The Schwarzschild Radius and the Scale Principle

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Earlier this year I wrote a paper entitled Scale Factors and the Scale Principle. In that paper I formulated a new law which describes nature at both quantum and cosmic scales. This article shows that the formula for the Schwarzschild radius (black hole radius) is a special case of the abovementioned formulation.

***Keywords**: Schwarzschild radius, Planck scale, Planck length, Planck mass.*

1. Introduction

In a previous article [1] I introduced the scale principle or scale law through the following mathematical relationship

(1)

Scale principle or scale law
$\left(\frac{Q_1}{Q_2}\right)^n \left[\leq \middle\vert = \middle\vert \geq\right] S \left(\frac{Q_3}{Q_4}\right)^m$

Where

a) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Temperature, etc), or

b) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.
The physical quantities can be variables, constants, dimensionless constants, or differentials.
(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length).

c) The relationship is one of three possibilities: **less than or equal to** inequation ($\leq$), or an equation (=), or a **greater than or equal to** inequation ($\geq$).

d) S is a dimensionless scale factor (this factor could be a real number, a complex number, a real function or a complex function)

e) n and m are integers 0, 1, 2, 3,… (In general these two numbers are different. e.g. 1: n =1 and m = 1. e.g. 2: n = 1 and m = 2. n and m cannot be both zero in the same relationship.)

2. Schwarzschild Radius

I shall show, through very simple mathematics, that the Schwarzschild radius $R_S = \frac{2GM}{c^2}$ obeys the scale principle.

We shall start the analysis by drawing the following scale table

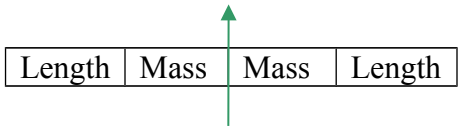
Length (Black Hole)	Mass (Planck Scale)	Mass (Black Hole)	Length (Planck Scale)
R_S	M_P	M	L_P

TABLE 1*: This simple scale table is used to show that the formula for the Schwarzschild radius obeys the scale law.*

Where

R_S = Schwarzschild radius of the black hole.
M_P = Planck mass
M = Mass of the black hole
L_P = Planck length

In order to get the right relationship we need to balance the table so that we place the Planck mass on one half of the table while the Planck length is placed on the other half. We also notice that there is symmetry about the vertical axis (shown in green) that divides the table into two halves:



Thus, according to the above scale table we write the following relationship

$$R_S M_P = S M L_P \tag{2}$$

As always we have introduced the scale factor S on the second side of the equation to complete the relationship.

Now we rewrite equation (2) in the form of the scale principle to get

$$\frac{R_S}{M} = S \frac{L_P}{M_P} \tag{3}$$

Comparing equation (3) with relationship (1) we find that equation (3) has the following form

$$\frac{Q_1}{Q_2} = S \frac{Q_3}{Q_4} \tag{4}$$

Where

$n = m = 1$
$Q_1 = R_S$
$Q_2 = M$
$Q_3 = L_P$
$Q_4 = M_P$
$S =$ to be determined by a separate analysis.

The Planck length and the Planck mass are defined respectively by

$$L_P = \sqrt{\frac{hG}{2\pi c^3}} \tag{5}$$

$$M_P = \sqrt{\frac{hc}{2\pi G}} \tag{6}$$

Substituting L_P and M_P in equation (3) with equations (5) and (6) yields

$$\frac{R_S}{M} = S \sqrt{\frac{hG}{2\pi c^3} \frac{2\pi G}{hc}} = S \sqrt{\frac{G^2}{c^4}} = S \frac{G}{c^2} \tag{7}$$

Solving for R_S

$$R_S = S\frac{GM}{c^2} \tag{8}$$

Thus we have proved that the formula for the Schwarzschild radius of a black hole is a special case of the scale principle. Based on Karl Schwarzschild solution [2] to the Einstein's general relativity's field equations we know that the scale factor is 2, thus we can write

$$R_S = \frac{2GM}{c^2} \tag{9}$$

We have to keep in mind that the purpose of this paper is not to find the scale factor for equation (8) but to prove that the Schwarzschild radius obeys the scale law.

3. Conclusions

Taking into account that the scale law describes several known laws of physics as I have shown both on previous papers [1, 3] and on this paper, we can consider that the scale law is a more general law than the specific laws it describes. Therefore the scale principle is a *law model* or *Meta law* nature applies to a wide range of phenomena.

REFERENCES

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[3] R. A. Frino, The Special Theory of Relativity and the Scale Principle, viXra: 1406.0144, (2014).