An Electrostatic Analogue for the Novel Temporary Magnetic Remanence Thermodynamic Cycles

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Abstract

This paper follows on from the exposition of the new temporary magnetic remanence cycles and looks at the similarities and subtle differences in the electrostatic analogue. Only the electromagnetics is analysed as the kinetic theory and thermodynamic analysis is almost exactly the same as the magnetic case.

1. Introduction

The author initially conducted enquiry into ferrofluids; these materials display temporary magnetic remanence[1]. This phenomenon lead to the development of thermodynamic cycles[2-4] by the author. Indeed the claims of violation of the 2nd Law of Thermodynamics in one of the papers seems to be reflected elsewhere[5] and it has become necessary to look at the general principle regarding electrostatic devices and whether there is any similarity with magnetostatic devices due to the near symmetry or duality with Maxwell's Classical Electrodynamic laws[6-8]:

$$\begin{aligned} &\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \\ &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &\nabla \cdot \mathbf{B} = 0 \\ &c^2 \nabla \times \mathbf{B} = \frac{\mathbf{j}}{\varepsilon_0} + \frac{\partial \mathbf{E}}{\partial t} \\ &\begin{pmatrix} \text{with the Lorentz force} \\ \mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \end{pmatrix} \end{aligned} \qquad \text{eqn. 1}$$

These equations are the so-called microscopic form of the laws and are always classically true, however in matter, a distinction is made between "free" and "bound" charges:

$$\rho = \rho_{bound} + \rho_{free} \qquad \text{eqn. 2}$$

And we define the "Polarisation field", **P**,

$$\rho_{free} = -\nabla \cdot \mathbf{P} \qquad \text{eqn. 3}$$

Where the polarisation vector represents the field from the charges inside the material that are polarised by the external electrical field:

$$\mathbf{P} = \varepsilon_0 \varepsilon_{r.xyz} \mathbf{E} \qquad \text{eqn. 4}$$

The relative permittivity $\varepsilon_{r.xyz}$ is written as a tensor such that the induced polarisation does not have to be collinear to the electric field. We write the "Displacement field", D:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \text{eqn. 5}$$

Substituting this concept into the first Maxwell equation yields,

$$\nabla \cdot \mathbf{D} = \rho_{free} \qquad \text{eqn. 6}$$

Similarly with the currents (and hence current density when considered over the same surface area) the same procedure can be followed:

$$\mathbf{j} = \mathbf{j}_{bound} + \mathbf{j}_{free} \qquad \text{eqn. 7}$$

The bound current is composed of "Ampèrian currents" from the spins of the magnetic material[6, 9-11], the "magnetisation", **M** *and* the displacement current due to the changing polarisation:

$$\mathbf{j}_{bound} = \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t} \qquad \text{eqn. 8}$$

Similarly to eqn. 4, a constitutive relationship between the induced polarised vector and the agent causing the polarisation is formed:

$$\mathbf{M} = \chi_{xyz} \mathbf{H} \qquad \text{eqn. 9}$$

However in this case the "magnetising field" **H** is considered fundamental (it isn't, there is only "B" field[6]) and this field is related to the free current:

$$\mathbf{H} = \mathbf{j}_{free} \qquad \text{eqn. 10}$$

Such that for magnetostatics we can write (from eqn. 7 and the static case of eqn. 1.3),

$$\mathbf{B} = \frac{1}{\varepsilon_0 c^2}\left(\mathbf{H} + \mathbf{M}\right) \equiv \mu_0 \left(\mathbf{H} + \mathbf{M}\right) \qquad \text{eqn. 11}$$

Which then leads to the way some of the literature writes the 4th Maxwell equation:

$$\varepsilon_0 c^2 \nabla\times\mathbf{B} = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\Rightarrow \cancel{\varepsilon_0 c^2}\nabla\times\left[\frac{1}{\cancel{\varepsilon_0 c^2}}\left(\mathbf{H} \cancel{+\mathbf{M}}\right)\right] = \mathbf{j}_{free} + \cancel{\nabla\times\mathbf{M}} + \frac{\partial \mathbf{P}}{\partial t} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\Rightarrow \nabla\times\mathbf{H} = \mathbf{j}_{free} + \frac{\partial \mathbf{D}}{\partial t} \qquad \text{eqn. 12}$$

Finally, the material response to the magnetising field (eqn. 9) is often hidden in a relative permeability term, thus:

$$\mathbf{B} = \mu_0\left(\mathbf{H} + \chi\mathbf{H}\right)$$

And,

$$\mu_r = 1 + \chi \qquad \text{eqn. 13}$$

The macroscopic Maxwell equations are quoted thus:

$$\begin{aligned}
&\nabla\cdot\mathbf{D} = \rho_{free}\\
&\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\\
&\nabla\cdot\mathbf{B} = 0\\
&\nabla\times\mathbf{H} = \mathbf{j}_{free} + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned} \qquad \text{eqn. 14}$$

These truthfully aren't "fundamental" for the hidden complexity of the extra **D** and **H** variables. However material characteristics in the form of **M** and **P** are introduced and we can begin to analyse the subtle differences between the magnetic thermo-cycle and the electrical variant.

