Theory of colorless and electrically neutral quarks and colored and electrically charged gauge bosons

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We propose a model of interactions in which two states of a quark, a colored and electrically charged state and a colorless and electrically neutral state, can transform into each other through the emission or absorption of a colored and electrically charged gauge boson. A novel feature of the model is that the colorless and electrically neutral quarks carry away the missing energy in decay processes as do neutrinos.

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I. INTRODUCTION

At the present stage of particle physics, there is an asymmetry between quarks and leptons: For every charged lepton, there is a corresponding type of neutral lepton, neutrino, which participates neither in strong interaction nor in electromagnetic interaction, whereas, for every charged quark, there is little known about a corresponding type of neutral quark, which participates neither in strong interaction nor in electromagnetic interaction.

The question can be raised as to whether such neutral quarks, which participate neither in strong interaction nor in electromagnetic interaction, exist. If they really exist, they should have neither color charge nor electric charge, and could carry away 'the missing energy' in decay processes as do neutrinos. One can build up a theory of such quarks, leaving the experimental verification of their existence blank.

In the present paper, this is the case, we postulate the existence of such quarks, i.e., the colorless and electrically neutral quarks, and build up a theory of the quarks. For the description of interaction between quarks, we also introduce another postulate that the transition of a quark from colored and electrically charged state to colorless and electrically neutral state or vice versa is accompanied by the emission or absorption of a colored and electrically charged gauge boson, just as the transition of a lepton from elctrically charged state to electrically neutral state or vice versa is accompanied by the emission or absorption of an electrically charged gauge boson.

In section II, we consider the transition of a quark from a colored and electrically charged state to a colorless and electrically neutral state with the emission of a colored and electrically charged gauge boson, and its reverse transition with the absorption of the gauge boson. In section III, we discuss the properties of the colorless and electrically neutral quarks. In section IV, we construct a model of interaction Lagrangian for the colorless and electrically neutral quark and the colored and electrically charged gauge boson fields. In section V, the possibility is explored of making an estimation of the mass of one of the colored and electrically charged gauge bosons from the measured $K^+ \to \pi^+ +$ 'missing energy' branching ratio.

We shall hereafter denote by $p_c^{c'}$ the state of a particle p, of which the color charge and electric charge are c and $c'e$ respectively, and denote by C_0 colorless. For any color c, the relation $c+\bar{c} = C_0$, $\bar{c}$ being the anti-c, holds .

II. QUARKS, AND COLORED AND ELECTRICALLY CHARGED GAUGE BOSONS

Let us consider a transition of a quark from a colored and electrically charged state q_{c1}^{e1} to another state q_{c2}^{e2} with the emission of a colored and electrically charged gauge boson $b_{c_b}^{e_b}$, $q_{c1}^{e1} \to q_{c2}^{e2} + b_{c_b}^{e_b}$, and its reverse transition with the absorption of the gauge boson, $q_{c2}^{e2} + b_{c_b}^{e_b} \to q_{c1}^{e1}$:

$$q_{c1}^{e1} \rightleftarrows q_{c2}^{e2} + b_{c_b}^{e_b}, \tag{1}$$

where

$$c_1 \neq C_0, \quad e_1 \neq 0, \quad (c_1 = r, g, b, \quad e_1 = -\frac{1}{3}, \quad +\frac{2}{3}), \tag{2}$$

$$c_b \neq C_0, \quad e_b \neq 0. \tag{3}$$

We shall now determine the color and electric charges of the gauge boson $b_{c_b}^{e_b}$, and those of the quark q_{c2}^{e2}. The transitions (1) must satisfy the law of conservation of color and electric charges:

$$c_1 = c_2 + c_b, \quad e_1 = e_2 + e_b, \tag{4}$$

which gives

$$c_b = c_1 + \bar{c}_2, \quad e_b = e_1 - e_2. \tag{5}$$

Substituting for c_b and e_b in (1) their values, we have

$$q_{c1}^{e1} \rightleftarrows q_{c2}^{e2} + b_{c1+\bar{c}_2}^{e1-e2}. \tag{6}$$

Among the various cases, we shall be concerned only with the case: The transition of a quark from a colored and electrically charged state q_{c1}^{e1} to a colorless and electrically neutral state $q_{C_0}^{0}$ with the emission of a colored and electrically charged gauge boson b_{c1}^{e1}, and its reverse transition with the absorption of the gauge boson:

