The Wave Medium and Special Relativity

Gary D. Simpson

email: gsim100887@aol.com

rev00 Jun 16, 2014

Abstract

A novel hypothesis concerning motion through the vacuum is presented. The hypothesis relates the complex time exponential of Quantum Mechanics (QM) to absolute motion through a wave medium. The hypothesis is combined with Special Relativity (SR). The two most significant results from this are that this combination produces a term that is consistent with angular momentum and that the direction associated with time is the direction of motion through the vacuum. These relations are presented in Equations 12.0/12.1 and Equation 4 respectively. Equation 12.4 then describes the electron-positron. These are a direct explanation for wave-particle duality and an explanation for the "arrow of time", thereby eliminating time as a fourth dimension. The hypothesis predicts a distinction between the emission and the absorption of energy. The hypothesis requires a tangible wave medium. A variation of the Stern-Gerlach experiment is proposed as empirical verification.

Preface

A general familiarity with Special Relativity is recommended. Knowledge of quaternions and vectors is required to understand the mathematics, although the text can be largely understood without the mathematics. The author recommends an initial reading for overall meaning, then a second reading, if desired, to focus on mathematical details. Please note that, for brevity, a generic **R** is used to represent the ratio between various quaternions.

"*Measure what is measureable, and make measureable what is not so*." - Galileo Galilee

Summary

Prior to beginning, the author will state clearly that this text is neither a criticism of Einstein nor a rejection of Special Relativity. The objective of this text is to review Special Relativity in consideration of a novel definition of motion through a wave medium. The quotation above from Galileo was selected because the hypothesis presented here makes it possible to "make measureable what is not so" in reference to motion through the vacuum. The author will rely heavily upon Einstein's 1905 paper titled "On the Electrodynamics of Moving Bodies". An English translation thereof can be found in reference [1]. The author will also reference a less technical text by Einstein presented in reference [2]. Wherever possible, the author will cite Einstein's own words and reasoning.

The hypothesis presented below in Equation 1 is a quaternion. Therefore, the quaternion mathematics of Hamilton is used throughout this text. Special Relativity is interpreted within this mathematical framework. To the best of his ability, the author has remained true to Einstein's intentions subject to the limits imposed by Hamilton. Of the equations presented, the most pertinent are: Equations 1, 4, 11, 12.0, 12.1, and 12.4.

One of the reasons for the rejection of the aether by Physics is that it is not required. Special Relativity does not require aether in order to work. Also, experimental attempts to measure motion with respect to the aether have been unsuccessful. Therefore, Occam's Razor implies that there is no aether. However, the author will present evidence that a tangible wave medium adds the following to the current understanding of Physics:

1. It explains QM spin. Absolute motion and QM spin are seen to be mathematically similar. A single transform (see Equation 12.0 and Equation 12.1) produces both linear momentum and angular momentum. This incorporates rotation into the inertial reference frame. This transform has dual configuration. One arrangement describes QM. The other arrangement describes SR. Eigen-values produce quantized values.
2. It requires a difference between absorption and emission of energy, and it thereby explains wave-particle duality. Energy is emitted as waves and absorbed as photons. Absorption occurs in the direction of motion. The act of emission or absorption is mathematically equivalent to multiplication of the transform by **i**.
3. It produces the total relativistic energy as part of the transform solutions.
4. It eliminates time as a fourth dimension. The direction associated with time is the direction of motion through the vacuum.
5. It suggests a conservation-type extension of the Pauli Exclusion Principle that encompasses both fermions and bosons.

This text is fairly lengthy. Therefore, the author will provide a brief synopsis of each section. The sections are presented in the order required for comprehension.

"Background" – briefly review SR, definition for simultaneity, definition for speed of light.

"Opportunities" - photon momentum, spin, vacuum energy, wave-particle duality vs matter and energy.

"Hypothesis" – present hypothesis, define relativistic phase angle, develop and present SR transforms.

"Vector Form" - present generalized method of producing Euler's Equation.

"Reference Frame" – apply SR transforms to a single reference frame, speculate regarding interpretation for energy emission and absorption, eliminate time as dimension, develop QM transforms, develop comparisons between SR and QM transforms.

"Collinear Motion" - develop transform for two collinear reference frames, present special cases.

"Non-Collinear Motion" - develop transform for two non-collinear reference frames, develop Eigen-value problem, develop QM/SR dual equation, relate absolute motion to angular momentum, develop scalar expression.

"Spin" - summarize circumstantial evidence, generalize Pauli Exclusion Principle.

"Electron-Positron" - present predecessor relation, apply Eigen-value solutions to produce electron-positron solution.

"Stern-Gerlach" - present proposed variation of experiment to test hypothesis.

Discussion

Background:

> "The introduction of a 'luminiferous ether' will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place." - Albert Einstein

The above quotation was taken from the end of the second paragraph of reference [1]. The portion of the quote with which the author of the present text is concerned is "nor assign a velocity-vector to a point of the empty space". The present text will focus upon how to "assign a velocity-vector to a point of the empty space".

In section I.1 of reference [1], Einstein defines a "stationary system" consisting of two points, A and B, then presents a definition of simultaneity that relies upon the propagation of light. The essence of that definition is that the transit time of light from A to B must equal the transit time of light in the reverse direction from B to A. Stated mathematically, this is:

$$t_B - t_A = t'_A - t_B$$

He also presents:

$$\frac{2AB}{t'_A - t_A} = c$$

The first equation defines synchronization of time between the reference points A and B. The second equation provides a definition for the velocity of light. In the second equation, the term AB should be understood to mean the distance from A to B. By defining "c" in this way, Einstein is defining the "round-trip" velocity as opposed to the "one-way" velocity. By this definition, he has intimately combined light with time. Therefore, the properties associated with light are built into SR. The "round-trip" that Einstein defines requires four events to occur. Light must be emitted. It must be absorbed. It must be re-emitted. And it must be re-absorbed. Therefore, the mechanisms of absorption and emission are also built into SR. These considerations will be revisited below.

Neither idea seems remarkable today, yet the work was considered revolutionary at the time it was published. To understand why, let us consider the prevailing thinking of that time regarding the "luminiferous ether". The key to this understanding is contained in the “rule” for the addition of velocities.

Suppose that points A and B are located on the x-axis and that they are both moving at velocity "v" in the direction of increasing "x". For clarity, let us assume that the "x" coordinate of B is greater than the "x" coordinate of A. If Euclidean geometry is true and if time and space are both absolute, it is easy to show that:

$$(t_B - t_A)\left(1 - \frac{v}{c}\right) = (t'_A - t_B)\left(1 + \frac{v}{c}\right)$$

If a reference frame is moving at velocity "v", then this relationship should describe the effect that motion has upon the transit times of light. After some consideration it becomes apparent, that if "v" and "c" are restricted to real scalar values, then perhaps the only way to reconcile this equation with Einstein's definition of simultaneity in the "stationary system" is for the velocity "v" to equal zero. The obvious comment to make is that "of course v = 0, it is a stationary system". The problem with this thinking is that every "moving" object is stationary in its own frame of reference, and every frame of reference must obey the same physical laws. The essence of Special Relativity is that every inertial frame of reference is equivalent to a rest frame. This realization would force a drastic re-thinking of Physics.

Opportunities:

There are several items in Physics that remain unresolved in the mind of the author. For example, the photon has momentum, but it has no rest mass. Its ability to carry momentum is considered an intrinsic property. Spin is considered to be an intrinsic property. The vacuum contains a residual energy that is an intrinsic property. The electron has the curious ability to be a point particle that has mass. The fact that this requires an infinite density is conveniently over-looked.

Wave-particle duality has been grudgingly accepted, yet mass and energy are considered to be distinct. There is no method to transition smoothly from mass (particle) to energy (wave). This is a serious problem, since the solution of differential equations requires that a function must be smooth and continuous. This text will provide a method of transitioning smoothly between wave and particle. In so doing, the text will also provide insight into QM spin and into the mechanism whereby light carries momentum. It will also hint at the source of the properties of the electron by characterizing the vacuum.

Hypothesis:

> "Moreover, what is meant here by motion 'in space'? From the considerations of the previous section the answer is self-evident. In the first place we entirely shun the vague word 'space', of which, we must honestly acknowledge, we cannot form the slightest conception, and we replace it by 'motion relative to a practically rigid body of reference'." - Albert Einstein

The above quotation was taken from the second paragraph of chapter 3 of reference [2]. This quotation captures the essence of the problem faced by Einstein. How does one measure motion with respect to empty space (i.e., the vacuum)? This is not an easy question. The author thinks that Quantum Mechanics holds the answer. An objective of this text is to provide a conception of "motion in space".

The hypothesis presented here is that the complex time exponential used in Quantum Mechanics can be expressed as follows:

Equation 1:

$$e^{i\alpha ct} = \cos(\alpha ct) + i\sin(\alpha ct) = \sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c}\mathbf{i}$$

This definition incorporates the Lorentz Transform into wave-particle behavior, and it does so in a manner that requires a one-to-one correspondence based upon absolute velocity. Most importantly, this definition allows a smooth transition between wave and particle. The left-hand side is the complex time exponential. The central portion is Euler's Equation. The right-hand side is the hypothesis. The italicized *i* is the complex *i* (i.e., the square root of -1). The bold faced **i** is the vector **i**. In principle, both the sine and cosine terms can be either positive or negative depending upon the value of αct.

The hypothesis is easily quantized by defining a relativistic phase angle ϕ such that:

Equation 1.1:

$$\phi = \sin^{-1}\left(\frac{v}{c}\right) = \alpha ct \pm 2\pi n\,; n = 0, 1, 2, etc.$$

The "luminiferous ether" equation from the previous section for a moving reference frame can be rearranged to produce the following:

$$\frac{(t_B - t_A)}{(t'_A - t_B)} = \frac{\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)}; v \neq c$$

Einstein's definition of simultaneity is then satisfied if:

$$\frac{(t_B - t_A)}{(t'_A - t_B)} = \frac{\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)} = 1; v \neq c$$

This equation does not present the simultaneity problem any differently. However, in view of the above hypothesis, the author proposes to revise this equation as follows:

Equation 2:

$$\frac{(t_B - t_A)}{(t'_A - t_B)} = \mathbf{R} = \frac{\left(1 + \frac{v}{c}\mathbf{i}\right)}{\left(1 - \frac{v}{c}\mathbf{i}\right)} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

Here **R** is the quaternion ratio produced by the division. One of the benefits of this formulation is that it is no longer necessary to state v ≠ c. This is solved in Appendix A with the solution being:

Equation 3.1:

$$\mathbf{R} = \left[\frac{1 - \left(\frac{v^2}{c^2}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right] + \left[\frac{2\left(\frac{v}{c}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right]\mathbf{i}$$

Next, let us suppose that **R*** is defined as follows:

$$\mathbf{R}^* = \frac{\left(1 - \frac{v}{c}\mathbf{i}\right)}{\left(1 + \frac{v}{c}\mathbf{i}\right)} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

Here, the direction of motion of the reference frame has been reversed, but the coordinate system has not changed.

This problem is also solved in Appendix A with the solution being:

Equation 3.2:

$$\mathbf{R}^* = \left[\frac{1 - \left(\frac{v^2}{c^2}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right] - \left[\frac{2\left(\frac{v}{c}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right]\mathbf{i}$$

It should be noted that these two solutions have the same scalar term and are complex conjugates. If the reader prefers, the r_0 and r_i terms can be rewritten slightly as:

$$r_0 = \frac{c^2 - v^2}{c^2 + v^2}$$

$$r_i = \pm\frac{2vc}{c^2 + v^2}$$

Vector Form:

The purpose of this section is to show how simple it is to relate electro-magnetism to the hypothesis presented in Equation 1. In a previous work[3], the author presented a method to formulate Euler's Equation in terms of two arbitrary vectors, provided those two vectors are restricted to the **j**-**k** plane. It is also possible to formulate Euler's Equation in terms of quaternions based upon those vectors. These formulations are as follows:

$$e^{\mathbf{i}\theta} = \frac{1}{|\mathbf{a}||\mathbf{b}|}(\mathbf{a}\cdot\mathbf{b} + \mathbf{a}\times\mathbf{b});\ a_i = b_i = 0$$

$$e^{\mathbf{i}\theta} = \frac{1}{|\mathbf{a}||\mathbf{b}|}[\mathbf{AB} - (a_0 b_0 - 2\mathbf{a}\cdot\mathbf{b} + a_0\mathbf{b} + b_0\mathbf{a})]\,;\ a_i = b_i = 0$$

In these formulations, the vectors **a** and **b** and the quaternions **A** and **B** do not correspond with the reference point A and B presented by Einstein. The task of this section is to determine vectors **a** and **b** and quaternions **A** and **B** that will produce the hypothesis presented above concerning the complex time exponential. It is easy to show that the following vectors satisfy this requirement:

$$\mathbf{a} = \sqrt{1 - \frac{v^2}{c^2}}\mathbf{j} - \frac{v}{c}\mathbf{k}$$

$$\mathbf{b} = \sqrt{1 - \frac{v^2}{c^2}}\mathbf{j}$$

Now suppose that these arbitrary vectors **a** and **b** are replaced by the **e** vector and the **b** vector of electro-magnetism. The vector form of Euler's Equation can then be written as:

$$e^{\mathbf{i}\theta} = \frac{1}{|\mathbf{e}||\mathbf{b}|}(\mathbf{e}\cdot\mathbf{b} + \mathbf{e}\times\mathbf{b});\ e_i = 0;\ b_i = 0$$

and the quaternion form of Euler's Equation can be written as:

$$e^{\mathbf{i}\theta} = \frac{1}{|\mathbf{e}||\mathbf{b}|}[\mathbf{EB} - (e_0 b_0 - 2\mathbf{e}\cdot\mathbf{b} + e_0\mathbf{b} + b_0\mathbf{e})]\,;\ e_i = 0;\ b_i = 0$$

In these equations, the reader must be aware that the "e" on the left-hand side is the natural logarithm "e" whereas the "**e**" on the right-hand side is the electric field. Typically, the dot product of the "**e**" vector and the "**b**" vector is zero. *The author has kept this term because he thinks that this term is necessary to recognize that the vacuum is the aether*.

Reference Frame:

Now let us consider the special case of light travelling at "c" (i.e., v = c). The two transforms presented above in Equation 3.1 and Equation 3.2 simplify to +**i** and -**i** respectively. When this is combined with Einstein's definition for time synchronization, the implication is as follows:

$$\mathbf{R} = \frac{(t_B - t_A)}{(t'_A - t_B)} = \pm\mathbf{i}$$

Equation 4:

$$(t_B - t_A) = \pm(t'_A - t_B)\mathbf{i}$$

Equation 4 is very interesting. It loosely satisfies Einstein's definition of synchronization. It does not change the increment between the time measurements. Instead, it gives time a direction for one-half of the round-trip. That direction coincides with the direction of absolute motion through the vacuum. Therefore, it is not necessary to consider time as a separate fourth dimension as is so frequently stated. The author interprets Equation 4 to mean that outgoing light is a scalar wave and that incoming light is a photon (i.e., a vector). Therefore, the processes of emission and absorption of energy are subtly different. Energy is emitted as a wave. Energy is absorbed as a vector. Energy absorption occurs in the direction of motion. Transfer of momentum does not occur until absorption occurs, and the acts of emission and absorption are mathematically equivalent to multiplication by the vector **i**. Therefore, the four events required by Einstein's definition for time synchronization have a net effect of $\mathbf{i}^4 = 1$. This also suggests to the author that the act of measurement does not make a wave-function collapse as is so frequently claimed. The ± symbol is included since the unit vector **i** can be in the direction of motion (+) or opposite the direction of motion (-). This will be clarified later in this text in the section titled "**Collinear Motion**". The author thinks that this accounts for the direction of the "arrow of time".

Prior to proceeding, the author will ask the reader a few rhetorical questions. Suppose that the universe is filled with scalar waves that are travelling in all directions and that have been emitted by countless sources. What are the necessary conditions for an observer to absorb one of these waves? Stated differently, is every observer capable of absorbing the photon associated with each of these waves? If the answer to this is "no", then what does that imply regarding the observable universe and more importantly, the un-observable universe?

The value of the complex time exponential (i.e., the hypothesis) for a true rest frame (i.e., v = 0) is the scalar value one. The essence of Special Relativity is that every inertial reference frame can be treated like a rest frame. Therefore, if the hypothesis can be transformed into the scalar value one, then it should be generally compatible with the various transformations of Special Relativity. This is easily done by multiplying the complex time exponential by its complex conjugate. Mathematically, this is represented as follows:

$$1 = \left[\sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c}\mathbf{i}\right]\left[\sqrt{1 - \frac{v^2}{c^2}} - \frac{v}{c}\mathbf{i}\right]$$

The hypothesis can also be converted into a scalar one by dividing it by itself.

$$1 = \frac{\left[\sqrt{1-\frac{v^2}{c^2}}+\frac{v}{c}\mathbf{i}\right]}{\left[\sqrt{1-\frac{v^2}{c^2}}+\frac{v}{c}\mathbf{i}\right]}$$

If these two relations are multiplied by each other, the following is produced:

$$1 = \left[\sqrt{1-\frac{v^2}{c^2}}+\frac{v}{c}\mathbf{i}\right]^2 \frac{\left[\sqrt{1-\frac{v^2}{c^2}}-\frac{v}{c}\mathbf{i}\right]}{\left[\sqrt{1-\frac{v^2}{c^2}}+\frac{v}{c}\mathbf{i}\right]}$$

This suggests that the following transformation might be significant:

$$\mathbf{R} = \frac{\left[\sqrt{1-\frac{v^2}{c^2}}-\frac{v}{c}\mathbf{i}\right]}{\left[\sqrt{1-\frac{v^2}{c^2}}+\frac{v}{c}\mathbf{i}\right]} = r_0 + r_i\mathbf{i}$$

Based upon Appendix A, the solution to this is:

$$r_i = \left(\frac{a_i(1+a_0)}{1+a_i^2}\right) = \frac{-\frac{v}{c}\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{1+\frac{v^2}{c^2}} = \frac{-vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}$$

$$r_0 = \frac{a_0 - a_i^2}{1+a_i^2} = \frac{\sqrt{1-\frac{v^2}{c^2}}-\frac{v^2}{c^2}}{1+\frac{v^2}{c^2}} = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}-v^2}{c^2+v^2}$$

Equation 5:

$$\mathbf{R} = \left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}}-v^2}{c^2+v^2}\right] \pm \left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right]\mathbf{i}$$

A ± symbol was included for the vector portion to include the transform if the denominator and numerator are transposed (i.e., the complex conjugate).

A reasonable question to ask at this point is "How do these transforms (i.e., Equations 3.1, 3.2, and 5) compare to each other?".

