	11	12	22	5	4
5	34	18	16	32	33	28	27	15	14	12	13	23	31	5
12	8	17	18	33	6	27	2	36	15	22	23	31	13	12
22	7	16	17	6	32	2	28	14	36	5	31	13	23	22
1	2	7	8	14	15	16	18	24	26	4	5	12	22	1

TABLE 124. Cayley subtable for G_{36}^{9}.

	7	10	17	20	28	29	12	22	6	14	15	24	25	26	32	33	36	1
7	11	32	22	15	31	25	27	8	19	3	29	9	10	35	20	21	30	7
10	33	13	14	23	25	31	30	35	18	28	2	34	7	8	16	17	27	10
17	31	36	11	26	22	33	34	18	35	21	19	30	3	29	9	10	20	17
20	36	31	24	13	32	23	35	21	34	18	16	28	2	27	7	8	17	20
28	22	24	31	6	11	14	16	2	29	10	35	20	21	19	30	3	9	28
29	26	23	6	31	15	13	21	30	28	34	7	18	16	17	2	27	8	29
12	34	35	16	21	27	30	31	13	36	26	24	33	6	32	14	15	25	12
22	18	30	2	35	8	21	13	23	33	36	14	26	24	25	6	32	15	22
6	9	8	20	17	30	27	25	32	1	4	5	11	12	13	22	23	31	6
14	19	27	29	8	35	17	32	15	11	1	22	4	5	31	12	13	23	14
15	30	18	9	28	20	34	33	36	13	23	1	31	4	5	11	12	22	15
24	3	17	10	27	21	8	15	25	4	11	12	1	22	23	5	31	13	24
25	35	34	19	18	29	28	36	26	31	13	11	23	1	22	4	5	12	25
26	20	2	30	7	9	16	14	24	5	12	13	22	23	1	31	4	11	26
32	21	28	3	34	10	18	26	33	23	31	4	13	11	12	1	22	5	32
33	29	16	35	2	19	7	24	6	22	5	31	12	13	11	23	1	4	33
36	10	7	21	16	3	2	6	14	12	22	23	5	31	4	13	11	1	36
1	7	10	17	20	28	29	12	22	6	14	15	24	25	26	32	33	36	1

TABLE 125. Cayley subtable for G_{36}^{10}.

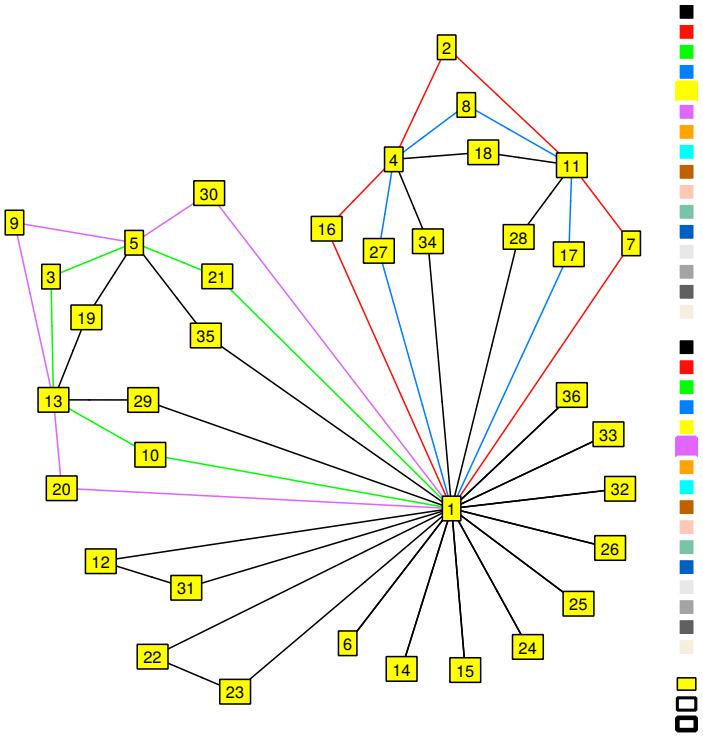

FIGURE 127. $G_{36}^{10} \cong S_3 \times S_3$. $(2^9, 3^2, 6^6)$. $(x_1^{36} + 15x_2^{18} + 8x_3^{12} + 12x_6^6)/36$.

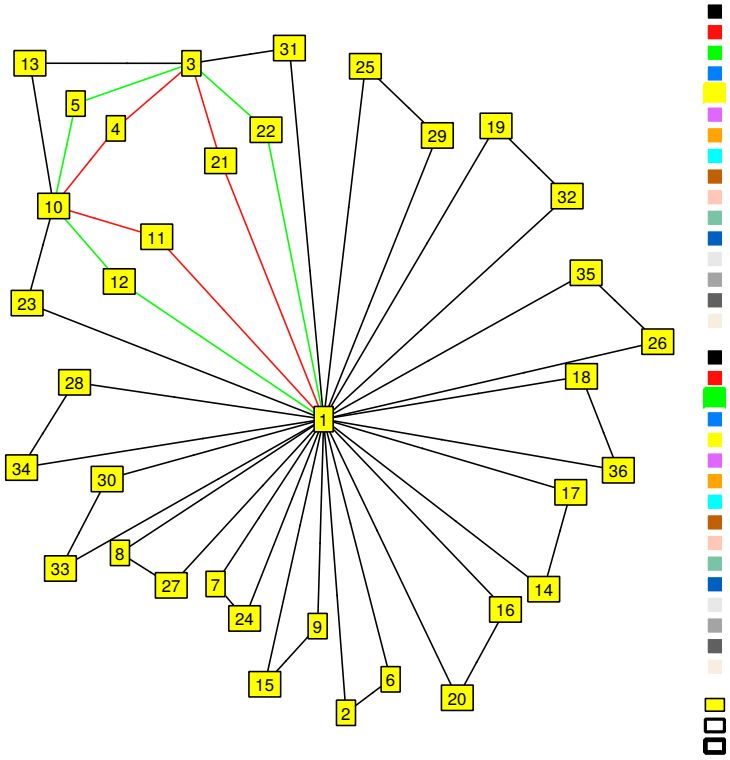

FIGURE 128. $G_{36}^{11} \cong C_3 \times A_4$. $(3^{12}, 6^3)$. $(x_1^{36} + 3x_2^{18} + 26x_3^{12} + 6x_6^6)/36$.

	11	12	23	2	7	8	9	14	16	18	19	25	26	28	30	1
11	10	31	22	19	29	30	7	36	25	35	17	33	32	20	28	11
12	31	10	21	30	35	19	18	32	34	29	28	24	36	9	17	12
23	22	21	10	18	28	7	30	33	14	17	35	36	24	2	29	23
2	18	19	30	6	14	15	16	3	5	25	26	11	12	32	34	2
7	28	29	35	14	24	25	26	10	12	32	33	21	22	15	36	7
8	7	30	19	16	26	27	6	23	4	34	14	12	11	36	25	8
9	30	7	18	27	34	16	15	11	13	26	25	3	23	33	14	9
14	32	33	36	3	10	11	12	17	19	21	22	28	29	4	31	14
16	34	14	25	13	23	5	4	18	20	12	11	7	30	22	3	16
18	17	35	29	26	33	34	14	31	11	36	24	22	21	27	32	18
19	35	17	28	34	36	26	25	21	23	33	32	10	31	16	24	19
25	24	36	33	12	22	23	3	35	18	31	10	29	28	13	21	25
26	36	24	32	23	31	12	11	28	30	22	21	17	35	5	10	26
28	2	20	9	33	16	36	24	13	21	27	6	5	4	34	15	28
30	29	28	17	25	32	14	34	22	3	24	36	31	10	6	33	30
1	11	12	23	2	7	8	9	14	16	18	19	25	26	28	30	1

TABLE 126. Cayley subtable for G_{36}^{11}.

	7	9	10	14	17	20	25	28	29	33	1
7	11	24	25	19	22	32	29	31	15	35	7
9	24	11	12	16	32	22	27	36	5	34	9
10	33	12	13	28	14	23	7	25	31	17	10
14	19	16	17	11	29	27	22	35	8	31	14
17	31	32	33	35	11	36	19	22	26	29	17
20	36	22	23	34	24	31	16	32	13	27	20
25	35	27	28	31	19	34	11	29	18	22	25
28	22	36	14	29	31	24	35	11	6	19	28
29	26	5	31	18	6	13	2	15	23	8	29
33	29	34	7	22	35	16	31	19	2	11	33
1	7	9	10	14	17	20	25	28	29	33	1

TABLE 127. Cayley subtable for G_{36}^{12}.