$$q_{c1}^{e1} \rightleftarrows q_{C_0}^{0} + b_{c1}^{e1}, \tag{7}$$

which means that the colored and electrically charged quark q_{c1}^{e1} and the colorless and electrically neutral quark $q_{C_0}^{0}$ transform into each other through the emission or absorption of the colored and electrically charged gauge boson b_{c1}^{e1}. The extension of the transition (7) to the cases of involving anti-particles may be made as follows:

$$q_{\bar{c}1}^{\bar{e}1} \rightleftarrows q_{C_0}^{\bar{0}} + b_{\bar{c}1}^{\bar{e}1}, \quad q_{c1}^{e1} + b_{\bar{c}1}^{\bar{e}1} \rightleftarrows q_{C_0}^{0}, \quad b_{c1}^{e1} \rightleftarrows q_{c1}^{e1} + q_{C_0}^{\bar{0}}, \quad q_{\bar{c}1}^{\bar{e}1} + b_{c1}^{e1} \rightleftarrows q_{C_0}^{\bar{0}}, \tag{8}$$

etc., where $\bar{A}$ denotes the anti-particle of A.

It should be noted that the colored and electrically charged gauge boson $b^{e_1}_{c_1}$ in (7) has the same color and electric charges as the colored and electrically charged quark $q^{e_1}_{c_1}$, i.e., $b^{e_b}_{c_b} = b^{e_1}_{c_1}$, it thus has the color charges r, g, b and the electric charges $-\frac{1}{3}e$, $+\frac{2}{3}e$.

The transitions (7) can be rewritten in the form

$$q_i^{Q/e} \rightleftarrows \kappa_q + W_i^{Q/e}, \tag{9}$$

where we have put $\kappa_q = q^0_{C_0}$, $W = b$, $i = c_1$, $Q/e = e_1$. The (9) takes the form when $q = u$, c or t, $(Q/e = +\frac{2}{3})$,

$$q_i^{+\frac{2}{3}} \rightleftarrows \kappa_q + W_i^{+\frac{2}{3}}, \tag{10}$$

and when $q = d$, s or b, $(Q/e = -\frac{1}{3})$,

$$q_i^{-\frac{1}{3}} \rightleftarrows \kappa_q + W_i^{-\frac{1}{3}}. \tag{11}$$

III. COLORLESS AND ELECTRICALLY NEUTRAL QUARKS

We may see from (10) and (11) that there can be six colorless and electrically neutral quarks, which we shall call simply '*neutral quarks*' or '*neutrino-like quarks*':

$$\kappa_q \ (q = u, c, t, d, s, b): \ \kappa_u, \ \kappa_c, \ \kappa_t, \ \kappa_d, \ \kappa_s, \ \kappa_b, \tag{12}$$

that is, there can be six pairs of (q, κ_q): (u, κ_u), (c, κ_c), (t, κ_t), (d, κ_d), (s, κ_s), (b, κ_b).

Neutral quarks are colorless and electrically neutral quarks, whereas neutrinos are colorless and electrically neutral leptons. Since neutral quarks have neither color charge nor electric charge, they participate neither in strong interactions nor in electromagnetic interactions. This means that neutral quarks can carry away the 'missing energy' as do neutrinos.

Let us consider two kinds of transitions

$$q_i^{Q/e} \rightleftarrows \kappa_q + W_i^{Q/e}, \tag{13}$$

$$l^- \rightleftarrows \nu_l + W^-, \tag{14}$$

where $Q/e = +\frac{2}{3}$ $(q = u, c, t)$, $Q/e = -\frac{1}{3}$ $(q = d, s, b)$, $l = e, \mu, \tau$. We may see that the transitions (13) take the same form as the transitions (14): A colored and electrically charged quark loses its color and electric charges completely through the emission of a colored and electrically charged gauge boson, just as an electrically charged lepton loses its electric charge completely through the emission of an electrically charged gauge boson. This suggests that the description of the transitions of a quark (13) can be made in the same way as the transitions of a lepton (14), that is, in the form of electroweak theory of leptons.

The neutral quarks must have spin $\frac{1}{2}$ and baryon number $\frac{1}{3}$. To describe within the framework of $SU(2)\times U(1)$ model, the left-handed neutral quarks have isospin $\frac{1}{2}$: Each left-handed colored and electrically charged quark and its left-handed neutral quark have the same magnitude of isospin charge but opposite in sign. Thus from the isospin charge T_3

$$T_3|q,L\rangle = \begin{cases} +\frac{1}{2}|q,L\rangle \;\text{ for }\; q=u,c,t, \\ -\frac{1}{2}|q,L\rangle \;\text{ for }\; q=d,s,b, \end{cases} \tag{15}$$

and $\frac{Q}{e} = T_3 + \frac{1}{2}Y$, the isospin charge T_3 and hypercharge Y of each left-handed neutral quark are

$$T_3|\kappa_q,L\rangle = \begin{cases} -\frac{1}{2}|\kappa_q,L\rangle \;\text{ for }\; q=u,c,t, \\ +\frac{1}{2}|\kappa_q,L\rangle \;\text{ for }\; q=d,s,b, \end{cases} \tag{16}$$

and

$$Y|\kappa_q,L\rangle = \begin{cases} +1|\kappa_q,L\rangle \;\text{ for }\; q=u,c,t, \\ -1|\kappa_q,L\rangle \;\text{ for }\; q=d,s,b. \end{cases} \tag{17}$$

Each right-handed neutral quark has isospin charge zero and hypercharge zero.

Accordingly, quarks can be classified into four types:

$$\begin{array}{lcccc} u-\text{type} & : & u, & c, & t \\ \kappa_d-\text{type} & : & \kappa_d, & \kappa_s, & \kappa_b \\ \kappa_u-\text{type} & : & \kappa_u, & \kappa_c, & \kappa_t \\ d-\text{type} & : & d, & s, & b \end{array} \tag{18}$$

and

$$G_{\text{I}} = \begin{pmatrix} u \\ \kappa_d \\ \kappa_u \\ d \end{pmatrix}, G_{\text{II}} = \begin{pmatrix} c \\ \kappa_s \\ \kappa_c \\ s \end{pmatrix}, G_{\text{III}} = \begin{pmatrix} t \\ \kappa_b \\ \kappa_t \\ b \end{pmatrix}. \tag{19}$$

We may see that neutral quarks resemble neutrinos in many respects. We utilize the resemblance between them by determining the masses of neutral quarks from the mass conditions of neutrinos: We assume that the mass of each neutral quark is either zero or very small in comparison to the mass of the corresponding colored and electrically charged quark, i.e., $m_{\kappa_q}=0$ or $m_{\kappa_q} \ll m_q$.

IV. MODEL

Let us consider ten column vectors

$$\Psi_{1i}^L = \begin{pmatrix} u_i^L \\ c_i^L \\ t_i^L \\ d_i^L \\ s_i^L \\ b_i^L \end{pmatrix}, \Psi_{2i}^L = \begin{pmatrix} u_i^L \\ c_i^L \\ t_i^L \\ \kappa_u^L \\ \kappa_c^L \\ \kappa_t^L \end{pmatrix}, \Psi_{3i}^L = \begin{pmatrix} \kappa_d^L \\ \kappa_s^L \\ \kappa_b^L \\ d_i^L \\ s_i^L \\ b_i^L \end{pmatrix}, \Psi_4^L = \begin{pmatrix} \kappa_d^L \\ \kappa_s^L \\ \kappa_b^L \\ \kappa_u^L \\ \kappa_c^L \\ \kappa_t^L \end{pmatrix}, \tag{20}$$

where $i = r, g, b$, $q_i^L = P_L q_i$, $\kappa_q^L = P_L \kappa_q$, $(q = u, c, t, d, s, b)$, $P_L = \frac{1-\gamma^5}{2}$.

By introducing the 6×6 isospin matrices $T_{j\alpha}$ $(j = 1, 2, 3,\ \alpha = 1, 2, 3, 4)$,

$$T_{1\alpha} = \frac{1}{2}\begin{pmatrix} 0 & U_\alpha \\ U_\alpha^\dagger & 0 \end{pmatrix}, T_{2\alpha} = \frac{1}{2}\begin{pmatrix} 0 & -iU_\alpha \\ iU_\alpha^\dagger & 0 \end{pmatrix}, T_{3\alpha} = \frac{1}{2}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{21}$$

the I being the 3×3 unit matrix, the U_α's 3×3 unitary matrices, which satisfy the commutation relations

$$[T_{i\alpha}, T_{j\alpha}] = i\epsilon_{ijk} T_{k\alpha},\ (\alpha : \text{unsummed}), \tag{22}$$

and the 6×6 diagonal hypercharge matrices Y_α

$$Y_1 = \begin{pmatrix} \frac{1}{3}I & 0 \\ 0 & \frac{1}{3}I \end{pmatrix}, Y_2 = \begin{pmatrix} \frac{1}{3}I & 0 \\ 0 & +I \end{pmatrix}, Y_3 = \begin{pmatrix} -I & 0 \\ 0 & \frac{1}{3}I \end{pmatrix}, Y_4 = \begin{pmatrix} -I & 0 \\ 0 & +I \end{pmatrix}, \tag{23}$$

we may construct the Lagrangian, which is invariant under $T_{j\alpha}$ and Y_α gauge transformations, being of the form

$$\mathscr{L} = \sum_{j=1}^{3} \sum_{i=r,g,b} i\bar{\Psi}_{ji}^L \gamma_\mu D^\mu \Psi_{ji}^L + i\bar{\Psi}_4^L \gamma_\mu D^\mu \Psi_4^L + \mathscr{L}', \tag{24}$$

where the $\mathscr{L}'$ involves the terms of right handed quark fields, free gauge fields, Higgs fields, etc. The covariant derivatives in (24) are defined as

$$D^\mu \Psi_{1i}^L = (\partial^\mu + \sum_{j=1}^{3} igT_{j1}W_{j1}^\mu + ig'\frac{1}{2}Y_1 B^\mu)\Psi_{1i}^L, \tag{25}$$

$$D^\mu \Psi_{2i}^L = (\partial^\mu + \sum_{j=1}^{2} igT_{j2}W_{j2i}^\mu + igT_{32}W_{32}^\mu + ig'\frac{1}{2}Y_2 B^\mu)\Psi_{2i}^L, \tag{26}$$

$$D^\mu \Psi_{3i}^L = (\partial^\mu + \sum_{j=1}^{2} igT_{j3}W_{j3i}^\mu + igT_{33}W_{33}^\mu + ig'\frac{1}{2}Y_3 B^\mu)\Psi_{3i}^L, \tag{27}$$

$$D^\mu \Psi_4^L = (\partial^\mu + \sum_{j=1}^{3} igT_{j4}W_{j4}^\mu + ig'\frac{1}{2}Y_4 B^\mu)\Psi_4^L. \tag{28}$$

The Lagrangian $\mathscr{L}$ defined in (24) involves not only the well-known term

$$\sum_{i=r,g,b} i\bar{\Psi}^L_{1i}\gamma_\mu D^\mu \Psi^L_{1i} \tag{29}$$

but also the newly introduced terms

$$\sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu D^\mu \Psi^L_{ji} + i\bar{\Psi}^L_{4}\gamma_\mu D^\mu \Psi^L_{4} + \mathscr{L}''. \tag{30}$$

Since the charged currents constructed out of Ψ^L_{1i} do not carry color charges, they are coupled not to colored and electrically charged gauge bosons, but to colorless and electrically charged gauge bosons like $W^\pm$. Whereas, the charged currents constructed out of Ψ^L_{2i} or Ψ^L_{3i} carry color and electric charges, and are coupled to colored and electrically charged gauge bosons.

The first term of (30) can be written in the form

$$\sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu D^\mu \Psi^L_{ji} = \sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu \partial^\mu \Psi^L_{ji} + \mathscr{L}_{\mathrm{IC}} + \mathscr{L}_{\mathrm{IN}}, \tag{31}$$

where

$$\mathscr{L}_{\mathrm{IC}} = -g\sum_{j=2}^{3}\sum_{k=1}^{2}\sum_{i=r,g,b} \bar{\Psi}^L_{ji}\gamma_\mu T_{kj} W^\mu_{kji}\Psi^L_{ji}, \tag{32}$$

$$\mathscr{L}_{\mathrm{IN}} = -\sum_{j=2}^{3}\sum_{i=r,g,b} \bar{\Psi}^L_{ji}\gamma_\mu (gT_{3j}W^\mu_{3j} + g'\frac{1}{2}Y_j B^\mu)\Psi^L_{ji}. \tag{33}$$

The $\mathscr{L}_{\mathrm{IC}}$ describes the interactons in which each current involving a neutral quark and a colored and electrically charged quark is coupled to a colored and electrically charged gauge boson:

$$\mathscr{L}_{\mathrm{IC}} = -\frac{g}{2\sqrt{2}}\sum_{i=r,g,b}(W^\mu_{2i}J_{2i\mu} + W^{\mu\dagger}_{2i}J^\dagger_{2i\mu} + W^{\mu\dagger}_{3i}J_{3i\mu} + W^\mu_{3i}J^\dagger_{3i\mu}), \tag{34}$$

where $W^\mu_{2i} = \frac{1}{\sqrt{2}}(W^\mu_{12i} - iW^\mu_{22i})$, $W^\mu_{3i} = \frac{1}{\sqrt{2}}(W^\mu_{13i} + iW^\mu_{23i})$,

$$J_{ji\mu} = 2\bar{\Psi}^L_{ji}\gamma_\mu H_j \Psi^L_{ji},\ H_j = \begin{pmatrix} 0 & U_j \\ 0 & 0 \end{pmatrix}. \tag{35}$$

We shall denote the quanta of the field W^μ_{2i} and those of the field W^μ_{3i} by $W_i^{+\frac{2}{3}}$ (or $W^{+\frac{2}{3}}$) and $W_i^{-\frac{1}{3}}$ (or $W^{-\frac{1}{3}}$) respectively. The $\mathscr{L}_{\mathrm{IC}}$ describes the processes such as

$$u_i \rightleftarrows \kappa_u + W_i^{+\frac{2}{3}},\ d_i \rightleftarrows \kappa_d + W_i^{-\frac{1}{3}}, \tag{36}$$

where $i = r, g, b$.

The $\mathscr{L}_{\mathrm{I_N}}$ describes the interactions in which neutral currents of quarks are coupled to colorless and electrically neutral gauge bosons:

$$\mathscr{L}_{\mathrm{I_N}} = -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu(gT_3W_{3j}^\mu + g'\frac{1}{2}Y_jB^\mu)\Psi_{ji}^L, \tag{37}$$

where $T_3 \equiv T_{3j}$. By introducing Hermitian fields Z_j^μ and A^μ

$$W_{3j}^\mu = \cos\theta_j Z_j^\mu + \sin\theta_j A^\mu, \tag{38}$$

$$B^\mu = -\sin\theta_j Z_j^\mu + \cos\theta_j A^\mu, \tag{39}$$

substituting for W_{3j}^μ and B^μ in (37) their values, we obtain

$$\begin{aligned}\mathscr{L}_{\mathrm{I_N}} &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu[gT_3(\cos\theta_j Z_j^\mu + \sin\theta_j A^\mu)\\ &\quad +g'\frac{1}{2}Y_j(-\sin\theta_j Z_j^\mu + \cos\theta_j A^\mu)]\Psi_{ji}^L\\ &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu[(g\cos\theta_j T_3 - g'\sin\theta_j\frac{1}{2}Y_j)Z_j^\mu\\ &\quad +(g\sin\theta_j T_3 + g'\cos\theta_j\frac{1}{2}Y_j)A^\mu]\Psi_{ji}^L.\end{aligned} \tag{40}$$

Since $\frac{Q_j}{e} = T_3 + \frac{1}{2}Y_j$ and $g\sin\theta_j T_3 + g'\cos\theta_j\frac{1}{2}Y_j = Q_j$, they agree if we take $g\sin\theta_j = g'\cos\theta_j = e$. It thus becomes

$$\begin{aligned}\mathscr{L}_{\mathrm{I_N}} &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu[\frac{g}{\cos\theta_j}(T_3 - \sin^2\theta_j\frac{Q_j}{e})Z_j^\mu + Q_jA^\mu]\Psi_{ji}^L\\ &= -\sum_{j=2}^{3}[\frac{g}{\cos\theta_j}(J_{\mu j}^{(T_3)} - \sin^2\theta_j\frac{J_{\mu j}^{(Q_j)}}{e})Z_j^\mu + J_{\mu j}^{(Q_j)}A^\mu],\end{aligned} \tag{41}$$

where $J_{\mu j}^{(T_3)} = \sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu T_3\Psi_{ji}^L$ and $J_{\mu j}^{(Q_j)} = \sum_{i=r,g,b}\bar{\Psi}_{ji}^{L}\gamma_\mu Q_j\Psi_{ji}^L$.

From (34) and (41), we have

$$\begin{aligned}\mathscr{L}_{\mathrm{I_C}} + \mathscr{L}_{\mathrm{I_N}} &= -\frac{g}{2\sqrt{2}}\sum_{i=r,g,b}(W_{2i}^\mu J_{2i\mu} + W_{2i}^{\mu\dagger}J_{2i\mu}^\dagger + W_{3i}^{\mu\dagger}J_{3i\mu} + W_{3i}^\mu J_{3i\mu}^\dagger)\\ &\quad -\sum_{j=2}^{3}[\frac{g}{\cos\theta_j}(J_{\mu j}^{(T_3)} - \sin^2\theta_j\frac{J_{\mu j}^{(Q_j)}}{e})Z_j^\mu + J_{\mu j}^{(Q_j)}A^\mu].\end{aligned} \tag{42}$$

A. Weak currents involving κ_u, κ_c, κ_t

From the three column vectors

$$\Psi_{2i}^L = \begin{pmatrix} u_i^L \\ c_i^L \\ t_i^L \\ \kappa_u^L \\ \kappa_c^L \\ \kappa_t^L \end{pmatrix}, \tag{43}$$

where $i = r, g, b$, $q_i^L = P_L q_i$, $\kappa_q^L = P_L \kappa_q$, $(q = u, c, t)$, $P_L = \frac{1-\gamma^5}{2}$, we obtain the interaction term from the gauge invariant Lagrangian

$$\begin{aligned}\mathscr{L}_{\mathrm{I}_2} = &-\frac{g}{2\sqrt{2}} \sum_{i=r,g,b} (W_{2i}^\mu J_{2i\mu} + W_{2i}^{\mu\dagger} J_{2i\mu}^\dagger) \\ &-[\frac{g}{\cos\theta_2}(J_{\mu 2}^{(T_3)} - \sin^2\theta_2 \frac{J_{\mu 2}^{(Q_2)}}{e}) Z_2^\mu + J_{\mu 2}^{(Q_2)} A^\mu],\end{aligned} \tag{44}$$

where $W_{2i}^\mu = \frac{1}{\sqrt{2}}(W_{12i}^\mu - i W_{22i}^\mu)$, of which the quanta correspond to $W_i^{+\frac{2}{3}}$ (or $W^{+\frac{2}{3}}$), and

$$J_{2i\mu} = 2\bar{\Psi}_{2i}^L \gamma_\mu H_2 \Psi_{2i}^L,\ H_2 = \begin{pmatrix} 0 & U_2 \\ 0 & 0 \end{pmatrix}, \tag{45}$$

where U_2 is a 3×3 unitary matrix, 0 the 3×3 null matrix.

The $\mathscr{L}_{\mathrm{I}_2}$ describes the interactions in which a charged current involving a neutral quark (κ_u, κ_c, κ_t) and a colored and electrically charged quark (u, c, t) is coupled to a colored and electrically charged gauge boson $W^{+\frac{2}{3}}$ or $\bar{W}^{+\frac{2}{3}}$, e.g.,

$$u_i \rightleftarrows \kappa_u + W_i^{+\frac{2}{3}}, \tag{46}$$

where $i = r, g, b$, and the interactions in which a neutral current is coupled to a colorless and electrically neutral gauge boson Z_2^0 corresponding to the quantum of the field Z_2^μ.

B. Weak currents involving κ_d, κ_s, κ_b

From the three column vectors

$$\Psi_{3i}^{L} = \begin{pmatrix} \kappa_d^L \\ \kappa_s^L \\ \kappa_b^L \\ d_i^L \\ s_i^L \\ b_i^L \end{pmatrix}, \tag{47}$$

where $i = r, g, b$, $q_i^L = P_L q_i$, $\kappa_q^L = P_L \kappa_q$, $(q = d, s, b)$, we obtain

$$\begin{aligned}\mathscr{L}_{\mathrm{I}_3} = & -\frac{g}{2\sqrt{2}} \sum_{i=r,g,b} (W_{3i}^{\mu\dagger} J_{3i\mu} + W_{3i}^{\mu} J_{3i\mu}^{\dagger}) \\ & -[\frac{g}{\cos\theta_3}(J_{\mu 3}^{(T_3)} - \sin^2\theta_3 \frac{J_{\mu 3}^{(Q_3)}}{e}) Z_3^{\mu} + J_{\mu 3}^{(Q_3)} A^{\mu}],\end{aligned} \tag{48}$$

where $W_{3i}^{\mu} = \frac{1}{\sqrt{2}}(W_{13i}^{\mu} + i W_{23i}^{\mu})$, of which the quanta correspond to $W_i^{-\frac{1}{3}}$ (or $W^{-\frac{1}{3}}$), and

$$J_{3i\mu} = 2\bar{\Psi}_{3i}^{L} \gamma_\mu H_3 \Psi_{3i}^{L}, \; H_3 = \begin{pmatrix} 0 & U_3 \\ 0 & 0 \end{pmatrix}, \tag{49}$$

where U_3 is a 3×3 unitary matrix.

The $\mathscr{L}_{\mathrm{I}_3}$ describes the interactions in which a charged current involving a neutral quark (κ_d, κ_s, κ_b) and a colored and electrically charged quark (d, s, b) is coupled to a colored and electrically charged gauge boson $W^{-\frac{1}{3}}$ or $\overline{W}^{-\frac{1}{3}}$, e.g.,

$$d_i \rightleftarrows \kappa_d + W_i^{-\frac{1}{3}}, \tag{50}$$

where $i = r, g, b$, and the interactions in which a neutral current is coupled to a colorless and electrically neutral gauge boson Z_3^0 corresponding to the quantum of the field Z_3^{μ}.

V. $K^+ \to \pi^+ +$ 'MISSING ENERGY'

For the description of the rare kaon decay mode $K^+ \to \pi^+ +$ 'missing energy' [1–18], beside the well-known process

$$\bar{s}_i \to \bar{d}_i \nu_l \bar{\nu}_l \; (l = e, \mu, \tau), \tag{51}$$

we may consider the quark level process as follows

$$\bar{s}_i \to \bar{d}_i \kappa'_d \bar{\kappa'_s}, \tag{52}$$

where $i = r, g, b$. The κ'_q $(q = d, s)$ is mixed states of κ_d, κ_s and κ_b. We may see that this process is very much similar to the muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ in many respects.

The interactions of the well-known process (51) are mediated by the gauge bosons $W^\pm$ and Z^0, whereas those of (52) are mediated by the gauge bosons $W^{-\frac{1}{3}}$, $\bar{W}^{-\frac{1}{3}}$ and Z_3^0. We may infer from extremely short range of the interactions of (52) that the mass of the gauge bosons $W^{-\frac{1}{3}}$ and Z_3^0 must be very massive.

If we now assume that the missing energy in the process $K^+ \to \pi^+ +$ 'missing energy' is mainly due to (51) and (52), the decay rate of the process can be written as

$$\Gamma(K^+ \to \pi^+ + \text{Nothing}) \approx \Gamma_{\text{(i)}} + \Gamma_{\text{(ii)}}, \tag{53}$$

where

$$\Gamma_{\text{(i)}} = \Gamma(\bar{s}_i \to \bar{d}_i \nu_l \bar{\nu}_l)_{l=e,\mu,\tau} \approx \Gamma(K^+ \to \pi^+ \nu\bar{\nu})_{\nu=\nu_e,\nu_\mu,\nu_\tau}, \tag{54}$$

$$\Gamma_{\text{(ii)}} = \Gamma(\bar{s}_i \to \bar{d}_i \kappa'_d \bar{\kappa'_s}) \approx \Gamma(K^+ \to \pi^+ \kappa'_d \bar{\kappa'_s}). \tag{55}$$

The interaction Lagrangian responsible for the process (52) is $\mathscr{L}_{\text{I}_3}$ in (48),

$$\mathscr{L}_{\text{I}_3} = -g_W \sum_{i=r,g,b} (W_{3i}^{\mu\dagger} J_{3i\mu} + W_{3i}^{\mu} J_{3i\mu}^\dagger) - [\frac{2\sqrt{2} g_W}{\cos\theta_3} (J_{\mu 3}^{(T_3)} - \sin^2\theta_3 \frac{J_{\mu 3}^{(Q_3)}}{e}) Z_3^\mu + J_{\mu 3}^{(Q_3)} A^\mu],$$

where $g_W = \frac{g}{2\sqrt{2}}$ and

$$J_{3i\mu} = 2\bar{\Psi}_\kappa^L \gamma_\mu U_3 \Psi_{q_i}^L = 2\bar{\Psi}_\kappa^{L\prime} \gamma_\mu \Psi_{q_i}^L, \tag{56}$$

$\Psi_\kappa^{L\prime} = U_3^\dagger \Psi_\kappa^L$, and

$$\Psi_\kappa^L = \begin{pmatrix} \kappa_d^L \\ \kappa_s^L \\ \kappa_b^L \end{pmatrix}, \Psi_{q_i}^L = \begin{pmatrix} d_i^L \\ s_i^L \\ b_i^L \end{pmatrix}. \tag{57}$$

In the limit $m_{W^{\pm\frac{1}{3}}} \to \infty$ and $m_{Z_3^0} \to \infty$, the $W^{\pm\frac{1}{3}}$ propagator and Z_3^0 propagator reduce to

$$i g^{\mu\nu} m_{W^{\pm\frac{1}{3}}}^{-2} \quad \text{and} \quad i g^{\mu\nu} m_{Z_3^0}^{-2} \tag{58}$$

respectively.

From (48), (56) and (57), we may construct the invariant amplitude of the lowest order for the process (52) which takes in the form

$$\mathscr{M} \approx -i\frac{4g_W^2}{m^2_{W^{\pm\frac{1}{3}}}}\bar{s}_i^L\gamma^\mu\kappa_s^{L\prime}\bar{\kappa}_d^{L\prime}\gamma_\mu d_i^L. \tag{59}$$

Assuming that $U_3 \approx I$, $m_{\kappa_d} \approx 0$, $m_{\kappa_s} \approx 0$, $m_d \ll m_s$, and that the process (52) is unaffected by strong and electromagnetic interactions except some negligible higher order corrections, we have from (59) and (55)

$$\Gamma(K^+ \to \pi^+\kappa_d'\bar{\kappa_s'}) \approx \frac{g_W^4 m_s^5}{96\pi^3 m^4_{W^{\pm\frac{1}{3}}}}, \tag{60}$$

where $g_W^2 \approx \frac{1}{\sqrt{2}}Gm^2_{W^\pm}$ well-known in electro-weak theory. This gives

$$B(K^+ \to \pi^+\kappa_d'\bar{\kappa_s'}) = \tau_{K^+}\Gamma(K^+ \to \pi^+\kappa_d'\bar{\kappa_s'}) \approx \frac{\tau_{K^+}G^2m^4_{W^\pm}m_s^5}{192\pi^3 m^4_{W^{\pm\frac{1}{3}}}}. \tag{61}$$

From this, we have

$$m_{W^{\pm\frac{1}{3}}} \approx \left(\frac{\tau_{K^+}G^2m^4_{W^\pm}m_s^5}{192\pi^3 B(K^+ \to \pi^+\kappa_d'\bar{\kappa_s'})}\right)^{\frac{1}{4}}, \tag{62}$$

where $G \approx 1.166379 \times 10^{-5}\mathrm{GeV}^{-2}$, $m_{W^\pm} \approx 80.385\mathrm{GeV}$, $m_s \approx 95\mathrm{MeV}$, $\tau_{K^+} \approx 1.238 \times 10^{-8}\mathrm{s}$ [19].

From the measured $K^+ \to \pi^+ +$ 'missing energy' branching ratio [9] and the predicted $K^+ \to \pi^+\nu\bar{\nu}(\nu = \nu_e, \nu_\mu, \nu_\tau)$ branching ratio[18],

$$B(K^+ \to \pi^+ + \mathrm{Nothing})_{\mathrm{Exp}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}, \tag{63}$$

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{Theo}} = (0.781^{+0.080}_{-0.071} \pm 0.029) \times 10^{-10}, \tag{64}$$

and from (53), we have

$$B(K^+ \to \pi^+\kappa_d'\bar{\kappa_s'}) \approx (0.95^{+\alpha'}_{-\alpha}) \times 10^{-10}. \tag{65}$$

Substituting this in (62), we obtain

$$m_{W^{\pm\frac{1}{3}}} \approx 6.18^{+\beta'}_{-\beta}\ \mathrm{TeV}. \tag{66}$$

Unfortunately, the range of the experimental uncertainty is so wide as we can see.

VI. CONCLUDING REMARKS

It should be noted that neutral quarks can be produced in non-leptonic decays, and they carry away 'missing energy' as do neutrinos. Thus we must take into account the processes involving neutral quarks as well as those involving neutrinos in non-leptonic weak interactions where missing energies occur.

To confirm our speculation, we should look for colored and electrically charged bosons with spin-1 and neutral quarks consistent with the properties described in this paper.

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