Equations 6 and 7 are simply restatements of Equations 3.1/3.2 and Equation 5 respectively. The subscripts SR and QM are used here to designate Special Relativity and Quantum Mechanics respectively.

Equation 6:

$$\mathbf{R}_{\mathrm{SR}} = \left[\frac{c^2 - v^2}{c^2 + v^2}\right] \pm \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}$$

For v = 0, $\mathbf{R}_{SR}$ = 1.

For v = c, $\mathbf{R}_{SR}$ = ±**i**.

Equation 7:

$$\mathbf{R}_{\mathrm{QM}} = \left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] \pm \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}$$

For v = 0, $\mathbf{R}_{QM}$ = 1.

For v = c, $\mathbf{R}_{QM}$ = -(1/2) ± (1/2)**i**.

These two (or four if the ± terms are considered) ratios can be related to each other as a difference or as a ratio. Both methods are solved in Appendix B and Appendix C respectively. The difference is defined as follows with the solutions being:

$$\mathbf{X} = \mathbf{R}_{\mathrm{SR}} - \mathbf{R}_{\mathrm{QM}}$$

Equation 8.1:

$$\mathbf{X} = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)(c \pm v\mathbf{i})$$

and

Equation 8.2:

$$\mathbf{X} = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\left(c \pm v\left[\frac{\left(3 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}\right]\mathbf{i}\right)$$

For v = 0, Equation 8.1 and 8.2 both simplify **X** = 0.

For v = c, Equations 8.1 and 8.2 simplify to

$$\mathbf{X} = \frac{1}{2}(1 \pm \mathbf{i}) = \frac{1}{2} \pm \frac{1}{2}\mathbf{i}$$

and

$$\mathbf{X} = \frac{1}{2}(1 \pm 3\mathbf{i}) = \frac{1}{2} \pm \frac{3}{2}\mathbf{i}$$

respectively.

The ratio is defined as follows with the solutions being:

$$\mathbf{R} = \frac{\mathbf{R}_{\text{SR}}}{\mathbf{R}_{\text{QM}}}$$

Equation 9.1:

$$\mathbf{R} = \left[\frac{(c^2 - v^2)\cos\theta + 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] \pm \left[\frac{-(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

Equation 9.2:

$$\mathbf{R} = \left[\frac{(c^2 - v^2)\cos\theta - 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] \pm \left[\frac{(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

For v = 0, Equations 9.1 and 9.2 both simplify to **R** = 1.

For v = c, Equations 9.1 and 9.2 simplify to the following:

$$\mathbf{R} = \pm 1 \pm \mathbf{i}$$

Therefore, for a rest-frame, these transforms make no change (i.e., **X** = 0 and **R** = 1). For a light-frame, things are more complicated. The vector portion of **R** is ±**i**, just as it was for Equations 3.1 and 3.2. Therefore, the reasoning associated with time and Equation 4 should remain valid.

When Equations 9.1 and 9.2 are developed in Appendix C, the term ($c^4 + v^2c^2$) appears and is presented in Equation C.7. The author thinks this is very significant since this is similar to the total relativistic energy term presented in Special Relativity. This term is then used as the basis for the cosine and sine terms presented in Equations 9.1 and 9.2. The reader is encouraged to review Appendix C. The main goal of doing so is to see how simply the problem can be structured as a system of simultaneous equations.

Collinear Motion:

Thus far, the author has only discussed a single reference frame in motion. Special Relativity was dedicated to examining the motion between reference frames. Therefore, relative motion is the next logical topic. Fortunately, the transformations have already been developed above.

The two reference frames will be defined as follows:

Equation 10.1:

$$\mathbf{R}_A = \sqrt{1 - \frac{v_i^2}{c^2}} + \frac{v_i}{c}\mathbf{i}$$

and

Equation 10.2:

$$\mathbf{R}_B = \sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{(v_i + \Delta v_i)}{c}\mathbf{i}$$

The transform relating them is then:

$$\mathbf{R} = \frac{\mathbf{R}_B}{\mathbf{R}_A} = \frac{\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{(v_i + \Delta v_i)}{c}\mathbf{i}}{\sqrt{1 - \frac{v_i^2}{c^2}} + \frac{v_i}{c}\mathbf{i}} = r_0 + r_i\mathbf{i}$$

This is solved in Appendix D with the solution being:

Equation 11:

$$\mathbf{R} = \cos(\phi_A - \phi_B) + \sin(\phi_A + \phi_B)\,\mathbf{i}$$

Here, ϕ_A and ϕ_B are the relativistic phase angles associated with reference frames A and B as defined by Equation 1.1. The reader is strongly encouraged to review Appendix D. The author thinks that Equation 11 is "largely consistent" with Special Relativity. For a small relative velocity, the cosine term is approximately one. As the absolute velocity of either reference frame is increased, the sine term becomes more significant and wave-particle behavior is observed.

The author qualified the statement above by using the term "largely consistent". What does this mean? Let us consider some examples. In Special Relativity, it does not matter if A is moving toward B or B is moving toward A. All that matters is the relative velocity between the two. In the hypothesis presented here, there is a distinction between these two cases, but it is subtle. Consider the cosine term. An identity of trigonometry is that cos(x) = cos(-x). Therefore, for the hypothesis, the two cases (i.e., A→B or B→A) result in the same value for the cosine. This is not true for the sine term. Another identity of trigonometry is sin(x) = -sin(-x). Also, Equation 11 uses the sum of the phase angles to determine the sine. The hypothesis will disagree with Special Relativity in situations where the sine term is significant. The author thinks that the sine term holds the key to resolving the various SR paradoxes such as the infamous "Twin Paradox".

Let us now briefly consider two special cases such that reference frames A and B have equal velocity magnitudes. In case one, let us assume they are moving in the same direction. In case two, let us assume that they are moving in opposite directions. For these cases, Equation 11 simplifies to [1+sin(2ϕ)**i**] for case one and to cos(2ϕ) for case two. The implication of the sine form of this is that if two reference frames are moving together then they "see" each other as a mixture of wave and particle. Wave-particle duality is a direct consequence of this interpretation. This sine form is also consistent with Equation 2. The implication of the cosine form of this is that if two reference frames are moving away from each other at equal magnitudes, then they "see" each other as scalar waves. The cosine form of these is consistent with pair production.

Next, let us consider two more special cases where a rest-frame and a light-frame "see" each other. The relativistic phase angles for these two reference frames are 0 and π/2 respectively. The direction of motion for the light-frame is first taken in the +**i** direction. The cosine term here is cos(±π/2) = 0. The sine term here is sin(π/2) = 1. In both of these cases, Equation 11 simplifies to +**i**. A light-frame and a rest-frame "see" each other in an identical fashion. If the direction of motion of the light-frame is in the –**i** direction, then Equation 11 simplifies to –**i** for both cases. Again, a light-frame and a rest-frame "see" each other in an identical fashion. The author thinks that this, combined with Equation 4, completely accounts for the "arrow of time", and that it is completely consistent with Einstein's definition of time synchronization in Special Relativity.

Further, two light-frames will "see" each other as scalar waves. If they are moving in the same direction then cos(0) = 1 and sin(π) = 0. Therefore, they "see" each other as scalar waves. If they are moving in opposite directions, then cos(π) = -1 and sin(0) = 0. Therefore, they "see" each other as opposite scalar waves.

Non-Collinear Motion:

Non-collinear motion is conceptually similar to collinear motion but the mathematical details are more difficult. Special Relativity does not provide a distinction between collinear and non-collinear motion. Therefore, a comparison between SR and the hypothesis will not be possible. Instead, the author will present a connection with Dirac.

Consider two reference frames A and B moving together through space such that their velocity vectors are as follows:

$$\boldsymbol{v}_A = v_i\mathbf{i}$$

$$\boldsymbol{v}_B = (v_i + \Delta v_i)\mathbf{i} + v_j\mathbf{j} + v_k\mathbf{k}$$

A bold-faced lower case letter such as "**v**" is used to designate a vector.

The mental image that the author is using here is the Sun-Earth system with the Sun being reference frame A and the Earth being reference frame B.

The transform for this system is:

$$\mathbf{R} = \frac{\sqrt{1 - \frac{(v_i + \Delta v_i)^2 + v_j^2 + v_k^2}{c^2}} + \frac{v_i + \Delta v_i}{c}\mathbf{i} + \frac{v_j}{c}\mathbf{j} + \frac{v_k}{c}\mathbf{k}}{\sqrt{1 - \frac{v_i^2}{c^2}} + \frac{v_i}{c}\mathbf{i}} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

This is solved in Appendix E with the solution being:

Equation 12.0:

$$\mathbf{R} = \left(\frac{a}{\gamma} + b_i b\right) + \left(ab_i + \frac{b}{\gamma}\right)\mathbf{i} + \left(ab_j + bb_k\right)\mathbf{j} + \left(ab_k + bb_j\right)\mathbf{k}$$

The new terms are defined as follows:

$$Let\ b = \frac{v_i}{c};\ Let\ a = \sqrt{1 - \frac{v_i^2}{c^2}} = \sqrt{1 - b^2}$$

$$Let\ b_i = \frac{v_i + \Delta v_i}{c}; Let\ b_j = \frac{v_j}{c}; Let\ b_k = \frac{v_k}{c}; Let\ \beta^2 = b_i^2 + b_j^2 + b_k^2; Let\ \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

What is interesting regarding this transform is that motion in the **j** direction produces an effect in the **k** direction and motion in the **k** direction produces an effect in the **j** direction. However, both effects are dependent upon motion in the **i** direction. The reader is encouraged to review Appendix E. What is noteworthy is that the system consists of four simultaneous equations but they are solved in pairs. Also, the matrix formulation is insightful. It is very similar to the matrix used to solve the Dirac Wave Equation.