2. The Electrical State Equations for the Magnetic Temporary Remanence Cycle

Previously[1, 2] it was shown that the magnetisation in the magnetic thermo-cycle very accurately followed a 1st-order differential equation:

$$\frac{dM}{dt} = -\frac{1}{\tau}\left(M - \chi\mu_r H\right) \qquad \text{eqn. 15}$$

The H-field is related to the magnetising current thus,

$$\frac{dM}{dt} = -\frac{1}{\tau}\left(M - \chi\mu_r \frac{N}{D} i\right) \qquad \text{eqn. 16}$$

A slight complication of the equation above is the inclusion of a "co-material" with high permeability μ_r to make the magnetising current smaller.

The author[2] analysed the dynamics and energetics of eqn. 15 in relation to the work done magnetising, the magnetisation energy, the magnetisation loss and the ultimate fate of the magnetisation energy, given that a temporary flux decays. Briefly, if the magnetisation process occurs at a rate substantially slower than the relaxation rate τ, then nearly all of the work of magnetisation will end up as magnetisation energy. If the process is much faster than τ, all the magnetisation work will end up as internal energy with no magnetic moment resulting. Similarly, if the magnetised material is disconnected from external circuitry, then neglecting radiative losses, the magnetisation energy gets converted to internal energy at a rate a function of dM/dt.

3. The "H-Field Cancellation" Method

The basis of the thermo-magnetic heat engine previously considered by the author[2, 3] was the conversion of heat energy directly into electricity. The mechanism was seen by Kinetic Theory to involve the micro-mechanical disruption of the dipoles that constitute the magnetisation, which by generator action (Faraday's Law, eqn. 1.2) did "dipole-work" Md*M* (*M* is the volume magnetisation). Dipole-work can be considered "micro-mechanical shaftwork" which makes the electrical system an *open system*. It directly lowers the internal energy of the magnetised material:

$$dU = TdS + \mu_0 H d\mathcal{M} + \mu_0 M d\mathcal{M} \qquad \text{eqn. 17}$$

The aim was then to make the dipole-work greater than the input magnetisation energy in a cycle:

$$\oint M d\mathcal{M} > \oint H d\mathcal{M}$$

This was shown to be impossible with a simple resistive load as only part of the magnetisation energy is returned. Cognition of this effect came by seeing the current waveforms induced into the electrical load: As the load resistance decreased, the circuit time constant became slower and slower, with a concomitant slowing in the rate of magnetisation decay. That is, the electrical work dumped into the load was not some simple function of τ,

$$W = \int_0^\infty \frac{V^2}{R} dt \Rightarrow \propto \int_0^\infty \frac{\left(\frac{dB}{dt}\right)^2}{R} dt \Rightarrow \propto \int_0^\infty \frac{\left(\frac{dM}{dt}\right)^2}{R} dt$$

Because τ was increasing. It was realised that the induced current was re-magnetising the working substance. This is the term $\chi\mu_r H$ in eqn. 15. A scheme was contrived to eliminate this re-magnetising H-field by superposition of a generated field at high frequency (so that the resultant is effectively invisible to the working substance) and this resulted in eqn. 15 being rendered:

$$\frac{dM}{dt} = -\frac{M}{\tau} \qquad \text{eqn. 18}$$

The energetics and method to do this showed that this was possible[2]. A graph of electrical work against 1/R for one cycle yielded the following graph:

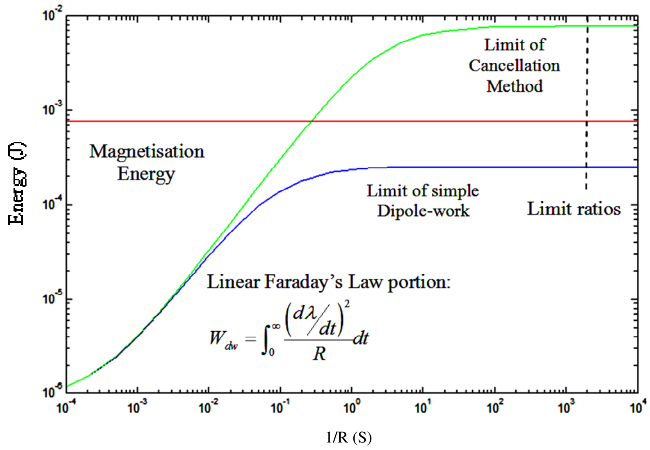

Figure 1

It can be seen that the simple dipole work into a resistive load never exceeds the magnetisation energy input (this is not due to losses, it simply reflects a flaw in the energy coupling process). The upper trace plateaus at the field energy of magnetic working substance that is,

$$W = \frac{1}{2}\varepsilon_0 c^2 \int_V B^2 dV \equiv \frac{1}{2}\mu_0 \int_V M^2 dV \qquad \text{eqn. 19}$$

The difference between this work, W and the magnetisation input energy represents heat energy converted to electrical energy.