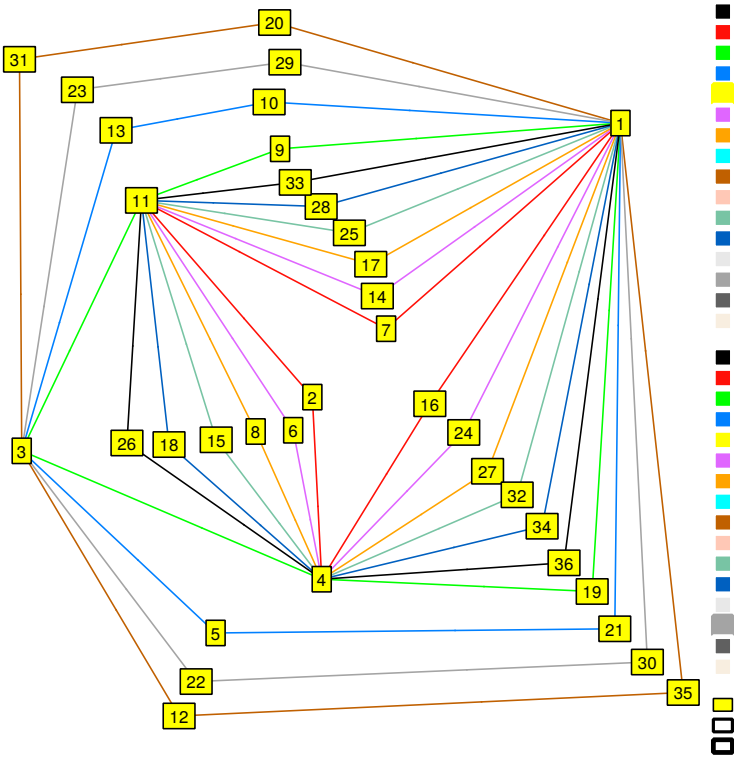

FIGURE 129. $G_{36}^{12} \cong C_6 \times S_3$. (6^{10}). $(x_1^{36} + 7x_2^{18} + 8x_3^{12} + 20x_6^6)/36$.

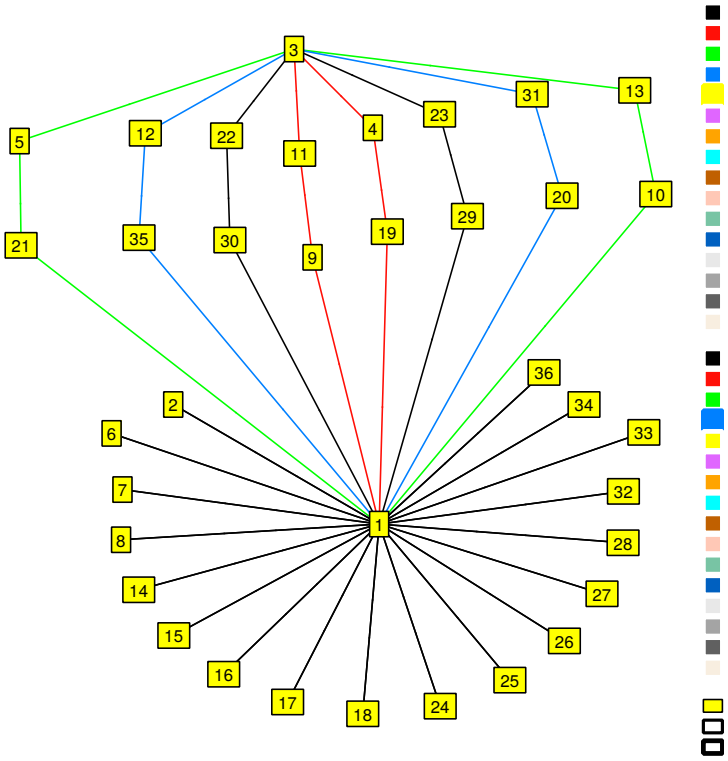

FIGURE 130. $G_{36}^{13} \cong C_2 \times ((C_3 \times C_3) \rtimes C_2)$. $(2^{18}, 6^4)$. $(x_1^{36} + 19x_2^{18} + 8x_3^{12} + 8x_6^6)/36$.

	9	10	20	29	2	6	7	8	14	15	16	17	18	24	25	26	27	28	32	33	34	36	1
9	11	12	22	5	24	16	6	32	2	27	14	15	36	7	8	34	25	26	17	18	33	28	9
10	12	13	23	31	26	18	33	6	28	2	36	14	15	34	7	8	24	25	16	17	32	27	10
20	22	23	31	13	36	34	26	24	18	16	33	6	32	28	2	27	14	15	7	8	25	17	20
29	5	31	13	23	33	28	36	14	34	7	26	24	25	18	16	17	6	32	2	27	15	8	29
2	14	15	25	32	1	3	4	5	9	10	11	12	13	19	20	21	22	23	29	30	31	35	2
6	7	8	17	27	3	1	9	10	4	5	19	20	21	11	12	13	29	30	22	23	35	31	6
7	24	25	32	15	11	19	1	22	3	29	4	5	31	9	10	35	12	13	20	21	23	30	7
8	25	26	33	36	13	21	23	1	30	3	31	4	5	35	9	10	11	12	19	20	22	29	8
14	16	17	27	8	19	11	3	29	1	22	9	10	35	4	5	31	20	21	12	13	30	23	14
15	17	18	28	34	21	13	30	3	23	1	35	9	10	31	4	5	19	20	11	12	29	22	15
16	6	32	15	25	4	9	11	12	19	20	1	22	23	3	29	30	5	31	10	35	13	21	16
17	32	33	36	26	31	35	13	11	21	19	23	1	22	30	3	29	4	5	9	10	12	20	17
18	33	6	14	24	5	10	12	13	20	21	22	23	1	29	30	3	31	4	35	9	11	19	18
24	2	27	8	17	9	4	19	20	11	12	3	29	30	1	22	23	10	35	5	31	21	13	24
25	27	28	34	18	35	31	21	19	13	11	30	3	29	23	1	22	9	10	4	5	20	12	25
26	28	2	7	16	10	5	20	21	12	13	29	30	3	22	23	1	35	9	31	4	19	11	26
27	15	36	26	33	23	30	31	4	35	9	13	11	12	21	19	20	1	22	3	29	5	10	27
28	36	14	24	6	22	29	5	31	10	35	12	13	11	20	21	19	23	1	30	3	4	9	28
32	8	34	18	28	30	23	35	9	31	4	21	19	20	13	11	12	3	29	1	22	10	5	32
33	34	7	16	2	29	22	10	35	5	31	20	21	19	12	13	11	30	3	23	1	9	4	33
34	26	24	6	14	12	20	22	23	29	30	5	31	4	10	35	9	13	11	21	19	1	3	34
36	18	16	2	7	20	12	29	30	22	23	10	35	9	5	31	4	21	19	13	11	3	1	36
1	9	10	20	29	2	6	7	8	14	15	16	17	18	24	25	26	27	28	32	33	34	36	1

