About Odd Perfect Numbers

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Abstract.

Any odd perfect number is unknown. Simple analysis valid almost for all combinations of odd prime divisors proves that odd numbers constituted of them cannot be perfect.

Following proof covers composite odd numbers with any number of prime divisors >2.
There are three possible cases concerning the power of the smallest divisor.

1) Its power is 1.
2) Its power is odd number >1.
3) Its power is even number.

1.Let $P = abc$ is a positive composite odd integer with a - the smallest of a, b, c prime divisors. It can be expressed as

$$P = (a-1)bc + bc \qquad (1)$$

If P is a perfect number then

$$P = 1 + a + b + c + a(b+c) + bc = (1+a)(1+b+c) + bc \qquad (2)$$

Obviously

$$(a-1)bc = (1+a)(1+b+c) \qquad (3)$$

There are two even numbers $(a-1) = 2^m p$ and $(a+1) = 2^n q$

$2^n q = 2^m p + 2 = 2(2^{m-1}p + 1)$

Here either m or n are equal 1 and p, q are coprime odd numbers, both < a.

Hence q cannot divide $(a-1)bc$

The same conclusion is true for any number of divisors and any power of , $b, c \ldots$ etc when the smallest divisor is to the power 1. For odd positive integer $P = ab^f c^k$ with a - the smallest divisor f, k - positive integers ≥ 1 the pattern

$$P = \sum_{i=1}^{f+k-1} (Q_i + aQ_i) + b^f c^k \qquad (4)$$

shown in Eq. (2) can be applied. Here Q_i is a sum of terms with the same summarized power i of divisors.

As an example let $f = 2, k = 3$. It can be expressed

$$P = (a-1)b^2c^3 + b^2c^3 \qquad (5)$$

$$P = 1 + a + b + c + a(b+c) + b^2 + bc + c^2 + a(b^2 + bc + c^2) + b^2c + bc^2 + c^3 +$$
$$+ a(b^2c + bc^2 + c^3) + b^2c^2 + bc^3 + a(b^2c^2 + bc^3) + b^2c^3 =$$
$$= (1+a)(1 + b + c + b^2 + bc + c^2 + b^2c + bc^2 + c^3 + b^2c^2 + bc^3) + b^2c^3 \qquad (6)$$

Obviously

$$(a-1)b^2c^3 = (1+a)\left(1 + b + c + b^2 + bc + c^2 + b^2c + bc^2 + c^3 + b^2c^2 + bc^3\right) \qquad (7)$$

2. The same pattern is applicable with any odd power of a.

For $P = a^{2k+1}bc$ even number of terms is added to the middle of Eq. (2) and on the right hand side divisor $(1 + a)$ still is kept.

As an example

$$P = a^3bc = (a-1)a^2bc + a^2bc \to 1 + a + b + c + a(b+c) + a^2 + bc + a(a^2 + bc) + \\ + a^2b + a^2c + a(a^2b + a^2c) + a^2bc \quad (8)$$

$$(a-1)a^2bc = (1+a)(1 + b + c + bc + a^2 + a^2b + a^2c) \quad (9)$$

Thus all composite odd numbers with odd power of smallest divisor cannot be perfect.

3.The pattern of Eq. (2) cannot be obtained when the power of smallest divisor is even.

If $P = a^2bc$

$$(a-1)abc + abc \to 1 + a + b + c + a(b+c) + bc + a^2 + a^2b + a^2c + abc \quad (10)$$

$$(a-1)abc = (1 + a + a^2)(1 + b + c) + bc \quad (11)$$

The foregoing method of proof cannot be applied to Eq. (11).