4. A device based on Electrical Remanence

There is an electrostatic analogue of temporary magnetic remanence and it would seem a simple matter to start from the dual of eqn. 15:

$$\frac{dP}{dt} = -\frac{1}{\tau}\left(P - \chi_e \varepsilon_0 \varepsilon_r E\right) \qquad \text{eqn. 20}$$

In constructing the electrostatic dual of the temporary remanence cycle, there are subtle similarities and differences. Both involve a charging and discharging phase: one with magnetic flux and an energy cost of the magnetising energy, the other an electric/polarisation flux and the energy cost of the polarisation energy. Both too would seem to have a "lossy" tank where this input energy is converted to internal energy at the rate a function of: $\frac{d\phi}{dt} = -\frac{\phi}{\tau}$.

However, there is no such analogue of the Faraday law eqn. 1.2 and changes in electrical flux cannot directly cause the generation of currents by an electromotive force – there can be no dipole-work term PdP similar to Md*M* (eqn. 17). Let us explore this. From the 1st Maxwell equation/Gauss' Law:

$$\nabla \cdot \left(E + \frac{P}{\varepsilon_0}\right) = \frac{\rho_{free}}{\varepsilon_0} \qquad \text{eqn. 21}$$

And hence,

$$\oint\!\!\oint_A \left(E + \frac{P}{\varepsilon_0}\right) \cdot dA = \frac{Q_{free}}{\varepsilon_0} \qquad \text{eqn. 22}$$

This represents a combination of the electric field at the plates of the capacitor and the electric field from the polarisation. The movement of the free charges is the circuit current, thus:

$$A\left(\frac{dE}{dt} + \frac{1}{\varepsilon_0}\frac{dP}{dt}\right) = \frac{1}{\varepsilon_0}\frac{di}{dt} \qquad \text{eqn. 23}$$

Multiplying both sides the voltage across the plates, i.e. $v = \int_d E \cdot dl$ yields,

$$Ed \cdot A\left(\frac{dE}{dt} + \frac{1}{\varepsilon_0}\frac{dP}{dt}\right) = \frac{1}{\varepsilon_0} v \frac{di}{dt}$$

Which upon integration w.r.t. time,

$$\Rightarrow \left(\varepsilon_0 EdE + EdP\right)V = vi \qquad \text{eqn. 24}$$

This is just seen to be the differential electrostatic work EdD and the instantaneous electrical power; its magnetic dual is HdB. If we neglect the polarising and the magnetic field energy (they are recoverable around a cycle), we have the usual work input terms:

$$EdP \cdot dV$$
$$and$$
$$HdM \cdot dV$$

In the first case, we do work on a lossy capacitor that seeks to turn this work into internal energy but we have no dipole-work term MdM, as in the second case with the magnetic system to surmount our losses.

Furthermore there can be no dual to the H-field cancellation method. No de-polarisation cancelling method can be made to strike out the term $\chi\varepsilon_0\varepsilon_r E$, when we realise that the potential across the load resistor is negative and acts to increase the rate of decay further. This only reflects energy leaving the capacitor "tank" (in competition to that being converted to heat), as it should.

In short, the difference between the electrical and magnetic systems is that, a circuit cannot be made with magnetic charges which could deliver electrical energy as electric charges do; they simply don't exist (eqn. 1.3). To make the electrostatic analogue requires an intermediate step of generation of temporary magnetic field from the collapse of the electric polarisation field and this then can generate electric current. This, of course, is an allusion to the displacement current term $\frac{\partial \mathbf{E}}{\partial t}$ of the fourth Maxwell equation (eqn. 1.4 or $\frac{\partial \mathbf{D}}{\partial t}$ eqn. 14.4).

The only way a capacitor-based heat engine could work is either:

1) Conventionally: Take the working substance around a cycle exposed to two or more reservoirs such that the polarisation is a function of temperature. The changes in entropy and heat flow relate to the electrostatic work:

$$\oint TdS = \oint EdP(T) \qquad \text{eqn. 25}$$

2) Unconventionally I: For devices purporting to deliver work from one thermodynamic reservoir[5], the electrical system must be *open*, that is externally generated charges must impinge on the electrical system.

3) Unconventionally II: Electrostatic dipole work must act via the intermediary of a magnetic field to induce a current, as discussed previously. This system, although electrically *closed* is thermodynamically *open*, as case 2.