TABLE 128. Cayley subtable for G_{36}^{13}.

	7	8	9	10	14	15	17	20	25	27	29	32	1
7	11	12	24	25	19	20	22	32	29	5	15	10	7
8	12	13	25	26	20	21	23	33	30	31	36	35	8
9	24	25	11	12	16	17	32	22	27	15	5	8	9
10	25	26	12	13	17	18	33	23	28	36	31	34	10
14	19	20	16	17	11	12	29	27	22	10	8	5	14
15	20	21	17	18	12	13	30	28	23	35	34	31	15
17	22	23	32	33	29	30	31	36	35	13	26	21	17
20	32	33	22	23	27	28	36	31	34	26	13	18	20
25	29	30	27	28	22	23	35	34	31	21	18	13	25
27	5	31	15	36	10	35	13	26	21	23	33	30	27
29	15	36	5	31	8	34	26	13	18	33	23	28	29
32	10	35	8	34	5	31	21	18	13	30	28	23	32
1	7	8	9	10	14	15	17	20	25	27	29	32	1

TABLE 129. Cayley subtable for G_{36}^{14}.

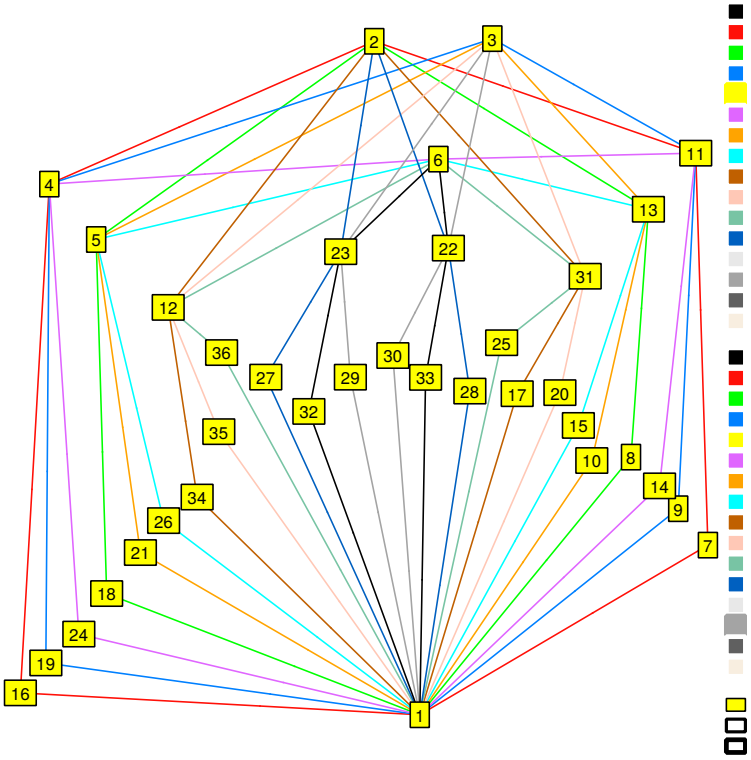

FIGURE 131. $G_{36}^{14} \cong C_6 \times C_6$. (6^{12}). $(x_1^{10} + 2x_1^8x_2 + 4x_1^7x_3 + x_1^6x_2^2 + 8x_1^5x_2x_3 + 4x_1^4x_3^2 + 4x_1^3x_2^2x_3 + 8x_1^2x_2x_3^2 + 4x_2^2x_3^2)/36$.

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