Now let us consider Equation 12.0 in some detail. Specifically, let us consider the **j** term and the **k** term. If these terms are both zero, then Equation 12.0 will simplify to:

$$\mathbf{R} = \left(\frac{a}{\gamma} + b_i b\right) + \left(ab_i + \frac{b}{\gamma}\right)\mathbf{i}$$

If the **j** term and the **k** term are both zero, then the following two equations must be true:

$$ab_j + bb_k = 0$$

$$ab_k + bb_j = 0$$

This is then represented in matrix form as:

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} b_j \\ b_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Homogeneous problems like this always have the trivial solution (i.e., $b_j = b_k = 0$) as a solution. There is also a possibility for non-trivial solutions. This is a problem solvable by using Eigen-values. The primary requirement of this method is that the determinant of the coefficient matrix must be zero. Therefore, ($a^2 - b^2 = 0$). The result of this is that the following must be true:

$$\frac{v_i}{c} = \pm\frac{1}{\sqrt{2}}; therefore\ a = \pm\frac{1}{\sqrt{2}}\ and\ b = \pm\frac{1}{\sqrt{2}}$$

The Eigen-value problem is expressed as follows:

$$(a - \lambda)^2 - b^2 = 0$$

This can be expanded and solved using the Quadratic Equation with the Eigen-values being:

$$\lambda = a \pm b = \pm\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}};\ \lambda = 0, \pm\sqrt{2}$$

The values for b_k and b_j are now:

Equation 12.0.1:

$$b_j = 0, \pm\sqrt{2};\ b_k = 0, \pm\sqrt{2};\ \frac{v_i}{c} = \pm\frac{1}{\sqrt{2}}$$

The next step was completely unforeseen. Equation 12.0 can be rearranged as follows:

Equation 12.1:

$$\mathbf{R} = a\left(\frac{1}{\gamma} + b_i\mathbf{i} + b_j\mathbf{j} + b_k\mathbf{k}\right) + b\left(b_i + \frac{1}{\gamma}\mathbf{i} + b_k\mathbf{j} + b_j\mathbf{k}\right)$$

The author is aware of two types of momentum (vectors) in Physics. These are linear momentum and angular momentum. The first group of terms on the right-hand side of Equation 12.1 can be converted into linear momentum plus a scalar term by multiplying by mc. Therefore, the author suspects that the second group of terms is a representation of angular momentum and perhaps Quantum Mechanical Spin. If the second group of terms is also multiplied by mc, it resembles angular momentum divided by distance. The necessary distance term can be incorporated by multiplying by cΔt. This converts the "b" term into a distance. Therefore, multiplying Equation 12.1 by $mc^2\Delta t$ will produce cΔt multiplied by linear momentum plus angular momentum. **The distance used in the angular momentum calculation is the**

distance traveled in the i direction over time increment Δt. The author interprets this in Figure 1 at the end of the main text. The author thinks that QM Spin is the limit as Δt→0. Equation 12.1 can be interpreted as a combination of Special Relativity with Mach's Principle.

There is something very interesting regarding Equations 12.0 and 12.1. Equation 12.0 is the time portion of a solution to the wave equations. It is applicable to the microscopic realm of Quantum Mechanics. Equation 12.1 is applicable to the macroscopic world of Special Relativity. Yet both equations consist of the same terms. The terms are simply rearranged.

Special Relativity is used to describe a non-rotating, inertial frame of reference. The author's focus here is upon the term "non-rotating". Physics has been largely unable to incorporate rotation into a reference frame. This has been a problem for several hundred years. Equation 12.1 accomplishes this. It does so, however, in a manner whereby linear momentum and angular momentum **are not** independent. The scalar terms can be made to cancel each other as follows:

Equation 12.1.1:

$$\frac{a}{\gamma} + bb_i = 0$$

Including this requirement makes it **appear** that linear momentum and angular momentum are independent. The author thinks that Equation 12.1.1 is the key to comprehension of absolute motion through the vacuum.

If the Eigen-value solutions are incorporated into Equation 12.1, the results are:

Equation 12.1.2: (λ = 0)

$$\mathbf{R} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i}\right)$$

and

Equation 12.1.3: (λ = ±sqrt2)

$$\mathbf{R} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i} \pm \sqrt{2}\mathbf{j} \pm \sqrt{2}\mathbf{k}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i} \pm \sqrt{2}\mathbf{j} \pm \sqrt{2}\mathbf{k}\right)$$

Spin:

The author thinks that Quantum Mechanical Spin is a manifestation of absolute motion through the wave medium. The author also thinks that Equation 6 is applicable to bosons and that Equation 7 is applicable to fermions. Consider the following pieces of circumstantial evidence regarding Equations 6 and 7:

1. For v = 0, the scalar terms are both one and the vector terms are both zero. Therefore, both transforms agree for a true rest frame.

2. For v = c, the vector term for SR is ±1 and for QM it is ±(1/2). These vector terms are consistent with the spin of the photon (boson) and the spin of the fermion respectively.

3. For v = c, the scalar term for SR is zero and for QM it is -(1/2). The scalar term is a little more subtle than the vector term. One of the properties of bosons is that any number of them can share the same space. This is consistent with a scalar term equal to zero, since zero added to zero equals zero. One of the properties of fermions is the Pauli Exclusion Principle. Only two electrons can share the same space. If the -(1/2) scalar value for QM is multiplied by two, the result is -1. This -1 scalar value exactly offsets the +1 scalar value associated with the reference frame. This behavior of bosons and fermions leads the author to think that there is a Principle more general than the Pauli Exclusion Principle. The Principle has two parts. The first part is that the reference frame is itself a particle or a wave-function. The second part is that the sum of all the scalar terms for all the particles or wave-functions associated with that reference frame - including the reference frame itself - must equal zero. This is a conservation law. This is consistent with using Equation 12.1.1 to set the scalar term to zero.

Electron-Positron:

Now let us again consider Equation 12.1. In previous works[4,5], the author developed the following relationship for an electron:

Equation 12.2:

$$m_E = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\mathbf{i}\right]$$

This was the equation that led to the hypothesis presented in Equation 1. Therefore, Equation 12.1 must be consistent with this predecessor relation. This equation was produced by solving the Schrödinger Wave Equation and the classical wave equation using spherical coordinates. This equation was based upon a simple model in one dimension, plus time. Setting v_j = v_k = 0 and Δv_i = 0 in Equation 12.1 produces the following:

$$\mathbf{R} = a\left(\frac{1}{\gamma} + b\mathbf{i}\right) + b\left(b + \frac{1}{\gamma}\mathbf{i}\right) = (a^2 + ab\mathbf{i}) + (b^2 + ba\mathbf{i}) = a^2 + b^2 + 2ab\mathbf{i}$$

$$\mathbf{R} = 1 + 2ab\mathbf{i}$$

The objective now is to multiply this expression for **R** by a quaternion **M** to produce Equation 12.2.

$$\mathbf{M}(1 + 2ab\mathbf{i}) = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right][a + b\mathbf{i}]$$

This is solved in Appendix F with the solution being:

Equation 12.3:

$$\mathbf{M} = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left\{\left[\frac{\cos(\phi) + \sin(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}\right] + \left[\frac{\sin(\phi) - \cos(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}\right]\mathbf{i}\right\}$$

Here, the ϕ term is as defined in Equation 1.1. It is the relativistic phase angle of the observer. The cosines and sines are as implied by Equation 1.

The next task is to apply Equation 12.3 for **M** to Equation 12.1.2 & 12.1.3 for **R**. This is done in Appendix G with the result being:

Equation 12.4:

$$\mathbf{MR} = \pm K\left[\mathrm{M_O}\left(\pm\frac{1}{\gamma} \pm b_i\right)(1 + \mathbf{i}) + \left(An_j - Bn_k\right)\mathbf{j} + \left(An_k + Bn_j\right)\mathbf{k}\right];\ n_j = 0, \pm 2;\ n_k = 0, \pm 2$$

where:

$$K = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]$$

$$A = \frac{\cos(\phi) + \sin(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}$$

$$B = \frac{\sin(\phi) - \cos(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}$$

$$\mathrm{M_O} = \frac{\cos(\phi) + \sin(\phi) + [\sin(\phi) - \cos(\phi)]\sin(2\phi)}{1 + sin^2(2\phi)} = A + B$$

The author thinks that Equation 12.4 is representative of the electron and positron. Equation 12.4 is as close as the author can come to matching the accepted solutions to the Dirac Wave Equation. Not surprisingly, it does not exactly match Dirac since those solutions are based upon matrices. The author will note that Equation 12.1.1 is qualitatively similar to the Dirac Spinors. Regarding the **j** and **k** terms in Equation 12.4, the author will note that the coefficients are produced by the following matrix multiplication:

$$\begin{bmatrix} A & -B \\ B & A \end{bmatrix}\begin{bmatrix} n_j \\ n_k \end{bmatrix} = \begin{bmatrix} \left(An_j - Bn_k\right) \\ \left(An_k + Bn_j\right) \end{bmatrix}$$

This is also similar to the methodology of Dirac.

Stern-Gerlach Experiment:

One of the first experimental observations regarding quantized spin was the Stern-Gerlach experiment. In that experiment, heated silver ions were discharged from a furnace and passed through a non-homogeneous magnetic field. The ions then struck a plate oriented perpendicular to the path of motion.

What was observed was that the ions clustered around two distinct points. This was interpreted to mean that the silver ions had angular momentum that corresponded with either up spin or down spin. More modern variations of the experiment are performed using electron beams. The author will note that there are three spin states predicted by Equation 12.4 for the **j** and **k** directions. These are zero and ±2. To the best of the author's knowledge, the zero spin state is not physically observed.

The author has presented an argument that spin and absolute motion are mathematically similar. If this argument is true, then the Stern-Gerlach experiment should be affected by motion. The author proposes that the experiment be performed using an apparatus that is moving with respect to the Earth. The predicted effect is that the separation between the split beams will vary with motion. The author thinks that the separation will be proportional to the velocity. It is possible that this effect can be observed as a result of variation in the Earth's motion about the Sun. The main uncertainty associated with this Sun-Earth proposal is that the velocity vector for absolute motion of the Sun-Earth system is unknown. In previous work[4,5], the author has presented evidence that the Sun-Earth system is travelling at 0.006136 c in a direction parallel to the axis of the Sun.

Conclusions

A hypothesis is presented regarding absolute motion through the vacuum. When combined with Special Relativity, the hypothesis explains wave-particle duality, energy emission and absorption, and the “arrow of time”. Most significantly, the combination links absolute motion to angular momentum and spin.

Acknowledgements

The author thanks Karoly Kehrer, Susan Barlow, and the rest of the crew at the WSM users group at Yahoo. The author is also grateful to viXra.org.

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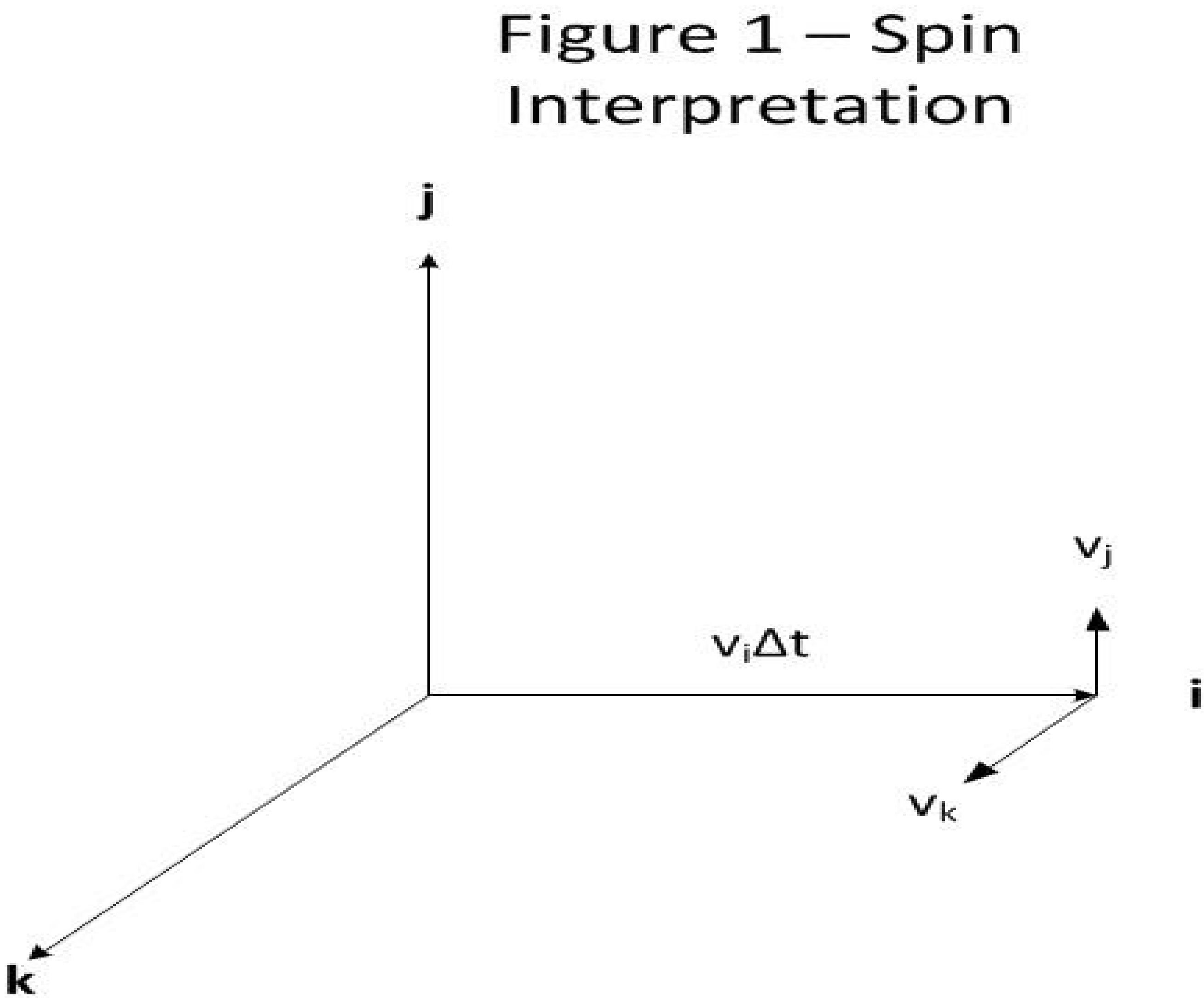

Appendix A

Begin by restating the equation to solve.

$$\mathbf{R} = \frac{\left(1 + \frac{v}{c}\mathbf{i}\right)}{\left(1 - \frac{v}{c}\mathbf{i}\right)} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

For the sake of generality, rewrite the above as follows:

$$\mathbf{R} = \frac{(a_0 + a_i\mathbf{i})}{(a_0 - a_i\mathbf{i})} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

Multiply both sides by the denominator.

$$(a_0 + a_i\mathbf{i}) = \left(r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}\right)(a_0 - a_i\mathbf{i})$$

Expand the right-hand side.

$$(a_0 + a_i\mathbf{i}) = \left(r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}\right)a_0 - \left(r_0\mathbf{i} - r_i - r_j\mathbf{k} + r_k\mathbf{j}\right)a_i$$

This produces the following four equations:

scalar:

$$a_0 = r_0 + a_i r_i$$

i:

$$a_i = r_i - a_i r_0$$

j:

$$0 = a_0 r_j - a_i r_k$$

k:

$$0 = a_0 r_k + a_i r_j$$

Solve the scalar and the **i** equations for r_0 and set them equal to each other.

$$a_0 - a_i r_i = r_0 = -(a_i - r_i)\frac{1}{a_i}$$

Solve for r_i.

$$a_0 - a_i r_i = -\left(1 - \frac{1}{a_i} r_i\right)$$

$$a_0 - a_i r_i = -1 + \frac{1}{a_i} r_i$$

$$1 + a_0 = \left(a_i + \frac{1}{a_i}\right) r_i$$

$$r_i = \left(\frac{1 + a_0}{a_i + \frac{1}{a_i}}\right)$$

Multiply by a_i/a_i to prevent the $1/a_i$ term from having division by zero error. Then rearrange slightly.

$$r_i = \left(\frac{a_i(1 + a_0)}{1 + a_i^2}\right)$$

Substitute r_i into scalar equation for r_0.

$$a_0 - a_i r_i = r_0 = a_0 - a_i^2 \left(\frac{1 + a_0}{1 + a_i^2}\right)$$

Multiply the a_0 term by $(1 + a_i^2)/(1 + a_i^2)$ and simplify.

$$r_0 = \frac{a_0\left(1 + a_i^2\right)}{1 + a_i^2} - a_i^2 \left(\frac{1 + a_0}{1 + a_i^2}\right) = \frac{a_0 - a_i^2}{1 + a_i^2}$$

Next, we must attempt to solve for r_j and r_k. Solve the **j** and **k** equations for r_j and set them equal to each other.

$$r_k \frac{a_i}{a_0} = r_j = -r_k \frac{a_0}{a_i}$$

If the coefficients a_0 and a_i are restricted to real numbers, then the author will accept this to mean that the solution is $r_j = r_k = 0$. However, this will be revisited in future work. The implication is that somehow, $a_i/a_0 = -a_0/a_i$. It is possible for this to be true if both a_0 and a_i are matrices rather than real numbers.

Therefore, **R** is expressed as follows:

$$\frac{(a_0 + a_i\mathbf{i})}{(a_0 - a_i\mathbf{i})} = \mathbf{R} = \left[\frac{a_0 - a_i^2}{1 + a_i^2}\right] + \left[\frac{a_i(1 + a_0)}{1 + a_i^2}\right]\mathbf{i}$$

The specific cases referenced from the main text are then satisfied by setting $a_0 = 1$ and $a_i = \pm(v/c)$. This produces the following:

$$\mathbf{R} = \left[\frac{1 - \left(\frac{v^2}{c^2}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right] \pm \left[\frac{2\left(\frac{v}{c}\right)}{1 + \left(\frac{v^2}{c^2}\right)}\right]\mathbf{i}$$

Appendix B

Begin by restating the quaternion ratios:

$$\mathbf{R}_{\mathrm{SR}} = \left[\frac{c^2 - v^2}{c^2 + v^2}\right] \pm \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}$$

$$\mathbf{R}_{\mathrm{QM}} = \left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] \pm \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}$$

The objective of this Appendix is to solve the following relation:

$$\mathbf{X} = \mathbf{R}_{\mathrm{SR}} - \mathbf{R}_{\mathrm{QM}}$$

Due to the ± terms in the $\mathbf{R}_{\mathrm{SR}}$ and $\mathbf{R}_{\mathrm{QM}}$ relations, there are potentially four solutions to the above. These will be designated as $\mathbf{X}_1$ through $\mathbf{X}_4$ respectively. The work sequence will be (++), (+-),(-+), and (--).

$\mathbf{X}_1$: (++)

$$\mathbf{X}_1 = \mathbf{R}_{SR} - \mathbf{R}_{QM} = \left\{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}\right\} - \left\{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_1 = \left\{\left[\frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] + \left[\frac{vc\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_1 = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)(c + v\mathbf{i})$$

The reason for factoring this as shown will be clear below.

$\mathbf{X}_2$: (+-)

$$\mathbf{X}_2 = \mathbf{R}_{SR} - \mathbf{R}_{QM} = \left\{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}\right\} - \left\{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_2 = \left\{\left[\frac{c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right] + \left[\frac{vc\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_2 = \left(\frac{c}{c^2+v^2}\right)\left\{\left[c\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\right] + \left[v\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)\right]\mathbf{i}\right\}$$

$$\mathbf{X}_2 = \left(\frac{c}{c^2+v^2}\right)\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\left(c+v\left[\frac{\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)}{\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)}\right]\mathbf{i}\right)$$

The author has factored this in this way to make $\mathbf{X}_2$ similar to $\mathbf{X}_1$. The denominator for the vector portion will not cause a division by zero error because it is multiplied by the same group of terms.

$\mathbf{X}_3$: (-+)

$$\mathbf{X}_3 = \mathbf{R}_{SR} - \mathbf{R}_{QM} = \left\{\left[\frac{c^2-v^2}{c^2+v^2}\right] - \left[\frac{2vc}{c^2+v^2}\right]\mathbf{i}\right\} - \left\{\left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}}-v^2}{c^2+v^2}\right] + \left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_3 = \left\{\left[\frac{c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right] - \left[\frac{vc\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_3 = \left(\frac{c}{c^2+v^2}\right)\left\{\left[c\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\right] - \left[v\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)\right]\mathbf{i}\right\}$$

$$\mathbf{X}_3 = \left(\frac{c}{c^2+v^2}\right)\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\left(c-v\left[\frac{\left(3+\sqrt{1-\frac{v^2}{c^2}}\right)}{\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)}\right]\mathbf{i}\right)$$

$\mathbf{X}_4$: (--)

$$\mathbf{X}_4 = \mathbf{R}_{SR} - \mathbf{R}_{QM} = \left\{\left[\frac{c^2-v^2}{c^2+v^2}\right] - \left[\frac{2vc}{c^2+v^2}\right]\mathbf{i}\right\} - \left\{\left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}}-v^2}{c^2+v^2}\right] - \left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2+v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_4 = \left\{\left[\frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] - \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}\right\} - \left\{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_4 = \left\{\left[\frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] + \left[\frac{vc\left(-1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_4 = \left\{\left[\frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] - \left[\frac{vc\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

$$\mathbf{X}_4 = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)(c - v\mathbf{i})$$

Summary for **X**:

$$\mathbf{X} = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)(c \pm v\mathbf{i})$$

and

$$\mathbf{X} = \left(\frac{c}{c^2 + v^2}\right)\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\left(c \pm v\left[\frac{\left(3 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}\right]\mathbf{i}\right)$$

For v = 0, the above all simplify to **X** = 0.

For v = c, the above simplify to:

$$\mathbf{X} = \frac{1}{2}(1 \pm \mathbf{i})\ and\ \mathbf{X} = \frac{1}{2}(1 \pm 3\mathbf{i})$$

Appendix C

Begin by restating the quaternion ratios:

$$\mathbf{R}_{\mathrm{SR}} = \left[\frac{c^2 - v^2}{c^2 + v^2}\right] \pm \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}$$

$$\mathbf{R}_{\mathrm{QM}} = \left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] \pm \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}$$

The objective of this Appendix is to solve the following relation:

$$\mathbf{R} = \frac{\mathbf{R}_{\mathrm{SR}}}{\mathbf{R}_{\mathrm{QM}}}$$

Due to the ± terms in the $\mathbf{R}_{\mathrm{SR}}$ and $\mathbf{R}_{\mathrm{QM}}$ relations, there are potentially four solutions to the above. These will be designated in this Appendix as $\mathbf{R}_1$ through $\mathbf{R}_4$ respectively. The work sequence will be (++), (+-), (-+), and (--).

$\mathbf{R}_1$: (++)

$$\mathbf{R}_1 = \frac{\mathbf{R}_{\mathrm{SR}}}{\mathbf{R}_{\mathrm{QM}}} = \frac{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}}{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}} = r_0 + r_i\mathbf{i}$$

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i} = (r_0 + r_i\mathbf{i})\left\{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right\}$$

Setting the scalar terms to be equal and the vector terms to be equal produces the following two equations:

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] = r_0\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - r_i\left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]$$

and

$$\left[\frac{2vc}{c^2 + v^2}\right] = r_0\left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] + r_i\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right]$$

It should be possible to determine a unique solution since there are two equations and two unknown values. The first obvious step is to multiply both equations by the common denominator term. This produces

$$c^2 - v^2 = r_0\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] - r_i\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

and

$$2vc = r_0\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right] + r_i\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

The next step will simplify the appearance of these equations. Define coefficients "a" and "b" as follows:

Equation C.1:

$$Let\ a = \left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

Equation C.2:

$$Let\ b = \left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

Substituting these coefficients back into the equations to solve makes them less intimidating.

Equation C.3:

$$c^2 - v^2 = ar_0 - br_i$$

and

Equation C.4:

$$2vc = br_0 + ar_i$$

These equations are easily solved by using the Method of Determinants. The resulting values for r_0 and r_i are:

Equation C.5:

$$r_0 = \frac{(c^2 - v^2)a + 2vcb}{a^2 + b^2}$$

Equation C.6:

$$r_i = \frac{2vca - (c^2 - v^2)b}{a^2 + b^2}$$

Next, determine expressions for a^2 and b^2 and their sum. Begin with a^2.

$$a = c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2$$

$$a^2 = \left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

$$a^2 = c^4\left(1 - \frac{v^2}{c^2}\right) - 2c^2v^2\sqrt{1 - \frac{v^2}{c^2}} + v^4$$

$$a^2 = c^4 - c^2v^2 - 2c^2v^2\sqrt{1 - \frac{v^2}{c^2}} + v^4$$

$$a^2 = c^4 - c^2v^2\left(1 + 2\sqrt{1 - \frac{v^2}{c^2}}\right) + v^4$$

Next, determine an expression for b^2.

$$b = vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)$$

$$b^2 = v^2c^2\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)$$

$$b^2 = v^2c^2\left(1 + 2\sqrt{1 - \frac{v^2}{c^2}} + 1 - \frac{v^2}{c^2}\right)$$

$$b^2 = v^2c^2\left(1 + 2\sqrt{1 - \frac{v^2}{c^2}}\right) + v^2c^2 - v^4$$

Now determine the sum of (a^2 + b^2).

$$a^2 + b^2 = \left\{c^4 - c^2v^2\left(1 + 2\sqrt{1 - \frac{v^2}{c^2}}\right) + v^4\right\} + \left\{v^2c^2\left(1 + 2\sqrt{1 - \frac{v^2}{c^2}}\right) + v^2c^2 - v^4\right\}$$

This simplifies to

Equation C.7:

$$a^2 + b^2 = c^4 + v^2c^2$$

The author thinks Equation C.7 is a very significant relationship. The right-hand side resembles the total energy equation from Special Relativity. The left-hand side resembles the trigonometric identity $\cos^2$ + $\sin^2$ = 1. Therefore, it might be consistent with the Copenhagen Interpretation of Quantum Mechanics. Also, it is possible to factor the left-hand side as (a+b**i**)(a-b**i**).

Does Equation C.7 imply that $a^2 = c^4$ and that $b^2 = v^2c^2$? The author does not think this is necessarily true, although it is true for v = 0. Instead, it is the scalar sum that is important. Of course, if it were possible to make one term on each side into a vector while the other term remained a scalar, then the one-to-one correspondence would apply.

Substituting C.7 into the values for r_0 and r_i (i.e., C.5 and C.6) allows the quaternion ratio $\mathbf{R}_1$ to be written as follows:

Equation C.8:

$$\mathbf{R}_1 = \left[\frac{(c^2 - v^2)\left(c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right) + 2v^2c^2\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^4 + v^2c^2}\right] + vc\left[\frac{2\left(c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right) - (c^2 - v^2)\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^4 + v^2c^2}\right]\mathbf{i}$$

For v = 0, this simplifies to

$$\mathbf{R}_1 = 1$$

For v = c, this simplifies to

$$\mathbf{R}_1 = 1 - \mathbf{i}$$

Given the trigonometric relationship, the author proposes these additional substitutions:

Equation C.9:

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1-\frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1-\frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

and

Equation C.10:

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1-\frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1-\frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

For v = 0, the cosine term is one and the sine term is zero. Therefore, the phase angle θ is zero.

For v = c, the cosine term is minus one divided by the square root of two, and the sine term is plus one divided by the square root of two. Therefore, the phase angle θ is 3π/4. The author thinks that this is a clue that absolute motion through the wave medium is related to QM spin.

Substitution of these trigonometric relations into C.5 and C.6 allows $\mathbf{R}_1$ to be expressed as follows:

Equation C.11:

$$\mathbf{R}_1 = \left[\frac{(c^2 - v^2)\cos\theta + 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] + \left[\frac{-(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

The main advantage to Equation C.11 is that it is more compact. The cosine and sine terms must be determined using C.9 and C.10.

There is something else that is noteworthy here. As an exercise, multiply Equation C.3 by equation C.4.

$$(c^2 - v^2)(2vc) = (ar_0 - br_i)(br_0 + ar_i)$$

$$(c^2 - v^2)(2vc) = abr_0^2 - b^2r_ir_0 + a^2r_0r_i - bar_i^2$$

Equation C.12:

$$(c^2 - v^2)(2vc) = ab\left(r_0^2 - r_i^2\right) + (a^2 - b^2)r_0r_i$$

Look at the symmetry of this equation! It is tempting to think that r_0 = a and that r_i = b but that is not correct. The author knows this because he did the multiplication and compared it to the left-hand side. Also, "a" and "b" are already defined by C.1 and C.2. Still, Equation C.12 is interesting and perhaps has a deeper meaning.

The author will solve the remaining three transforms using less text than the first.

R$_2$: (+-)

$$\mathbf{R}_2 = \frac{\mathbf{R}_{\text{SR}}}{\mathbf{R}_{\text{QM}}} = \frac{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}}{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}} = r_0 + r_i\mathbf{i}$$

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] + \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i} = (r_0 + r_i\mathbf{i})\left(\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right)$$

$$[c^2 - v^2] + [2vc]\mathbf{i} = (r_0 + r_i\mathbf{i})\left(\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] - \left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]\mathbf{i}\right)$$

Setting the scalar part equal to the scalar part and the vector part equal to the vector part gives

$$[c^2 - v^2] = r_0\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] + r_i\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

and

$$[2vc] = -r_0\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right] + r_i\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

$$Let\ a = \left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

$$Let\ b = \left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

Substituting these values gives

$$c^2 - v^2 = ar_0 + br_i$$

and

$$2vc = -br_0 + ar_i$$

These two equations are then solved using the Method of Determinants with the solution being

$$r_0 = \frac{(c^2 - v^2)a - 2vcb}{a^2 + b^2}$$

and

$$r_i = \frac{2vca + (c^2 - v^2)b}{a^2 + b^2}$$

$$\mathbf{R}_2 = \left[\frac{(c^2 - v^2)\left(c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right) - 2v^2c^2\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^4 + v^2c^2}\right] + vc\left[\frac{2\left(c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right) + (c^2 - v^2)\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^4 + v^2c^2}\right]\mathbf{i}$$

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

$$\mathbf{R}_2 = \left[\frac{(c^2 - v^2)\cos\theta - 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] + \left[\frac{(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

For v = 0, θ = 0 and

$$\mathbf{R}_2 = 1$$

For v = c, θ = 3π/4 and

$$\mathbf{R}_2 = -1 - \mathbf{i}$$

$\mathbf{R}_3$: (-+)

$$\mathbf{R}_3 = \frac{\mathbf{R}_{\mathrm{SR}}}{\mathbf{R}_{\mathrm{QM}}} = \frac{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] - \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}}{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}} = r_0 + r_i\mathbf{i}$$

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] - \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i} = (r_0 + r_i\mathbf{i})\left(\left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + \left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right)$$

Setting the scalar part equal to the scalar part and the vector part equal to the vector part gives

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] = r_0\left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - r_i\left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]$$

and

$$-\left[\frac{2vc}{c^2 + v^2}\right] = r_0\left[\frac{vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] + r_i\left[\frac{c^2\sqrt{1-\frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right]$$

Multiplication by common denominator gives

$$[c^2 - v^2] = r_0\left[c^2\sqrt{1-\frac{v^2}{c^2}} - v^2\right] - r_i\left[vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)\right]$$

and

$$-[2vc] = r_0\left[vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)\right] + r_i\left[c^2\sqrt{1-\frac{v^2}{c^2}} - v^2\right]$$

$$Let\ a = \left[c^2\sqrt{1-\frac{v^2}{c^2}} - v^2\right]$$

$$Let\ b = \left[vc\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)\right]$$

Substituting these gives

$$c^2 - v^2 = ar_0 - br_i$$

and

$$-2vc = br_0 + ar_i$$

These equations are solved using the Method of Determinants with the solution being

$$r_0 = \frac{(c^2 - v^2)a - 2vcb}{a^2 + b^2}$$

and

$$r_i = \frac{-2vca - (c^2 - v^2)b}{a^2 + b^2}$$

$$\mathbf{R}_3 = \left[\frac{(c^2 - v^2)a - 2vcb}{a^2 + b^2}\right] + \left[\frac{-2vca - (c^2 - v^2)b}{a^2 + b^2}\right]\mathbf{i}$$

$$\mathbf{R}_3 = \left[\frac{(c^2 - v^2)\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] - 2vc\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]}{c^4 + v^2c^2}\right] + \left[\frac{-2vc\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] - (c^2 - v^2)\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]}{c^4 + v^2c^2}\right]\mathbf{i}$$

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

$$\mathbf{R}_3 = \left[\frac{(c^2 - v^2)\cos\theta - 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] - \left[\frac{(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

For v = 0, θ = 0 and

$$\mathbf{R}_3 = 1$$

For v = c, θ = 3π/4 and

$$\mathbf{R}_3 = -1 + \mathbf{i}$$

R$_4$: (--)

$$\mathbf{R}_4 = \frac{\mathbf{R}_{\text{SR}}}{\mathbf{R}_{\text{QM}}} = \frac{\left[\frac{c^2 - v^2}{c^2 + v^2}\right] - \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i}}{\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}} = r_0 + r_i\mathbf{i}$$

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] - \left[\frac{2vc}{c^2 + v^2}\right]\mathbf{i} = (r_0 + r_i\mathbf{i})\left(\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] - \left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]\mathbf{i}\right)$$

Setting the scalar part equal to the scalar part and the vector part equal to the vector part gives

$$\left[\frac{c^2 - v^2}{c^2 + v^2}\right] = r_0\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right] + r_i\left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right]$$

and

$$-\left[\frac{2vc}{c^2 + v^2}\right] = -r_0\left[\frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{c^2 + v^2}\right] + r_i\left[\frac{c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2}{c^2 + v^2}\right]$$

Multiplication by the common denominator gives

$$[c^2 - v^2] = r_0\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] + r_i\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

and

$$-[2vc] = -r_0\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right] + r_i\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

$$Let\ a = \left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right]$$

$$Let\ b = \left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]$$

Making these substitutions gives

$$c^2 - v^2 = ar_0 + br_i$$

and

$$-2vc = -br_0 + ar_i$$

The solution to this system is

$$r_0 = \frac{(c^2 - v^2)a + 2vcb}{a^2 + b^2}$$

and

$$r_i = \frac{-2vca + (c^2 - v^2)b}{a^2 + b^2}$$

$$\mathbf{R}_4 = \left[\frac{(c^2 - v^2)a + 2vcb}{a^2 + b^2}\right] + \left[\frac{-2vca + (c^2 - v^2)b}{a^2 + b^2}\right]\mathbf{i}$$

$$\mathbf{R}_4 = \left[\frac{(c^2 - v^2)\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] + 2vc\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]}{c^4 + v^2c^2}\right] + \left[\frac{-2vc\left[c^2\sqrt{1 - \frac{v^2}{c^2}} - v^2\right] + (c^2 - v^2)\left[vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)\right]}{c^4 + v^2c^2}\right]\mathbf{i}$$

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

$$\mathbf{R}_4 = \left[\frac{(c^2 - v^2)\cos\theta + 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] - \left[\frac{-(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

For v = 0, θ = 0 and

$$\mathbf{R}_4 = 1$$

For v = c, θ = 3π/4 and

$$\mathbf{R}_4 = 1 + \mathbf{i}$$

Summary:

$$\mathbf{R}_1 = \left[\frac{(c^2 - v^2)\cos\theta + 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] + \left[\frac{-(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

$$\mathbf{R}_2 = \left[\frac{(c^2 - v^2)\cos\theta - 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] + \left[\frac{(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

$$\mathbf{R}_3 = \left[\frac{(c^2 - v^2)\cos\theta - 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] - \left[\frac{(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

$$\mathbf{R}_4 = \left[\frac{(c^2 - v^2)\cos\theta + 2vc\sin\theta}{\sqrt{c^4 + v^2c^2}}\right] - \left[\frac{-(c^2 - v^2)\sin\theta + 2vc\cos\theta}{\sqrt{c^4 + v^2c^2}}\right]\mathbf{i}$$

$$\cos\theta = \frac{a}{\sqrt{c^4 + v^2c^2}} = \frac{c^2\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{c\left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2}\right)}{\sqrt{c^2 + v^2}}$$

$$\sin\theta = \frac{b}{\sqrt{c^4 + v^2c^2}} = \frac{vc\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^4 + v^2c^2}} = \frac{v\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}{\sqrt{c^2 + v^2}}$$