The discussion shall continue with the third case and the displacement current. We use the fourth Maxwell equation with a dielectric so the current density term is left out:

$$c^2\nabla \times B = \frac{\partial E}{\partial t} \qquad \text{eqn. 26}$$

The electric field results from the polarisation along the x-axis (figure 2), which is the axis of the capacitor, so we can write:

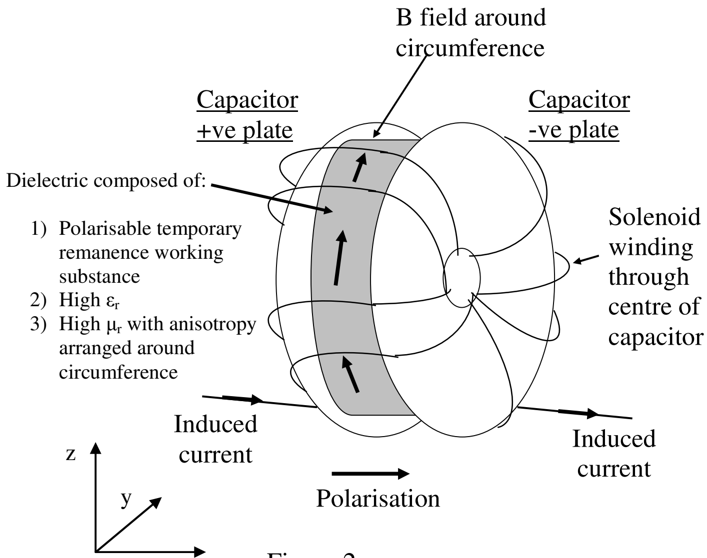

Figure 2

$$c^2\nabla \times B = \varepsilon_r \frac{\partial P_x}{\partial t}\mathbf{i} \qquad \text{eqn. 27}$$

Where the curl operator in this context can only have components in the yz plane and a co-material ε_r has been included to boost the field. For simplicity we shall consider cylindrical symmetry and we know that the B-field will circulate around the changing P_x vector. Using Stoke's Identity to relate the line integral of the curl of B to the surface integral of the flux from P, we find:

$$c^2\int_0^r B\cdot d(2\pi l) = \int_0^r \varepsilon_r \frac{\partial P_x}{\partial t}\cdot d(\pi l^2)$$

Thus the temporary independent magnetic flux is:

$$B_{yz}(t) = \varepsilon_r \frac{r}{c^2}\frac{\partial P_x(t)}{\partial t} \qquad \text{eqn. 28}$$

This B-field is itself changing and will lead to E_{temp} (eqn. 29) and so on, as a series in powers of $1/c^2$, so we safely truncate it to first order in $1/c^2$. The E-field is given by Maxwell's 2nd equation:

$$\nabla \times E_{temp} = -\frac{\partial B_{yz}(t)}{\partial t} \qquad \text{eqn. 29}$$

Which we know from Stoke's Identity will lead to an E-field perpendicular to the plane yz, that is, in the anti- x axis direction, increasing with magnitude with the radius (our line integral path is an axially aligned loop through the centre of the capacitor, see figure 2):

$$v_{temp} = \oint_{path} E_{temp}\cdot dl = -\frac{\partial \lambda}{\partial t} \qquad \text{eqn. 30}$$

The path at the centre contributes nothing, so we can write (V is the volume, n is the turns per unit length):

$$v_{temp} = -\mu_r nV \frac{\varepsilon_r}{c^2} \frac{\partial^2 P_x(t)}{\partial t^2} \qquad \text{eqn. 31}$$

A magnetic co-material μ_r has been included to boost the magnetic field (which must be non-conductive, e.g. ferrite).

The end result of this arrangement is, once again, a temporary magnetic flux. The polarising energy of the electret has been transferred to magnetising energy of the ferrite. This seems to accrue no benefit, until we realise that the H-field cancellation technique can applied, to liberate dipole-work in excess of the magnetising energy. The difference represents heat energy of the electret being converted into electrical energy.

5. Conclusion

The Maxwell Equations have a certain amount of symmetry which leads to the concept of electrical duality. However slight differences exist and this is manifest in making a temporary electrical remenance cycle as a dual to temporary magnetic remenance cycle. What has been found, for the former case, is that an intermediary step of generation of a temporary magnetic field is required. Once this has been achieved, the same techniques of the H-field cancellation method[2, 3] are applied as to the latter method.

This paper hopefully explains the misconception about the devices that the system is closed and akin to filling and emptying a tank – either with polarisation energy or with magnetisation energy. It has been shown that electrical dipole work PdP can only return the polarisation energy, EdP. However magnetic dipole work MdM can, via the H-field cancellation method return more than the input magnetisation energy. In this case, the Brownian disruption of both the magnetic case (and the correctly configured electrical case) constitutes micro-mechanical shaftwork and an open system.

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