

Link between Special relativity and general relativity

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Abstract

No doubt, there is a continuity of physical laws. Here I show a link between general and special relativity and especially a kind of gravitational trace or signature in a moving frame, means a memorising of initial process.

Let R' be an orthonormal frame in uniform translation with speed \vec{V} along Ox with respect to a referential R . We can consider that R' has instantly gone from the speed $V = 0$ to $V \neq 0$ at certain time $t = 0$ which imply bearing an acceleration, $\vec{\gamma} = \vec{V}\delta(\tau)$ where τ is R' proper time. At this time R' is subject equivalently to gravitational field ϕ .

Let m be a fixed mass in R , it is view as accelerating one from an observer of R' , which means bearing a force $\vec{f} = -m\vec{\gamma} = -m\vec{V}\delta(\tau)$, which force is equivalently an expression of ϕ .

It then develops a power $\frac{dw}{d\tau} = \vec{f} \cdot \vec{v}$ where \vec{v} is its velocity in R' and w is its mechanical energy, so

$$\frac{dw}{d\tau} = \vec{f} \cdot \vec{v} = -m\vec{V} \cdot \vec{v}(\tau)\delta(\tau) = -m\vec{V} \cdot \vec{v}(0)\delta(\tau) = mV^2\delta(\tau) \quad \text{for} \quad \vec{v}(\tau)\delta(\tau) = \vec{v}(0)\delta(\tau) = -\vec{V}\delta(\tau)$$

According to energy conservation principle,

$$\frac{dw}{d\tau} = -m \frac{d\phi}{d\tau} \Rightarrow \frac{d\phi}{d\tau} = -V^2\delta(\tau) \quad \text{then for any } \tau > 0 \quad \frac{d\phi}{d\tau} = 0 \quad \text{or} \quad \phi = C\tau = \phi^+$$

But things are the same for the observer of R' if the frame has gone opposite direction, in ox^- sense. This case is simply equivalent to inverse time and put $-\vec{V}$ in place of \vec{V} then

$$\frac{d\phi(-\tau)}{d(-\tau)} = -(-V)^2\delta(-\tau) = -V^2\delta(\tau) = \frac{d\phi(\tau)}{d\tau} \Rightarrow \text{for any } \tau < 0 \quad \phi = C\tau = \phi^-.$$

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Elsewhere

$$\frac{d\varphi(-\tau)}{d(-\tau)} = -\frac{d\varphi(-\tau)}{d\tau} = \frac{d\varphi(\tau)}{d\tau} \Rightarrow \frac{d(\varphi(\tau)+\varphi(-\tau))}{d\tau} = 0 \Rightarrow \text{for any } \tau \quad \varphi(\tau) + \varphi(-\tau) = 0 \Rightarrow \varphi^+ = -\varphi^- = \varphi$$

$$\text{Conclusion} \begin{cases} \varphi(\tau) = +\varphi & \text{for } \tau > 0 \\ \varphi(\tau) = -\varphi & \text{for } \tau < 0 \end{cases}$$

$$\lim_{\varepsilon \rightarrow 0} \int_{-\varepsilon}^{+\varepsilon} d\varphi = \lim_{\varepsilon \rightarrow 0} \int_{-\varepsilon}^{+\varepsilon} -V^2 \delta(\tau) d\tau \Leftrightarrow 2\varphi = -V^2 \quad \text{i.e. an uniform gravitational field } \varphi .$$

We learn from general relativity that for uniform and weak field .

$$\tau = \frac{x_0}{c} \sqrt{1 + \frac{2\varphi}{c^2}} = t \sqrt{1 - \frac{V^2}{c^2}}$$

This is the exact formula from special relativity of the relation between reference time and proper time.