Proof of Beal's Conjecture

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Abstract : Using visualization of the pattern by providing examples and an elementary proof, we are able to prove and show that A,B and C will always have a common prime factor.

1 Introduction

We sometimes feel hesitant to reach out to mathematicians and understand their thoughts, if you are non-mathematician. Likewise it is somewhat difficult to teach and share what mathematicians know to their counterpart. If I may say, I am a little bit of both, meaning I do have difficulties understanding many aspects but I have the passion and I can understand math somehow. But I also have the difficulty sharing my thoughts and ideas regarding math to others. But with patience, hard work and determination, one may fully understand the concept. And then we all can share the beauty of Mathematics. Hoping this discovery in solving the Beal's Conjecture would encourage us more to be mathematically inclined.

2 Definitions

First, let's define the problem. ***The Beal Conjecture*** states that

Let A, B, C, x, y, and z be positive integers with x, y, z > 2. If $A^x + B^y = C^z$, then A, B, and C have a common factor. [1]

In number theory, the fundamental theorem of arithmetic, also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of prime numbers, and that, although the order of the primes in the second case is arbitrary, the primes themselves are not. [2]

3 Statement of Main Results

By studying the results, it made me believe that because of the unique-prime-factorization concept, we can't factor out C^z into $A^x + B^y$ with exponents greater than 2 and not have a prime factor. While doing this research and analysis, trying to factor out C^z is the key to the solution. Having those primes separate without $A, B\ and\ C$ having a common prime factor, seemed it is just impossible to do so. But we were able to determine that there is a certain pattern and that tells us $A, B\ and\ C$ will always have a common prime factor.

4 Indication of Methodology

Using elementary method of substitution and a direct approach, we can show the connection between the pattern and the equation. The key approach in solving the problem is the decomposition or factoring out the primes in C^z. Visualizing some examples and getting the relationship between the proof and the example arrives to the conclusion that there is indeed a pattern. So if I may say, the proof itself is the algorithm in factoring C^z thereby producing $A^x + B^y$.

5 Details of Proof

First, let's visualize and introduce the pattern or equation that seems to hold true and let's provide some examples. Assume that there is *p* (which is to denote a prime number or distinct set of primes), *u* and *v* are positive integers such as $0 < u < v$

$$A^x + B^y = C^z$$

$$p^n(u) + p^n(v-u) = p^n(u + (v-u))$$

where

$A^x = p^n(u)$
$B^y = p^n(v-u)$
$C^z = p^n(u + (v-u))$

Let's check the pattern with few examples involving different primes as our basic cases.

Example 1

$2^3 + 2^3 = 2^4$
Factoring.
$2^3 + 2^3 = 2^3(2)$
Pattern matched.
$$2^3(1) + 2^3(2-1) = 2^3(1 + (2-1))$$
To simplify.
$2^3(1) + 2^3(1) = 2^3(1 + (2-1))$
$2^3 + 2^3 = 2^3(1+1)$
$2^3 + 2^3 = 2^3(2)$
$2^3 + 2^3 = 2^4$
$2^4 = 2^4$

Example 2

$3^3 + 6^3 = 3^5$
Factoring.
$3^3 + 3^3(2^3) = 3^3(3^2)$
Pattern matched.
$$3^3(1) + 3^3(9-1) = 3^3(1 + (9-1)$$
To simplify.
$3^3(1) + 3^3(9-1) = 3^3(1 + (9-1)$
$3^3(1) + 3^3(8) = 3^3(1+8)$
$3^3 + 3^3(2^3) = 3^3(9)$
$3^3 + 6^3 = 3^3(3^2)$
$3^3 + 6^3 = 3^5$
$3^5 = 3^5$

Example 3

$7^3 + 7^4 = 14^3$

Factoring.

$7^3(1) + 7^3(7) = 7^3(2^3)$

$7^3(1) + 7^3(8-1) = 7^3(8)$

Pattern matched.

$$7^3(1) + 7^3(8-1) = 7^3(1 + (8-1)$$

To simplify.

$7^3 + 7^3(8-1) = 7^3(1 + 7)$

$7^3 + 7^3(7) = 7^3(8)$

$7^3 + 7^4 = 7^3(2^3)$

$7^3 + 7^4 = 14^3$

$14^3 = 14^3$

Let's now recall the identity, state the conjecture and its proof.

$$A^x + B^y = C^z$$

Conjecture. Let A, B, C, x, y, and z be positive integers with x, y, z > 2 such that $A^x + B^y = C^z$ have a common prime factor.

Proof.

Let's recall the identity

$$A^x + B^y = C^z$$

1. Let's assume that x, y, z > 2. We can split C^z into

   $C^z = C^{z-2}\ (C^2\ )$

2. Furthermore, without loss of generality, to factor C^2 further into

   $C^2 = 1 + (C^2 - 1)$

3. Replacing C^2 back into the equation

   $C^z = C^{z-2}\ (1 + (C^2 - 1)\ )$

4. Using substitution to assume the following values for $A^x, B^y and\ C^z$. Let

   $A^x = C^{z-2}(1)$

   $B^y = C^{z-2}(C^2 - 1)$

   $C^z = C^{z-2}(1 + (C^2 - 1))$

5. By substitution, we have

   $C^{z-2}(1) + C^{z-2}(C^2 - 1)\ = C^{z-2}(1 + (C^2 - 1))$

6. To simplify

   $C^{z-2} + C^z - C^{z-2}\ = C^{z-2}(1 + C^2 - 1)$

7. C^{z-2} cancels out on the left hand side. 1 cancels out on the right hand side. To simplify further the right hand side

$$C^z = C^{z-2}\ (\ C^2\ )$$

8. Now it shows that the equality holds.

$$C^z = C^z$$

9. Finally, we arrive to our conclusion that there will always be a common factor which from the given statement is C^{z-2}.

Q.E.D.

We have visualized both the pattern and the proof. And to further establish the connection for both.

From our proof

$$A^x = C^{z-2}(1)$$
$$B^y = C^{z-2}(C^2 - 1)$$
$$C^z = C^{z-2}(1 + (C^2 - 1))$$

From our visualization

$$A^x = p^n(u)$$
$$B^y = p^n(v - u)$$
$$C^z = p^n(u + (v - u))$$

By substitution, we are able to arrive at the pattern we have generalized earlier using examples.

$$p^n = C^{z-2}$$
$$v = C^2$$
$$u = 1$$

6 Concluding Remarks

We have clearly shown using elementary proof that A, B and C will always have a common factor. Since A, B and C are positive integers, and have a common prime factor, as stated previously; therefore we can conclude that the Beal's Conjecture is true.

References :

[1] R. Daniel Muldin, ***A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem***, Notices of the AMS Volume 44 Number 11, 1436.

[2] http://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic