Gravitational interaction in the medium of non-zero density

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The paper presents the novel results obtained by more comprehensively analyzing well-known and clearly visible physical processes associated with gravitational interaction in the system of material bodies in the medium of non-zero density. The work is based on the statement that the "buoyancy" (Archimedes) force acting on the material body located in a medium of non-zero density is of the gravitational nature. Due to this approach, we managed, staying in the framework of classical physics and mechanics definitions, to introduce the concept of the body's gravitating mass as a mass determining the gravitational interaction intensity and also to establish an analytical relation between the gravitating and inertial masses of the material body. Combining the direct and indirect (Archimedes force) gravitational effect on the material body in the medium, we succeeded in distinguishing from the total gravitational field of this system a structure with a dipole-like field line distribution. This fact allows us to assert that, along with the gravitational attraction, there exists also gravitational repulsion of material bodies, and this fact does not contradict the meanings of existing basic definitions and concepts of classical physics and mechanics.

"Generally known — the fact that something is *known*
does not mean that it is *realized*"

George Wilhelm Friedrich Hegel [1, p. 22]

1. Problem definition

Gravitational interaction in a system of material bodies is conventionally considered ignoring the presence of the medium where the system is located. Therefore, the problem of the medium effect upon gravitational interaction between material bodies is of significant interest.

Let there be a static medium $\mathcal{F}$ with uniform distribution of the substance density $\rho_{\mathcal{F}} > 0$ (e.g., non-compressible liquid), which is limited by a spherical surface with radius $R_{\mathcal{F}}$. Put into an arbitrary fixed point of this medium a rigid uniform sphere $\mathcal{G}$ of constant density $\rho_{\mathcal{G}} > 0$ and radius $R_{\mathcal{G}} \ll R_{\mathcal{F}}$. It is necessary to derive a relation describing the material body gravitational interaction with its surrounding medium $\mathcal{F}$ in the absence of any external impacts of the gravitational or other nature. Here we consider the problem in the simplest form[1].

The problem of the body $\mathcal{G}$ gravitational field effect upon the medium $\mathcal{F}$ physical characteristics, namely, density, will be considered below; let us assume for a while that distribution of the medium $\mathcal{F}$ density does not change after inserting the anomaly (body $\mathcal{G}$) into it.

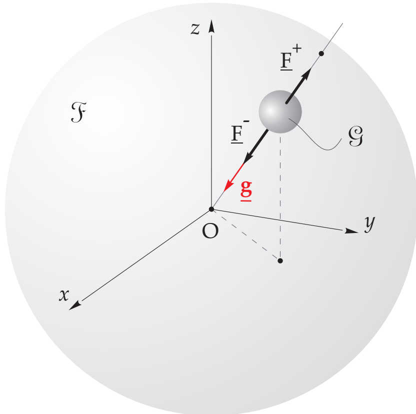


Figure 1.

Let us consider a Cartesian coordinate system $\mathbf{O}xyz$ (Fig. 1) whose origin is at the center of the sphere of radius $R_{\mathcal{F}}$. Set the body $\mathcal{G}$ center of mass at distance δ from point $\mathbf{O}$. The gravitational field at an arbitrary point of medium $\mathcal{F}$ is characterized by the field vector $\underline{\mathbf{g}}_{\mathcal{F}}(\underline{r})$, where $\underline{r}$ is the radius-vector of the point under consideration. Due to the symmetry of the substance shape and volume distribution, the gravitational field intensity vector $\underline{\mathbf{g}}_{\mathcal{F}}$ at each selected point of the $\mathcal{F}$ domain is directed to the "center of attraction", i.e., to the sphere center (point $\mathbf{O}$).

Designate the force of gravitational interaction between body $\mathcal{G}$ and the domain $\mathcal{F}$ matter as $\underline{F}^{-}$. Force $\underline{F}^{-}$ direction is determined by the intensity vector of the centrally-symmetrical gravitational field $\underline{\mathbf{g}}_{\mathcal{F}}$, which is directed towards the spherical domain center $\mathbf{O}$. In its turn, the centrally symmetric structure of the gravitational field inside spherical domain $\mathcal{F}$ causes also a centrally symmetrical distribution of the pressure of matter in the $\mathcal{F}$ medium. The presence of a pressure gradient in the medium surrounding body $\mathcal{G}$ gives rise to the so-called "buoyancy" force $\underline{F}^{+}$.

Hence, body $\mathcal{G}$ is simultaneously exposed to two quasi mutually "independent" forces that, however, both depend on the gravitational field of medium $\mathcal{F}$ surrounding the body. The vector sum of these forces is:

$$\underline{F} = \underline{F}^{+} + \underline{F}^{-} \ , \tag{1}$$

where $\underline{F}^{-}$ is the force of the direct gravitational effect on the $\mathcal{G}$ body from medium $\mathcal{F}$ and $\underline{F}^{+}$ is the "buoyancy" force caused by the existence of a pressure gradient in medium $\mathcal{F}$.

[1]The imperative of the substance density coordinate and time constancy in the $\mathcal{F}$ domain, the material body $\mathcal{G}$ rigidity and immobility with respect to medium $\mathcal{F}$, and selection of their spherical geometry is not mandatory; however, it allowed us to simplify the problem without loss of generality and focus on the main task that is to define the gravitational force acting upon body $\mathcal{G}$ from medium $\mathcal{F}$ surrounding it.

One of real cases of the considered problem is the experiment with a "soaring" liquid drop under zero gravity[2]. This is just the case when the liquid drop contains a density *anomaly*[3] (e.g., a "pellet") in the absence of any external forces. Under these conditions, the "pellet" can "sink" towards the drop center or "emerge" to the drop surface depending on the ratio between its own density and that of the liquid.

Nevertheless, despite the defined problem triviality and obviousness of its solution, let us consider in detail each of the relation (1) force components.

2. Direct medium action on the body

Since the central symmetry of the matter density distribution in spherical domain $\mathcal{F}$ makes all the radial directions equivalent, let us set the body $\mathcal{G}$ center of mass $\mathbf{O}'$ so that it is on the $\mathbf{O}z$ axis at distance δ from the gravitational attraction center $\mathbf{O}$ in the $\mathcal{F}$ domain (Fig. 2).

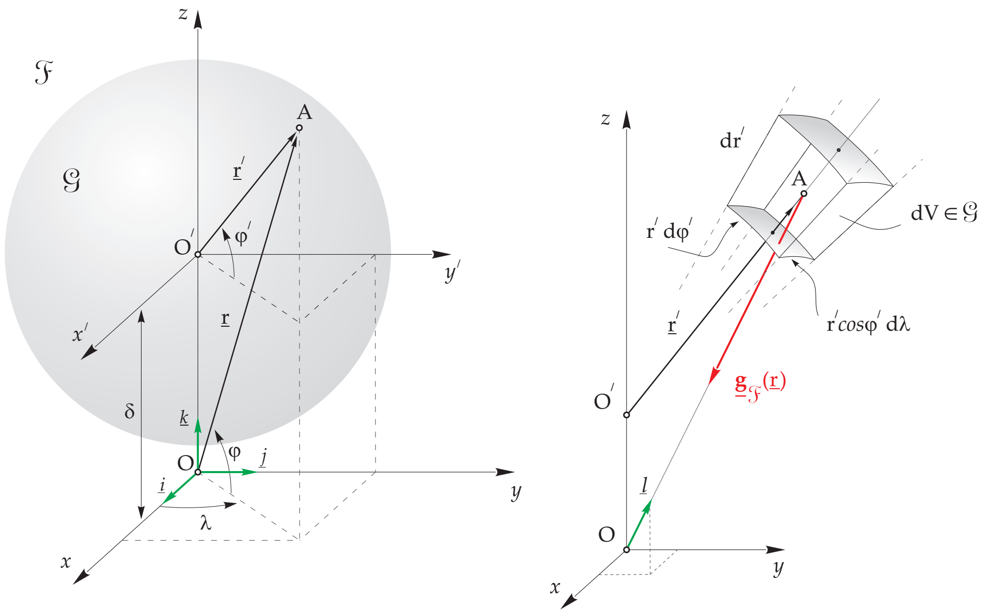


Figure 2.

The auxiliary rectangular coordinate system $\mathbf{O}'x'y'z$ associated with the body $\mathcal{G}$ center of mass is oriented so that axis $\mathbf{O}'x'$ is parallel to $\mathbf{O}x$ while

[2] A space domain where gravitational forces are on the average counterpoised by centrifugal forces or forces of other nature.

[3] Term *anomaly* is used in gravimetry to define an inclusion (foreign body) with density different from that of the surrounding medium [2, 3].

axis $\mathbf{O}'y'$ is parallel to $\mathbf{O}y$.

The gravitational field vector $\underline{\mathbf{g}}_{\mathcal{F}}$ is defined at each point of the limited-size domain $\mathcal{F}$ under consideration in the following way:

$$\underline{\mathbf{g}}_{\mathcal{F}}(\underline{r}) = -\mathbf{G}\,\frac{V_{\mathcal{F}}(|\underline{r}|)\rho_{\mathcal{F}}}{|\underline{r}|^3}\,\underline{r} = -\mathbf{G}\,\frac{4}{3}\pi\rho_{\mathcal{F}}\,\underline{r}\ , \qquad 0 \leqslant |\underline{r}| \leqslant R_{\mathcal{F}}\ , \tag{2}$$

where $\underline{r}$ is the radius-vector of an arbitrary point $\mathbf{A}$ of the $\mathcal{F}$ domain; $V_{\mathcal{F}}$ is the spherical domain $\mathcal{F}$ volume as a function of radius $|\underline{r}|$; $\rho_{\mathcal{F}}$ is the medium $\mathcal{F}$ density; $\mathbf{G}$ is the gravitational constant. The procedure of deriving relation (2) for the gravitational field intensity is considered in detail in, e.g., [2, 4]. As relation (2) shows, gravitational field gradient $\underline{\mathbf{g}}_{\mathcal{F}}$ at the point with radius-vector $\underline{r}$ is always directed towards the geometrical center (generally, towards the center of attraction) of the medium $\mathcal{F}$ domain under consideration; in this case, this is point $\mathbf{O}$.

Now, knowing the gravitational field intensity $\underline{\mathbf{g}}_{\mathcal{F}}$ at each point of the $\mathcal{F}$ domain, we can determine the resultant gravitational effect on body $\mathcal{G}$ from medium $\mathcal{F}$ as an integral over the body volume $V_{\mathcal{G}}$:

$$\underline{F}^{-} = \int\limits_{V_{\mathcal{G}}} d\underline{F}^{-}\ , \quad d\underline{F}^{-} = \rho_{\mathcal{G}} dV\,\underline{\mathbf{g}}_{\mathcal{F}}\ . \tag{3}$$

Since the task stipulates the symmetry of geometric and physical parameters about axis $\mathbf{O}z$, it is evident that

$$F_x^{-} = \underline{i}\cdot\underline{F}^{-} = 0, \qquad F_y^{-} = \underline{j}\cdot\underline{F}^{-} = 0, \qquad F_z^{-} = \underline{k}\cdot\underline{F}^{-} = \int\limits_{V_{\mathcal{G}}} \underline{k}\cdot d\underline{F}^{-}\ .$$

Hence, taking into account relations (2) and (3), obtain:

$$F_z^{-} = \int\limits_{V_{\mathcal{G}}} \underline{k}\cdot d\underline{F}^{-} = \rho_{\mathcal{G}}\int\limits_{V_{\mathcal{G}}} \underline{k}\cdot\underline{\mathbf{g}}_{\mathcal{F}}\,dV = -\frac{4}{3}\pi\mathbf{G}\rho_{\mathcal{G}}\rho_{\mathcal{F}}\int\limits_{V_{\mathcal{G}}} \overbrace{\underline{k}\cdot\underline{r}}^{r\sin\varphi}\ dV\ . \tag{4}$$

Based on the preset geometry of the task (Fig. 2), it is possible to write the following equality:

$$r\sin\varphi = \delta + r'\sin\varphi'\ , \tag{5}$$

and a relation for elementary volume dV constructed around point $\mathbf{A}(r',\varphi',\lambda)\in\mathcal{G}$:

$$dV = r'd\varphi'\cdot r'\cos\varphi'\ d\lambda\cdot dr' = r'^2\cos\varphi'\ dr'\ d\varphi'\ d\lambda\ . \tag{6}$$

Substituting (5) and (6) into expression (4) for F_z^{+}, obtain:

$$F_z^{-} = -\frac{4}{3}\pi \mathbf{G}\rho_{\mathcal{G}}\rho_{\mathcal{F}} \left(\delta V_{\mathcal{G}} + \overbrace{\int\limits_{V_{\mathcal{G}}} r' \sin\varphi' \, dV}^{\mathbf{I}} \right) , \tag{7}$$

where $\mathbf{I} = 0$ because

$$\mathbf{I} = \int\limits_{V_{\mathcal{G}}} r' \sin\varphi' \, \overbrace{r' d\varphi' r' \cos\varphi' d\lambda dr'}^{dV} = \int\limits_{0}^{2\pi} d\lambda \int\limits_{0}^{R_{\mathcal{G}}} r'^3 dr' \cancelto{0}{\int\limits_{-\pi/2}^{\pi/2} \sin\varphi' \cos\varphi' d\varphi'} = 0 \; .$$

Grouping the factors in relation (7), obtain

$$F_z^{-} = \overbrace{-\frac{4}{3}\pi \mathbf{G}\rho_F \delta}^{\mathbf{g}_{\mathcal{F}}(\delta)} \; \rho_{\mathcal{G}} V_{\mathcal{G}}$$

or, in the vector form,

$$\boxed{\underline{F}^{-} = \rho_{\mathcal{G}} V_{\mathcal{G}} \; \underline{\mathbf{g}}_{\mathcal{F}}(\delta) \; .} \tag{8}$$

Thus, force $\underline{F}^{-}$ of the medium $\mathcal{F}$ gravitational effect on body $\mathcal{G}$ whose mass is presented as product $\rho_{\mathcal{G}} V_{\mathcal{G}}$ is co-directed with the gravitational field vector at the point where the body is located. Relation (8) is valid also in the case when point $\mathbf{O}$ that is the medium $\mathcal{F}$ center of attraction is located inside the $\mathcal{G}$ body, namely, when $\delta < R_{\mathcal{G}}$.

3. The medium action on the body via the pressure induced by the medium inherent gravitational field

Consider now in relation (1) the second component of the force impact upon the body, namely, $\underline{F}^{+}$. Spherical domain of medium $\mathcal{F}$ with uniformly distributed density $\rho_{\mathcal{F}}$ induces centrosymmetrical gravitational field with the center of attraction at point $\mathbf{O}$. In its turn, this field generates in medium $\mathcal{F}$ under consideration a centrosymmetrical pressure distribution with a corresponding radial gradient. Since body $\mathcal{G}$ is an object with a non-zero volume, integration of the medium $\mathcal{F}$ pressure effect over the body surface gives us a force tending to "push" body $\mathcal{G}$ out into the domain where the medium $\mathcal{F}$ pressure is minimal (in other words, the *Archimedes's force*). The problem definition does not stipulate the existence of gradients of any other nature except for gravitational.

How is the pressure distributed in gravitating medium $\mathcal{F}$? To answer this question, select in the $\mathcal{F}$ medium an elementary volume $dV \in \mathcal{F}$ around point **A** with spherical coordinates r, φ and λ (Fig. 3).

$$dV = dS\,dr = \overbrace{r\,d\varphi \cdot r\cos\varphi\,d\lambda}^{dS}\,dr\,.$$

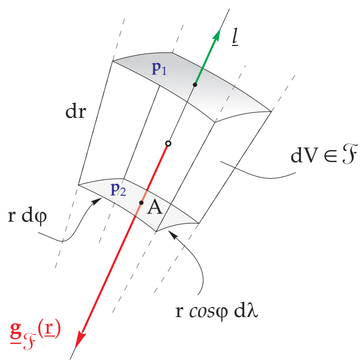


Figure 3.

Elementary volume dV is in equilibrium. The bottom side of elementary volume dV is subject to pressure p_2 that is counterpoised by pressure p_1 acting on the top side plus gravitational force of the elementary volume. The gravitational interaction force is of the same direction as the medium $\mathcal{F}$ gravitational field vector. Forces due to the pressure on the elementary volume lateral sides are mutually counterpoised. Taking into account all these factors, we can write the balance equation for elementary volume dV in the form of a projection on the unit vector $\underline{l}$ direction:

$$p_2 dS\,\underline{l} = p_1 dS\,\underline{l} + \rho_{\mathcal{F}} dV\,\underline{\mathbf{g}}_{\mathcal{F}}\,, \quad \underline{l} = \frac{\underline{r}}{|\underline{r}|}\,. \tag{9}$$

Multiplying the left and right sides of the equality by unit vector $\underline{l}$, obtain the pressure increment dp:

$$dp = p_2 - p_1 = \rho_{\mathcal{F}} dr\,\underline{\mathbf{g}}_{\mathcal{F}} \cdot \underline{l}\,. \tag{10}$$

Integrating (10) with respect to r and taking into account relation (2), obtain for the gravitational field vector $\underline{\mathbf{g}}_{\mathcal{F}}$ the following relation:

$$p(r) = -\frac{2}{3}\pi\mathbf{G}\rho_{\mathcal{F}}^2 r^2 + const\,. \tag{11}$$

The constant may be determined from the condition at the domain $\mathcal{F}$ boundary:

$$p\Big|_{r=R_{\mathcal{F}}} = p_{\mathcal{F}}\,, \tag{12}$$

where $p_{\mathcal{F}}$ is the external pressure at the medium $\mathcal{F}$ boundary. Now the relation for pressure at an arbitrary point of domain $\mathcal{F}$ gets the following form:

$$\boxed{p(r) = p_{\mathcal{F}} + \frac{2}{3}\pi \mathbf{G} \rho_{\mathcal{F}}^2 \left(R_{\mathcal{F}}^2 - r^2\right) \ .} \tag{13}$$

The force caused by the existence of the gravitation-induce pressure gradient. Now we know the pressure distribution in the $\mathcal{F}$ domain. Let us determine "buoyant" force $\underline{F}^{+}$ acting upon body $\mathcal{G}$, which is caused by the presence of pressure gradient (10) in medium $\mathcal{F}$:

$$\underline{F}^{+} = \int\limits_{S_{\mathcal{G}}} p(\underline{r}) \underline{dS}, \quad \text{where} \qquad \underline{dS} = \underline{n}\, dS \ . \tag{14}$$

Here $p(\underline{r})$ is the medium $\mathcal{F}$ pressure upon elementary area dS of the body $\mathcal{G}$ surface; $\underline{n}$ is the normal to the body surface elementary area dS at point $\mathbf{A}(\underline{r})$. Fig. 3 shows that

$$dS = R_{\mathcal{G}} \cos\varphi' d\lambda \; R_{\mathcal{G}} d\varphi' \ .$$

In calculating integral (14), take into account that the problem defined here possesses the geometric and field symmetry about axis $\mathbf{O}z$; hence,

$$F_x^{+} = \underline{i} \cdot \underline{F}^{+} = 0 \ , \quad F_y^{+} = \underline{j} \cdot \underline{F}^{+} = 0 \ , \quad F_z^{+} = \underline{k} \cdot \underline{F}^{+} = \int\limits_{S_{\mathcal{G}}} p(\underline{r}) \overbrace{\underline{k} \cdot \underline{n}}^{\sin\varphi'} dS \ .$$

Thus, taking into account the dS expression, obtain

$$F_z^{+} = \int\limits_{S_{\mathcal{G}}} p(r) \sin\varphi' dS = \int\limits_{-\pi/2}^{\pi/2} p(r) \sin\varphi' \overbrace{R_{\mathcal{G}} \cos\varphi' d\lambda \; R_{\mathcal{G}} d\varphi'}^{dS} =$$

$$= R_{\mathcal{G}}^2 \int\limits_0^{2\pi} d\lambda \int\limits_{-\pi/2}^{\pi/2} p(r) \sin\varphi' \cos\varphi' d\varphi' \ .$$

Substituting relation (13) for $p(r)$ and assuming that $p_{\mathcal{F}} = 0$, obtain as a result of integration the force:

$$F_z^{+} = \frac{4}{3}\pi R_{\mathcal{G}}^2 \, \pi \mathbf{G} \rho_{\mathcal{F}}^2 \int\limits_{-\pi/2}^{\pi/2} \left(R_{\mathcal{F}}^2 - r^2\right) \sin\varphi' \cos\varphi' d\varphi' \ . \tag{15}$$

The expression for r^2 follows from the problem geometry (Fig. 2)

$$r^2 = \delta^2 + R_{\mathcal{G}}^2 + 2\delta R_{\mathcal{G}} \sin\varphi' \ , \tag{16}$$

where δ is the shift of the anomaly $\mathcal{G}$ center; r is the distance between the medium $\mathcal{F}$ attraction center and a point on the body $\mathcal{G}$ surface. Substituting (16) into the expression for F_z^+, obtain:

$$F_z^+ = \frac{4}{3}\pi R_{\mathcal{G}}^2 \, \pi \mathbf{G} \rho_{\mathcal{F}}^2 \int\limits_{-\pi/2}^{\pi/2} \left(R_{\mathcal{F}}^2 - \overbrace{\left(\delta^2 + R_{\mathcal{G}}^2 + 2\delta R_{\mathcal{G}} \sin\varphi'\right)}^{r^2} \right) \sin\varphi' \cos\varphi' d\varphi' \ . \tag{17}$$

Let us calculate auxiliary integrals:

$$\mathbf{I}_1 = \int\limits_{-\pi/2}^{\pi/2} \sin\varphi' \cos\varphi' \, d\varphi' = \frac{1}{2} \int\limits_{-\pi/2}^{\pi/2} d\sin^2\!\varphi' = 0 \ ,$$

$$\mathbf{I}_2 = \int\limits_{-\pi/2}^{\pi/2} \sin^2\!\varphi' \cos\varphi' \, d\varphi' = \frac{1}{3} \int\limits_{-\pi/2}^{\pi/2} d\sin^3\!\varphi' = \frac{2}{3} \ .$$

Now, taking into account $\mathbf{I}_1$ and $\mathbf{I}_2$, expression (17) may be rewritten as:

$$F^+ = \frac{4}{3}\pi R_{\mathcal{G}}^2 \, \pi \mathbf{G} \rho_{\mathcal{F}}^2 2\delta R_{\mathcal{G}} \cdot \mathbf{I}_2 = \overbrace{\frac{4}{3}\pi R_{\mathcal{G}}^3}^{V_{\mathcal{G}}} \, \rho_{\mathcal{F}} \, \overbrace{\frac{4}{3}\pi \mathbf{G} \rho_{\mathcal{F}} \delta}^{-\mathbf{g}_{\mathcal{F}}(\delta)} \tag{18}$$

or, in the vector form,

$$\boxed{\underline{F}^+ = -\rho_{\mathcal{F}} V_{\mathcal{G}} \, \underline{\mathbf{g}}_{\mathcal{F}}(\delta) \ .} \tag{19}$$

Thus, body $\mathcal{G}$ located in medium $\mathcal{F}$ with non-zero density undergoes a "buoyancy" action via force F^+ applied to the body's center of mass. Formula (19) shows that the "buoyancy" force $\underline{F}^+$ acting upon body $\mathcal{G}$ is always counter-directed to the gravitational field vector $\underline{\mathbf{g}}_{\mathcal{F}}$. Emphasize that, at the preset gravitational field intensity, "buoyancy" force $\underline{F}^+$ depends only on the medium density $\rho_{\mathcal{F}}$ and volume $V_{\mathcal{G}}$ of the body under consideration and is independent of the pressure magnitude and distribution in medium $\mathcal{F}$; this is fully consistent with the classical definition of the *Archimedes law*[4] for the force acting upon a body submerged in a liquid medium.

[4] "... all the pressures the liquid applies to the body submerged in it have an upright resultant force equal to the volume displacement weight; its application point is the center of gravity of the volume submerged in the liquid." N. E. Zhukovsky [5, p. 654].

4. Resultant effect upon the body

Let us now return to the main problem, namely, to revealing the medium $\mathcal{F}$ response to material body $\mathcal{G}$ included in it. Substitute into (1) forces (8) and (19) acting upon body $\mathcal{G}$ from medium $\mathcal{F}$:

$$\underline{F} = \underline{F}^{+} + \underline{F}^{-} = -\rho_{\mathcal{F}} V_{\mathcal{G}}\, \underline{\mathbf{g}}_{\mathcal{F}} + \rho_{\mathcal{G}} V_{\mathcal{G}}\, \underline{\mathbf{g}}_{\mathcal{F}} = \overbrace{\underbrace{\rho_{\mathcal{G}} V_{\mathcal{G}}}_{M_{\mathcal{G}}} \left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)}^{m_{\mathcal{G}}} \underline{\mathbf{g}}_{\mathcal{F}} \;, \tag{20}$$

or

$$\underline{F} = m_{\mathcal{G}} \underline{\mathbf{g}}_{\mathcal{F}} \;, \quad \text{where} \quad m_{\mathcal{G}} = M_{\mathcal{G}} \left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right) \;, \tag{21}$$

Here $M_{\mathcal{G}}$ designates the body $\mathcal{G}$ mass in its classical meaning: a product of the body density $\rho_{\mathcal{G}}$ and volume $V_{\mathcal{G}}$; $m_{\mathcal{G}}$ designates the portion of the body $\mathcal{G}$ mass $M_{\mathcal{G}}$, which participates in the body gravitational interaction with medium $\mathcal{F}$; $\underline{\mathbf{g}}_{\mathcal{F}}$ is the vector of the medium $\mathcal{F}$ gravitational field at the point where body $\mathcal{G}$ is located.

Why we offer to join these forces together instead of considering them individually? The reasons for this are as follows. First, the forces considered are of one and the same physical nature — gravitational; i.e., they depend on the gravitational field intensity in the vicinity of the body. Second, these forces are tangent to the field line of gravitational field induced by the medium $\mathcal{F}$ matter.

Things came around so that the gravitational attraction force and buoyancy Archimedes force were discovered in different historical epochs[5]; as a result, they are regarded and perceived as "independent" force factors acting from outside on a material body in a non-zero density medium.

For the above reasons, we can assert that, in the absence of any non-gravitational external factors, body $\mathcal{G}$ located in medium $\mathcal{F}$ is subject to only one force, namely, the force of gravitational interaction defined by (21).

This force sign (direction of action) depends only on the ratio between densities of body $\mathcal{G}$ and medium $\mathcal{F}$ surrounding it. Fig. 4 illustrates graphically the transition from the classical two-force concept to the concept of one force.

The body curvilinear motion along the gravitational field line implies arising of extra forces as a result of the body motion in a dissipating medium;

[5]The time interval between the moments of recognizing these forces by the scientific community is somewhat shorter than 2000 years. Archimedes (227–212 B.C.) — a buoyancy force acting on the body submerged in a medium, and Isaac Newton (1643–1727) — the law on two-body gravitational attraction.

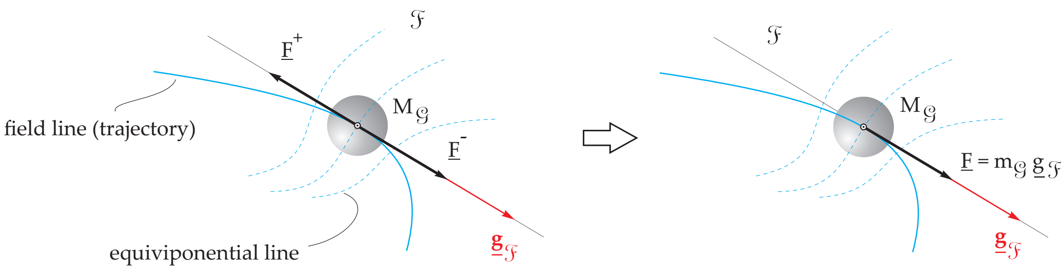


Figure 4. Schematic diagram of the transition from the two-component interpretation of the medium $\mathcal{F}$ gravitational effect on body $\mathcal{G}$ to the single-component concept.

these forces deflect the body from the preset trajectory. However, they are of other nature, and thus are beyond the scope of our problem.

Formula (21) shows that the gravitational interaction in medium $\mathcal{F}$ is determined by not the entire mass of the body but only by its part $m_{\mathcal{G}}$ referred to as *gravitating mass*. Term *gravitating mass* describes the cause-and-effect relation between the material body and its gravitational field more exactly than term "gravitation mass"[6]. As for the total mass of body $\mathcal{G}$ designated above as $M_{\mathcal{G}}$, we call it *inertial* and define as the product of the body $\mathcal{G}$ volume $V_{\mathcal{G}}$ by its density $\rho_{\mathcal{G}}$.

Thus, omitting the symbols designating the body and medium in (21), obtain a formula establishing a functional relation between inertial mass M and gravitating mass m of the body located in the medium with density $\rho_0 \geqslant 0$:

$$\boxed{m = M\left(1 - \frac{\rho_0}{\rho}\right) \;, \quad \text{where} \qquad M = \rho V \;.} \tag{22}$$

Analysis of relation (22) allows us to define two important properties of the material body gravitating mass provided it is possible to use the concept of matter density:

1) The gravitating mass is always lower than the inertial mass:

$$|m| < M \,. \tag{23}$$

[6]In the literature (e.g., [6]), the gravitational mass is defined in many different manners. Two variants of gravitational interaction between the material body and medium, which were considered earlier, gave rise to a novel definition of the gravitating mass that, we believe, better fits the gravitational interaction phenomena observed in Nature: *the body's gravitating mass is a product of the body's volume and difference between its density and the medium density*. The gravitating mass may be also defined more utilitarianly: the body's gravitating mass is a factor determining the possibility of its interaction with other material objects.

2) The equality of the gravitating and inertial masses is valid only in the limit case when the density of medium surrounding the body is zero:

$$\lim_{\rho_0 \to 0} m = M \,. \tag{24}$$

These properties of the body gravitating mass are illustrated in Fig. 5 where ρ_* designates the minimum possible density of the medium. This

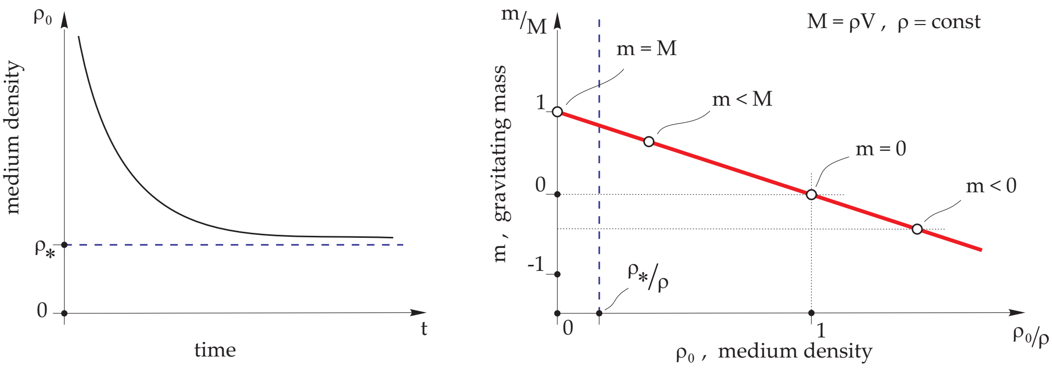


Figure 5. The body gravitating mass versus the surrounding medium density.

minimal density corresponds to the limiting amount of matter at which in the selected representative volume the concept of density remains physically meaningful within the scope of the specific problem. The Fig. 5 left plot shows that the up-to-date methods for creating high vacuum[7] fundamentally cannot reduce the medium density to zero. Multiple experiments [7, 8, 9] on validation of the postulate on the equality of the material body gravitating and inertial masses unambiguously confirm the absolutely theoretical character of the mass equivalency principle realizable only in the zero-density medium.

The gravitating mass expression (22) was used in studying the nature of the Earth's center of mass motion under the action of external gravitational forces [10] and the Earth's core motion under the influence of the Moon's perigee mass [11], as well as in the paper on the gravitating mass [12].

5. The additivity principle

Since the zero-density medium is a medium free of matter, the additivity principle for gravitational forces acting in the system of material bodies is

[7]Material medium of the minimum possible density

strictly valid and does not need proving. And what is about the *additivity principle* in the non-zero density medium?

Formula (22) that establishes the relation between the body's inertial and gravitating masses shows that the gravitating mass of the body with fixed geometric and physical parameters depends only on the density of medium surrounding the body.

Hence, assuming the possibility of gravitational compression of medium $\mathcal{F}$ surrounding body $\mathcal{G}$, we face the impossibility of applying the *additivity principle* and necessity of allowing for the influence of each body on density variation in the entire medium $\mathcal{F}$ domain under consideration in analyzing the two-body mutual gravitational interaction in the medium with $\rho_{\mathcal{F}} > 0$.

Nevertheless, let us show that in practice the *additivity principle* may be used also in the problems of gravitational interaction of bodies in the non-zero density medium.

Let us define the law according to which medium $\mathcal{F}$ will be compressed around body $\mathcal{G}$. For this purpose, assume that medium $\mathcal{F}$ is an ideal gas able to significantly compress (change its density) under external forces of various nature. Here we ignore the effect of the medium gravitational self-compression. Consider again the above problem (see Figs. 1, 2 and 3) but put the gravitational field source, namely, body $\mathcal{G}$, into the domain $\mathcal{F}$ geometrical center ($\delta = 0$). From the condition of the medium elementary volume equilibrium (Fig. 3), obtain the formula for pressure increment dp:

$$dp = p_2 - p_1 = \rho_{\mathcal{F}}(r)\mathbf{g}_{\mathcal{G}}(r)dr \ , \tag{25}$$

where

$$\mathbf{g}_{\mathcal{G}}(r) = -\mathbf{G}\frac{V_{\mathcal{G}}\rho_{\mathcal{G}}}{(R_{\mathcal{G}}+r)^2} \ , \quad V_{\mathcal{G}} = \frac{4}{3}\pi R_{\mathcal{G}}^3 \ . \tag{26}$$

Here p_2 is the pressure at the lower (closest to the body) side; p_1 is the pressure upon the upper side of the elementary volume; r is the distance from the body surface to the medium elementary volume under consideration; $\mathbf{g}_{\mathcal{G}}$ is the body $\mathcal{G}$ gravitational field intensity; $R_{\mathcal{G}}$ and $\rho_{\mathcal{G}}$ are the body's radius and density, respectively; $\rho_{\mathcal{F}}$ is the medium density; $\mathbf{G}$ is the gravitational constant.

Use the following equation of the ideal gas kinetic theory [13, 14] to interrelate the pressure, density and temperature of the gaseous medium under consideration:

$$pV = \frac{m}{\mathcal{M}}\mathbf{R}T \ , \quad m = V\rho \quad \Longrightarrow \quad p = \frac{\rho}{\mathcal{M}}\mathbf{R}T \quad \Longrightarrow \quad dp = \frac{\mathbf{R}T}{\mathcal{M}}d\rho \ , \tag{27}$$

here V is the gas volume under consideration; ρ is the density; $\mathcal{M}$ is the molar mass; $\mathbf{R}$ is the universal gas constant; p is the pressure; T is the temperature.

Excluding dp from equations (27) and (25)

$$d\rho_{\mathcal{F}} = \frac{\mathcal{M}}{\mathbf{R}T}\,\rho_{\mathcal{F}}\mathbf{g}_{\mathcal{G}}(r)\,dr \quad \Longrightarrow \quad \frac{d\rho_{\mathcal{F}}}{\rho_{\mathcal{F}}} = -\frac{\mathcal{M}}{\mathbf{R}T}\,\mathbf{G}\frac{V_{\mathcal{G}}\rho_{\mathcal{G}}}{(R_{\mathcal{G}}+r)^2}\,dr \tag{28}$$

and integrating (28), obtain:

$$\ln\rho_{\mathcal{F}} = -\frac{\mathcal{M}}{\mathbf{R}T}\,\mathbf{G}V_{\mathcal{G}}\rho_{\mathcal{G}}\int\frac{dr}{(R_{\mathcal{G}}+r)^2} = \frac{\mathcal{M}}{\mathbf{R}T}\,\mathbf{G}\frac{V_{\mathcal{G}}\rho_{\mathcal{G}}}{R_{\mathcal{G}}+r} + const\;. \tag{29}$$

Find the integration constant from the boundary condition for medium $\mathcal{F}$ density on the body $\mathcal{G}$ surface:

$$\rho_{\mathcal{F}}\Big|_{r=0} = \rho_0\;. \tag{30}$$

As a result, obtain an exponential function characterizing the medium $\mathcal{F}$ density variation with distance from the body $\mathcal{G}$ surface:

$$\rho_{\mathcal{F}} = \rho_0\,\exp\left(-\frac{\mathcal{M}}{\mathbf{R}T}\,\mathbf{g}_0R_{\mathcal{G}}\left(1-\frac{1}{1+r/R_{\mathcal{G}}}\right)\right)\,, \tag{31}$$

where $\mathbf{g}_0$ is the gravitational field on the body surface;
$\mathbf{R} = 8.3144621\,J/(K\cdot mole)$.

Now, using relation (31), construct the medium $\mathcal{F}$ density distribution around body $\mathcal{G}$. Let us take as medium $\mathcal{F}$ an ideal gas whose main physical parameters are adequate to the Earth's standard atmosphere ($T = 288.15\,°$K, $\mathcal{M} = 0.02898\,kg/mole$, $\rho_0 = 1.225\,kg/m^3$). The material body $\mathcal{G}$ characteristics are listed in Table 1. Fig. 6 illustrates the medium $\mathcal{F}$ density variation

Table 1. Body parameters

	density, kg/m^3	radius, m	mass, kg	$\mathbf{g}_0$, m/s^2
Earth	5514	6.371×10^6	5.973×10^{24}	9.821
test body	7200	10	3.016×10^7	2.013×10^{-5}

with distance from the body surface for the system of two gravitating bodies $\mathcal{G}$ (Table 1).

For the Earth, the density of gaseous medium $\mathcal{F}$ at distance $r = 10\,km$ from the surface is $0.37412\,kg/m^3$; for the test body, the medium $\mathcal{F}$ density remains almost constant, namely, $1.224999997\,kg/m^3$. This results from that

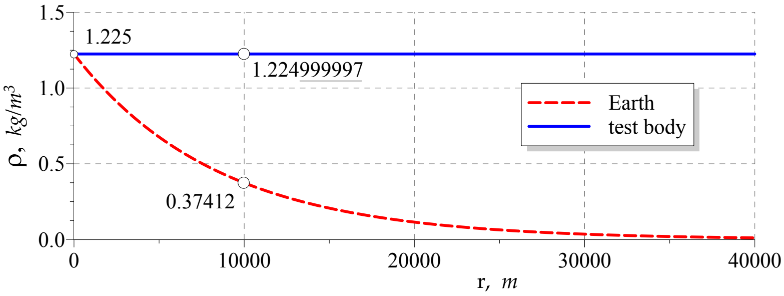


Figure 6. Medium $\mathcal{F}$ (air medium) density variation with the distance from the surface of the Earth and test body.

the test body mass is far lower than the Earth's mass and, hence, its gravitational field is far less intense. In the scope of this problem, gravitational field value $\mathbf{g}_0$ on the test body surface is about 487900 times less than the gravitational field on the Earth's surface.

Thus, estimation of the effect of the gaseous medium gravitational compression in the vicinity of the test body having considerable gravitating mass[8] showed that the medium density has formally changed but quite insignificantly. This means that in the case of a physically realizable system of material bodies located in an ideal gas we can ignore the medium density variation due to introducing gravitating bodies into it. This is valid also for liquid, loose, deformable and solid media since their compressibility is far lower than that of the gaseous medium.

6. Gravitational interaction of a pair of bodies

All the above mentioned allows one to describe gravitational interaction of two and more material bodies in a non-zero density medium by using the universal gravitation law in its conventional form but involving the *gravitating mass* concept. For simplicity, exclude the medium gravitational effect on the material bodies located in it. This implies that medium $\mathcal{F}$ is uniform in density and unlimited in volume, i.e., that it has neither center of mass or center of attraction.

In this case, the gravitational interaction force $\underline{F}$ for two bodies $\mathcal{G}_1$ and $\mathcal{G}_2$ whose centers of mass are $\underline{r}$ apart each other (Fig. 7) in uniform medium $\mathcal{F}$ of constant density ρ_0 gets the following form:

$$\underline{F} = \mathbf{G}\,\frac{m_1 m_2}{|\underline{r}|^3}\,\underline{r} \quad \text{or} \quad F = \mathbf{G}\,\frac{m_1 m_2}{r^2}\ , \tag{32}$$

[8] Realizable under laboratory conditions.

where $r = |\underline{r}|$; $F = |\underline{F}|$; m_1 and m_2 are the gravitating masses of bodies $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively. According to (22), the gravitating masses will be defined as:

$$m_1 = M_1\left(1 - \frac{\rho_0}{\rho_1}\right) , \quad m_2 = M_2\left(1 - \frac{\rho_0}{\rho_2}\right) .$$

Here $M_1 = \rho_1 V_1$ and $M_2 = \rho_2 V_2$ are the bodies' inertial masses.

When the medium $\mathcal{F}$ density tends to zero, formula (32) gets the classical form:

$$\lim_{\rho_0 \to 0} F = \mathbf{G} \, \frac{M_1 M_2}{r^2} . \tag{33}$$

Thus, using expression (22) for the material body gravitating mass, it is possible to determine the conditions under which the body gravitating mass may be either positive or negative. We can see that just the existence of the gravitating mass, its sign and value are indissolubly related to the medium density.

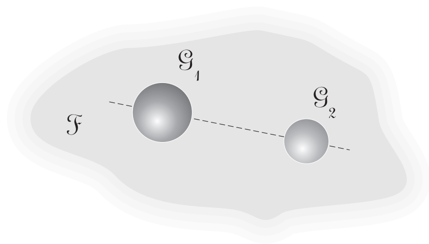


Figure 7.

Due to the gravitating mass alternation, it is possible to reasonably use such terms as gravitational "attraction" and "repulsion" for the system of material bodies in the non-zero density medium in studying physical processes[9] observed in Nature.

7. Gravitational dipole

Here we consider a problem of finding equipotential and force lines of the gravitational field created by two material bodies $\mathcal{G}_1$ and $\mathcal{G}_2$ located in motionless uniform medium $\mathcal{F}$ with density ρ_0; the gravitational effect on the bodies from medium $\mathcal{F}$ itself is ignored. Assume that the bodies are uniform rigid spheres R_1 and R_2 in radii, whose centers of mass are at fixed

[9]For instance, the submarine submerging or emerging process is nothing but controlling the gravitational mass value and sign by varying the ballast; at the zero running speed, this makes the submarine moving along the Earth's gravity field line. The submarine suspension at a preset depth means that its gravitational mass is zero. The same principle is valid also for aircrafts. A dirigible or air balloon rises (pushes from the Earth) not because of the pressure gradient in the Earth's atmosphere but because it has a negative gravitating mass (this means that the dirigible mean density is lower than the atmosphere density). Varying the aircraft average density relative to that of the environment, we change the sign and magnitude of its gravitating mass.

distance $L = 30\,mm$ from each other. The radii and densities of the bodies are given in Table 2.

Table 2. Parameters of bodies $\mathcal{G}_1$ and $\mathcal{G}_2$

	density, kg/m^3	radius, mm
$\mathcal{G}_1$	$\rho_1 = 7200$	$R_1 = 7$
$\mathcal{G}_2$	$\rho_2 = 6700$	$R_2 = 5$

The problem is considered in the three-dimensional version. Spheres $\mathcal{G}_1$ and $\mathcal{G}_2$ are motionless, and the medium $\mathcal{F}$ volume is sufficiently large to exclude the possible influence of edge effects. Assume that $\rho_1 > \rho_2$. The parameter of our task will be the medium density ρ_0. Let us construct a set of Poisson's equations for medium $\mathcal{F}$ and bodies $\mathcal{G}_1$, $\mathcal{G}_2$ included in it:

$$\begin{cases} \Delta\mathbf{U}_{\mathcal{F}} &= -4\pi\mathbf{G}\rho_0\ , \\ \Delta\mathbf{U}_{\mathcal{G}_1} &= -4\pi\mathbf{G}\rho_1\ , \\ \Delta\mathbf{U}_{\mathcal{G}_2} &= -4\pi\mathbf{G}\rho_2\ . \end{cases} \quad \Rightarrow \quad \begin{cases} \Delta\mathbf{U}_{\mathcal{F}} &= 0\ , \\ \Delta\mathbf{U}_{\mathcal{G}_1} &= -4\pi\mathbf{G}(\rho_1 - \rho_0)\ , \\ \Delta\mathbf{U}_{\mathcal{G}_2} &= -4\pi\mathbf{G}(\rho_2 - \rho_0)\ . \end{cases} \tag{34}$$

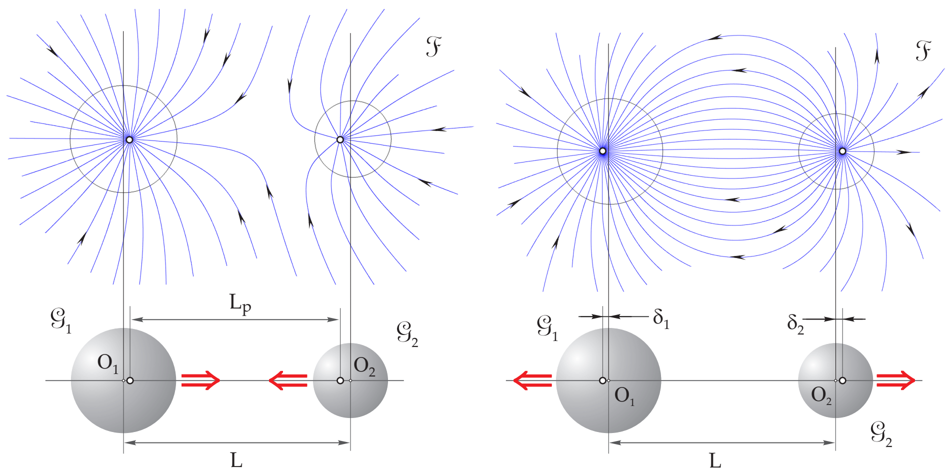
Fig. 8 presents the set (34) solution in the form of distribution of the gravitational field $\mathbf{U} = \mathbf{U}_{\mathcal{G}_1} + \mathbf{U}_{\mathcal{G}_2}$ lines. The Fig. 8(a) illustrates the task (34) solution for the zero density medium. Fig. 8(b) presents the field lines distribution in medium $\mathcal{F}$ whose density satisfies inequality $\rho_1 > \rho_0 > \rho_2$ and is equal to $\rho_0 = 7000\,kg/m^3$.

Analyzing the character of the two-body gravitational field lines distribution (Fig. 8), we can see that the field lines are closed at peculiar points that do not coincide with the bodies' centers of mass. At these points, the total gravitational force acting on the test mass is zero. Hereinafter we will refer to these equilibrium points as *gravitational poles*. The gravitational pole position in each body of the system under consideration depends on the bodies' geometric and physical characteristics as well as on their mutual arrangement and orientation. Table 3 lists the shifts of the gravitational poles from centers of mass obtained by solving set (34).

Table 3. Shift of gravitational poles.

ρ_0, kg/m^3	δ_1, mm	δ_2, mm
0	0.126	0.493
7000	-0.198	-0.250

Note that gravitating masses $\mathcal{G}_1$ and $\mathcal{G}_2$ (Fig. 8(b)) exhibit a distinctive field lines distribution characteristic of an electrical or magnetic dipole. The



(a) Attraction effect.

(b) Repulsion effect, ($\rho_1 > \rho_0 > \rho_2$)

Figure 8. The gravitational field lines structure for a system of two bodies ($\mathcal{G}_1$ and $\mathcal{G}_2$) in the medium ρ_0 in density for $\rho_1 > \rho_2$.

only difference is that having the same character of the field lines distribution as a magnetic dipole or a pair of oppositely charged particles, the gravitating masses exhibit completely opposite character of the physical effect, namely, a pair of material bodies with opposite gravitational poles repulse instead of attracting.

Taking into account the above, we can now formulate a number of axioms defining the *gravitational monopole and dipole* concept:

Axioma 7.1. *A material body of an arbitrary shape and finite volume surrounded by a non-zero density medium exhibits properties of a positive monopole provided the body's average density is higher than the medium density.*

Axioma 7.2. *A material body of an arbitrary shape and finite volume surrounded by a non-zero density medium exhibits properties of a negative monopole provided the body's average density mean density is lower than the medium density.*

Axioma 7.3. *A material body of an arbitrary shape and finite volume located at the interface of two mediums forms a gravitational dipole provided its mean density is lower than that of one of the media and higher than that of the other.*

Axioma 7.4. *Two material bodies of an arbitrary shape and finite volume located in the non-zero density medium form a gravitational dipole provided* *medium density is lower than that of one of the bodies and higher than that of the other.*

8. The gravitating mass in mechanics

8.1. Body "falling" in the Earth's gravitational field

Let us consider a problem of the material body "falling" in the Earth's atmosphere; to say more exactly, let us compare the "falling" times of two bodies and answer the question: *Which of two bodies of identical geometry but different densities "falls" quicker, all other initial conditions being equal?*

Assume that the body's "falling" onto the Earth's surface is the mutual gravitational attraction between the Earth and material body $\mathcal{G}$. Assume that the Earth is a uniform-density sphere with radius $R_\oplus$ and mass $M_\oplus$.

The Earth's gravitational field intensity is $\underline{\mathbf{g}}$. Assume that the Earth remains motionless, and only body $\mathcal{G}$ moves towards the Earth (Fig. 9). In addition, assume that during "falling" the Earth's gravitational field intensity $\underline{\mathbf{g}}$ remains constant both in direction and magnitude:

$$\underline{\mathbf{g}} = -\underline{k}\mathbf{g} \ , \quad \text{where} \quad \mathbf{g} = \mathbf{G}\,\frac{M_\oplus}{R_\oplus^2} \ . \tag{35}$$

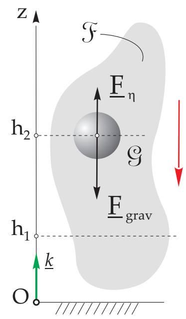


Figure 9.

Let us take as a "falling" body $\mathcal{G}$ a uniform sphere $R_\mathcal{G}$ in radius and $\rho_\mathcal{G}$ in density. The body "falls" in air medium $\mathcal{F}$ characterized by a constant temperature, density $\rho_\mathcal{F}$ and viscosity η. At the initial time moment, the body's initial velocity is zero, and its distance from the Earth's surface is h. In the framework of our task, body $\mathcal{G}$ is subject in "falling" to only two forces: $\underline{F}_{\rm grav}$ that is the force of mutual gravitational attraction of sphere $\mathcal{G}$ and the Earth, and $\underline{F}_\eta$ that is the medium $\mathcal{F}$ resistance force. Therefore, the equation for force $\underline{F}$ that makes the body moving may be written in the following form:

$$\underline{F} = \underline{F}_{\rm grav} + \underline{F}_\eta \ , \tag{36}$$

where

$$\underline{F}_\mathcal{G} = M\ddot{\underline{r}} \ , \quad \underline{F}_{\rm grav} = m\underline{\mathbf{g}} \ , \quad \underline{F}_\eta = \alpha\eta\dot{\underline{r}} \ . \tag{37}$$

Here $\underline{r} = \underline{k}z$ is the radius-vector of the "falling" body position; M and m are the body $\mathcal{G}$ inertial and gravitating masses, respectively; α is the body shape factor.

Further we will not consider the medium resistance force $\underline{F}_\eta$ since our task is to compare the "falling" times of two bodies of identical geometries but different densities in a preset medium under identical initial conditions. Multiplying the left and right sides of equation (36) by ort $\underline{k}$, obtain the motion equation in the $\mathbf{O}z$-axis projection

$$M\ddot{z} = -m\mathbf{g}\ , \quad \text{where} \quad m = M\left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)\ . \tag{38}$$

According to this problem definition, the main parameters are the medium $\mathcal{F}$ and body $\mathcal{G}$ densities. Let us solve equation (38) under the following initial conditions:

$$\dot{z}\Big|_{t=0} = 0\ , \quad z\Big|_{t=0} = h_2\ . \tag{39}$$

Integrating (39), obtain:

$$\dot{z} = -\left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)\mathbf{g}\,t\ , \quad z = -\frac{1}{2}\left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)\mathbf{g}\,t^2 + h_2\ . \tag{40}$$

Herefrom find the time of body "falling" from altitude h_2 to mark h_1 in the Earth's gravitational field as a function of medium density $\rho_{\mathcal{F}}$ and average density $\rho_{\mathcal{G}}$ of the "falling" body:

$$t = \sqrt{\frac{2(h_2 - h_1)}{\left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)\mathbf{g}}} \tag{41}$$

In the limit case, when the medium density $\rho_{\mathcal{F}}$ tends to zero, the "falling" time will be minimal, i.e.,

$$t_{min} = \lim_{\rho_{\mathcal{F}} \to 0} t = \sqrt{\frac{2(h_2 - h_1)}{\mathbf{g}}}\ . \tag{42}$$

Equation (41) also shows that as the medium density $\rho_{\mathcal{F}}$ approaches the body's average density $\rho_{\mathcal{G}}$, "falling" time t increases and becomes infinite when the densities become equal; this is the case of the body $\mathcal{G}$ "suspension" in medium $\mathcal{F}$, namely, the case of the body's zero buoyancy.

Now, using (41), answer the above question: which of two "falling" bodies of identical geometries but different densities will be the first that reaches mark h_1 ? From (41) it follows that, when $\rho_{\mathcal{G}_1} > \rho_{\mathcal{G}_2}$, inequality $t_{\mathcal{G}_1} < t_{\mathcal{G}_2}$ is valid and vice versa. Thus, we can conclude that when two geometrically identical bodies "fall" in a non-zero density medium in the Earth's gravitational field in the absence of other forces, the body of a higher density always "falls" quicker.

Table 4. Body $\mathcal{G}$ "falling" time in medium $\mathcal{F}$.

	vacuum $\rho_{\mathcal{F}} = 0\,kg/m^3$	**air** $\rho_{\mathcal{F}} = 1.225\,kg/m^3$	**water** $\rho_{\mathcal{F}} = 1000\,kg/m^3$
plumbum $\rho_{\mathcal{G}} = 11336\,kg/m^3$	$0.45127\,s$	$0.45129\,s$	$0.47259\,s$
ironwood $\rho_{\mathcal{G}} = 1170\,kg/m^3$	$0.45127\,s$	$0.45150\,s$	$1.18386\,s$

Calculate the "falling" time of a spherical body $R_{\mathcal{G}} = 5\,mm$ in radius and $\rho_{\mathcal{G}}$ in density from altitude $h_2 = 1\,m$ to mark $h_1 = 0\,m$ in the medium $\rho_{\mathcal{F}}$ in density. The calculations for various combinations of the body and medium densities are listed in Table 4. We can see that in the non-zero density medium the heavier body will be the first to "fall", since its gravitating mass is greater.

Now it is reasonable to make a note on the frequently used concept "free falling" with respect to a body "falling" onto the Earth's surface. This conventional expression contains a semantic error and contradicts the physics of the observed process. Does "free falling" really exist? The answer is obvious: "No". The body moves forcedly due to its gravitational interaction with the Earth.

8.2. Pendulum motion equation

This case will be considered in order to demonstrate the role of the gravitating and inertial masses in the pendulum oscillation. Consider the pendulum oscillatory motion (Fig. 10) in the gravitational field of constant intensity $\underline{\mathbf{g}}$ taking into account that the process takes place in medium $\mathcal{F}$ with density $\rho_{\mathcal{F}} > 0$. Here we ignore the medium viscosity since our goal is to demonstrate the medium $\mathcal{F}$ density effect on the pendulum oscillation period. In this case, the balance of moments may be expressed as:

$$J\ddot{\varphi} = -L\,F_{\text{grav}} \sin\varphi\ , \qquad (43)$$

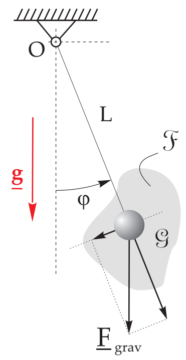


Figure 10.

where

$$F_{\text{grav}} = m\mathbf{g}\ , \qquad J = ML^2\ , \quad M = \rho_{\mathcal{G}} V_{\mathcal{G}}\ .$$

Here M and m are the inertial and gravitating masses of sphere $\mathcal{G}$ represented by expression (22); L is the length of a massless rigid suspension; $\rho_{\mathcal{G}}$ is the sphere $\mathcal{G}$ average density. Therefore, the pendulum oscillation equation gets the following form:

$$\ddot{\varphi} + \left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right) \frac{\mathbf{g}}{L} \sin\varphi = 0 \; . \tag{44}$$

The equation shows that the oscillation process remains active also when the medium density $\rho_{\mathcal{F}}$ exceeds the body density $\rho_{\mathcal{G}}$. However, in this case we obtain the inverse pendulum configuration. If the condition of equal densities is fulfilled, oscillation cannot take place, namely, the pendulum remains in the state of equilibrium at any preset initial angle φ.

8.3. Body equilibrium at the interface of two media

Consider the interface of two semi-infinite media mutually balanced in the Earth's gravitational field: water and atmosphere. The gravitational field vector $\underline{\mathbf{g}}$ is directed perpendicular to the water surface that is just regarded as the media interface. Let a wooden cube to be in the statically equilibrium state, being partially submerged in water (Fig. 11). Let us estimate the depth it is submerged to. It is known that the air density (the medium above the water surface) is $\rho_{air} = 1.29\,kg/m^3$, water density is $\rho_{water} = 1000\,kg/m^3$, and the wooden (oak) cube density is $\rho_{cube} = 800\,kg/m^3$. The cube edge length is $a = 1\,m$. The media interface conditionally divides the cube into two parts.

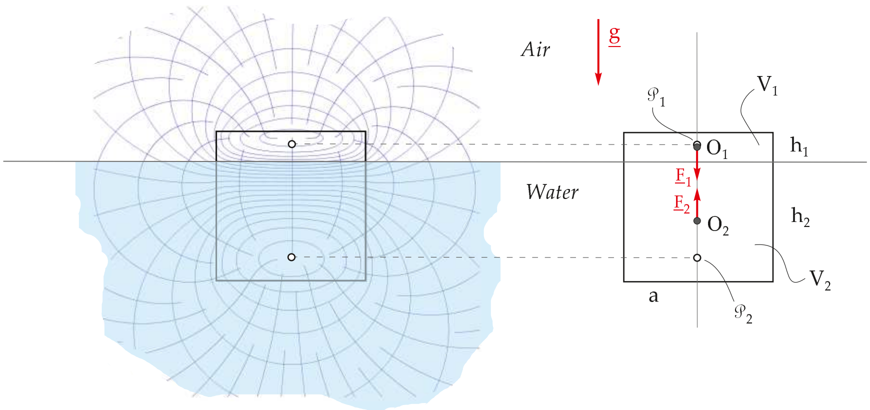


Figure 11. Body equilibrium at the two media interface.

One of those parts is regarded as an anomaly in air, the other is an anomaly in water. Each anomaly is exposed to the Earth's gravitational field. Since the cube is in equilibrium, this means that the sum of gravitational forces acting on its parts is zero. Taking into account that the Earth's gravitational field vector $\underline{\mathbf{g}}$ has the same direction in air and water, we can write that

$$\underline{F}_1 + \underline{F}_2 = 0 \;, \quad \text{or} \quad (m_1{+}m_2)\,\underline{\mathbf{g}} = 0 \;.$$

From this relation, obtain the equilibrium condition: the sum of gravitating masses of the cube above-water and sub-water parts is zero:

$$m_1{+}m_2 = 0 \;, \tag{45}$$

where

$$m_1 = \overbrace{\rho_{\text{cube}} V_1}^{M_1} \left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{cube}}}\right) \;, \quad m_2 = \overbrace{\rho_{\text{cube}} V_2}^{M_2} \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{cube}}}\right) \;.$$

Substituting gravitating masses m_1, m_2 into (45) and taking into account that $V_1 = a^2 h_1$ and $V_2 = a^2 h_2$, obtain a system of equations:

$$h_1(\rho_{\text{cube}} - \rho_{\text{air}}) + h_2(\rho_{\text{cube}} - \rho_{\text{water}}) = 0 \;, \quad h_1 + h_2 = a$$

Hence, the cube submersion depth in water is

$$h_2 = a \,\frac{\rho_{\text{cube}} - \rho_{\text{air}}}{\rho_{\text{water}} - \rho_{\text{air}}} \approx 0.8\, m \;.$$

This example clearly demonstrates the validity of Axiom 7.4 mentioned in page 17.

9. Summary

Considering the motion of material bodies in non-zero density media in the gravitational field in the absence of force factors of other nature, we succeeded in revealing new manifestations of the gravitating mass in the Universe we live in. The basic results of the paper are:

1) Applicability of the *additivity principle* for gravitational interaction between two or more material bodies in the non-zero density medium has been justified.

2) Definition of the *gravitating mass* of a finite-volume material body has been suggested; the definition fully complies with the processes caused by the gravitational interaction, which are observed in Nature.

3) Conditions have been defined, under which the material body or a two-body system forms a *gravitational dipole* whose field lines configuration is inherent in the classical concept of an electrical or magnetic dipole. The only distinctive feature of the gravitational dipole is that opposite-polar gravitating masses repulse, while similarly polar masses attract to each other.

4) The validity of the postulate on the *mass equivalency*, namely, on equality of the material body inertial and gravitating masses, has been analytically proved for the zero-density medium.

The main idea is that, joining the Archimedes law and the Newton's law of universal gravitation, we have revealed the existence in Nature of such a missing manifestation of the gravitational interaction as the effect of repulsion.

The fact that the Universe's matter has to have such a fundamental property as gravitational repulsion (the antithesis to attraction) was pointed to as far back as by Friedrich Engels:

> *"It is commonly accepted that weight is the most general definition of materiality, namely, attraction is the integral property of matter rather than repulsion. However, attraction and repulsion are inseparable from each other to the same extent as positive and negative; dialectically, it can be predicted that the true theory of matter should pay to repulsion the same attention as to attraction; moreover, a theory of matter, which is based only on attraction, shall be false, erroneous, scanty, evasive."* [15, p. 210–211]

References

[1] *Hegel, G.W.F.* Phenomenology of spirit / G.W.F. Hegel. "— Moscow: NAUKA, 2000. "— P. 495.

[2] *Mironov, V. S.* Course of gravimetric prospecting / V. S. Mironov. "— Leningrad: NEDRA, 1980. "— P. 543.

[3] *Torge, W.* Gravimetry / W. Torge. "— Berlin: Walter de Gruyter, 1989. "— P. 465.

[4] *Grushinsky, N. P.* Basic principles of gravimetry / N. P. Grushinsky. "— Moscow: NAUKA, 1983. "— P. 352.

[5] *Zhukovsky, N. E.* Theoretical mechanics / N. E. Zhukovsky. "— 1950. "— P. 812.

[6] *Jammer, Max.* Concepts of Mass : in classical and modern physics / Max Jammer. "— Cambridge, MA: Harvard University Press, 1961. "— P. 230.

[7] *Braginsky, V. B.* Physical experiments with test bodies / V. B. Braginsky. "— NAUKA, 1970. "— P. 136.

[8] *Braginsky, V. B.* Verification of the equivalence of inertial and gravitational mass / V. B. Braginsky, V. I. Panov // *Sov. Phys.-JETP.* "— 1972. "— Vol. 34. "— Pp. 463–76.

[9] *Tian Chen, Y.* Gravitational Experiments in the Laboratory / Y. Tian Chen, Alan Cook. "— Cambridge University Press, 2005. "— July. "— P. 282.

[10] *Kiryan, D. G.* Motion of the Earth's centre of mass. Physical principles / D. G. Kiryan, G. V. Kiryan // *Kinematika i Fizika Nebesnykh Tel Supplement.* "— 2005. "— June. "— Vol. 5. "— Pp. 376–380.

[11] *Kiryan, D. G.* Moon's perigee mass as a missing component of the Earth's precession-nutation theory / D. G. Kiryan, G. V. Kiryan // *ArXiv e-prints (1109.4969v6).* "— 2013. "— June.

[12] *Kiryan, D. G.* On the gravitational mass / D. G. Kiryan, G. V. Kiryan // *International Journal Of Applied And Fundamental Research.* "— 2013. "— Vol. 1. "— `http://www.science-sd.com/452-24059`.

[13] *Dushman, S.* The Scientific Foundations of Vacuum Techniques / S. Dushman. "— John Wiley & Sons, 1949.

[14] *Kikoin, I. K.* Molecular Physics / I. K. Kikoin, A. K. Kikoin. "— Fizmatgiz, 1963. "— P. 500.

[15] *Engels, Friedrich.* Dialectics of Nature / Friedrich Engels. "— Moscow, 1953. "— P. 328.