For all of these transforms at v = 0, **R** = 1. At v = c, they simplify to

$$\mathbf{R} = \pm 1 \pm \mathbf{i}$$

Appendix D

Begin by restating the problem to solve

$$\mathbf{R} = \frac{\mathbf{R}_B}{\mathbf{R}_A} = \frac{\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{v_i + \Delta v_i}{c}\mathbf{i}}{\sqrt{1 - \frac{{v_i}^2}{c^2}} + \frac{v_i}{c}\mathbf{i}} = r_0 + r_i\mathbf{i}$$

$$\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{v_i + \Delta v_i}{c}\mathbf{i} = (r_0 + r_i\mathbf{i})\left(\sqrt{1 - \frac{{v_i}^2}{c^2}} + \frac{v_i}{c}\mathbf{i}\right)$$

Set the scalar part equal to the scalar part and the vector part equal to the vector part.

$$\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} = r_0\sqrt{1 - \frac{{v_i}^2}{c^2}} - r_i\frac{v_i}{c}$$

and

$$\frac{v_i + \Delta v_i}{c} = r_i\sqrt{1 - \frac{{v_i}^2}{c^2}} + r_0\frac{v_i}{c}$$

This is essentially the same problem structure as in Appendix C.

$$Let\ a = \sqrt{1 - \frac{{v_i}^2}{c^2}}$$

and

$$Let\ b = \frac{v_i}{c}$$

The two equations are then

$$ar_0 - br_i = \sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}}$$

and

$$br_0 + ar_i = \frac{v_i + \Delta v_i}{c}$$

This is represented as a matrix as follows

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} r_0 \\ r_i \end{bmatrix} = \begin{bmatrix} \sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} \\ \frac{v_i + \Delta v_i}{c} \end{bmatrix}$$

This system is then solved using the Method of Determinants with the solution being

$$r_0 = \frac{a\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + b\frac{v_i + \Delta v_i}{c}}{a^2 + b^2}$$

and

$$r_i = \frac{a\frac{v_i + \Delta v_i}{c} + b\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}}}{a^2 + b^2}$$

Substituting back in "a" and "b" gives

$$r_0 = \sqrt{1 - \frac{{v_i}^2}{c^2}}\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{v_i(v_i + \Delta v_i)}{c^2}$$

and

$$r_i = \sqrt{1 - \frac{{v_i}^2}{c^2}}\left(\frac{v_i + \Delta v_i}{c}\right) + \frac{v_i}{c}\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}}$$

The denominator (a^2 + b^2) simplified to the value one. The transform can therefore be expressed as

Equation D.1:

$$\mathbf{R} = \left[\sqrt{1 - \frac{{v_i}^2}{c^2}}\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}} + \frac{v_i(v_i + \Delta v_i)}{c^2}\right] + \left[\sqrt{1 - \frac{{v_i}^2}{c^2}}\left(\frac{v_i + \Delta v_i}{c}\right) + \frac{v_i}{c}\sqrt{1 - \frac{(v_i + \Delta v_i)^2}{c^2}}\right]\mathbf{i}$$

This equation seems to be correct but is unsatisfactory to the author. The author hopes to simplify this expression by combining terms. The key to this exercise rests with several trigonometric identities and with the hypothesis presented in the main text.

The hypothesis presented in the main text is as follows:

$$\cos\phi + i\sin\phi = \sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\mathbf{i}$$

Therefore, the cosine term corresponds with the square root term and the sine term corresponds with the (v/c) term. If the two reference frames are taken to be A and B then their respective phase angles are ϕ_A and ϕ_B respectively. Inserting these relations into Equation D.1 produces the following

Equation D.2:

$$\mathbf{R} = [\cos\phi_A\cos\phi_B + \sin\phi_A\sin\phi_B] + [\cos\phi_A\sin\phi_B + \sin\phi_A\cos\phi_B]\mathbf{i}$$

The trigonometric identities are as follows

$$\cos u\cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u\sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u\sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\sin u\cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

Inserting these identities into Equation D.2 results in the following

Equation D.3:

$$\mathbf{R} = \cos(\phi_A - \phi_B) + \sin(\phi_A + \phi_B)\,\mathbf{i}$$

If the two reference frames are travelling at velocities that are approximately equal, then the cosine term is approximately one. This is completely consistent with the predictions of Special Relativity. As the relative velocity between the reference frames increases, the sine term becomes more significant and is perceived as wave-particle behavior. This is also completely consistent with Special Relativity.

Appendix E

The objective of this Appendix is to determine the transform associated with generic 3-dimensional motion between two reference frames. Begin by restating the problem

$$\mathbf{R} = \frac{\sqrt{1 - \frac{(v_i + \Delta v_i)^2 + v_j^2 + v_k^2}{c^2}} + \frac{v_i + \Delta v_i}{c}\mathbf{i} + \frac{v_j}{c}\mathbf{j} + \frac{v_k}{c}\mathbf{k}}{\sqrt{1 - \frac{v_i^2}{c^2}} + \frac{v_i}{c}\mathbf{i}} = r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$

Multiply both sides by the denominator.

$$\sqrt{1 - \frac{(v_i + \Delta v_i)^2 + v_j^2 + v_k^2}{c^2}} + \frac{v_i + \Delta v_i}{c}\mathbf{i} + \frac{v_j}{c}\mathbf{j} + \frac{v_k}{c}\mathbf{k}$$
$$= (r_0 + r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k})\left(\sqrt{1 - \frac{v_i^2}{c^2}} + \frac{v_i}{c}\mathbf{i}\right)$$

The scalar terms on each side of the equation must be equal and each of the vector terms on each side must be equal. This produces the following four simultaneous equations:

Scalar:

$$\sqrt{1 - \frac{(v_i + \Delta v_i)^2 + v_j^2 + v_k^2}{c^2}} = r_0\sqrt{1 - \frac{v_i^2}{c^2}} - r_i\frac{v_i}{c}$$

i:

$$\frac{v_i + \Delta v_i}{c} = r_0\frac{v_i}{c} + r_i\sqrt{1 - \frac{v_i^2}{c^2}}$$

j:

$$\frac{v_j}{c} = r_j\sqrt{1 - \frac{v_i^2}{c^2}} + r_k\frac{v_i}{c}$$

k:

$$\frac{v_k}{c} = r_k\sqrt{1 - \frac{v_i^2}{c^2}} - r_j\frac{v_i}{c}$$

This problem consists of four simultaneous equations with four unknown values. The system can be expressed in matrix form as follows:

$$Let\ b = \frac{v_i}{c}$$

$$Let\ a = \sqrt{1 - \frac{v_i^2}{c^2}} = \sqrt{1 - b^2}$$

$$Let\ b_i = \frac{v_i + \Delta v_i}{c}; Let\ b_j = \frac{v_j}{c}; Let\ b_k = \frac{v_k}{c}; Let\ \beta^2 = b_i^2 + b_j^2 + b_k^2$$

$$Let\ \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\begin{bmatrix} a & -b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix} \begin{bmatrix} r_0 \\ r_i \\ r_j \\ r_k \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma} \\ b_i \\ b_j \\ b_k \end{bmatrix}$$

This system is easily solved using the Method of Determinants. It is even simpler than a typical four variable system because there are two pairs of two variable systems that are independent of each other. That is the meaning of the 8 zeros in the coefficient matrix. The coefficient matrix is very similar to that produced when solving the Dirac Wave Equation.

The solution to the system is then

$$r_0 = \frac{\frac{a}{\gamma} + b_i b}{a^2 + b^2} = \frac{a}{\gamma} + b_i b$$

$$r_i = \frac{ab_i + \frac{b}{\gamma}}{a^2 + b^2} = ab_i + \frac{b}{\gamma}$$

$$r_j = \frac{ab_j + bb_k}{a^2 + b^2} = ab_j + bb_k$$

$$r_k = \frac{ab_k + bb_j}{a^2 + b^2} = ab_k + bb_j$$

For these four relations, the ($a^2 + b^2$) term equals one. The transform is therefore

$$\mathbf{R} = \left(\frac{a}{\gamma} + b_i b\right) + \left(ab_i + \frac{b}{\gamma}\right)\mathbf{i} + \left(ab_j + bb_k\right)\mathbf{j} + \left(ab_k + bb_j\right)\mathbf{k}$$

Appendix F

Restate the problem to solve

$$\mathbf{M}(1 + 2ab\mathbf{i}) = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right][a + b\mathbf{i}]$$

$$Let\ \mathbf{M} = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right](m_0 + m_i\mathbf{i})$$

$$(m_0 + m_i\mathbf{i})(1 + 2ab\mathbf{i}) = (a + b\mathbf{i})$$

$$Where\ a = \sqrt{1 - \frac{v_i^2}{c^2}}\ and\ b = \frac{v_i}{c}$$

Setting the scalar parts to be equal and the vector parts to be equal produces the following

$$m_0 - 2m_i ab = a$$

and

$$m_i + 2m_0 ab = b$$

This is then represented in matrix form as

$$\begin{bmatrix} 1 & -2ab \\ 2ab & 1 \end{bmatrix}\begin{bmatrix} m_0 \\ m_i \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

This is then solved using the Method of Determinants with the solution being

$$m_0 = \frac{a + 2ab^2}{1 + 4a^2b^2}\ and\ m_i = \frac{b - 2a^2b}{1 + 4a^2b^2}$$

The denominator is simplified by using the hypothesis from the main text and a trigonometric identity. From the hypothesis, a = cos(ϕ) and b = sin(ϕ). Therefore, the denominator is $(1 + [2\cos(\phi)\sin(\phi)]^2)$. An identity from trigonometry is that sin(2u) = 2sin(u)cos(u). Therefore, the denominator simplifies to $[1 + \sin^2(2\phi)]$. The trigonometric identity can also be used in the numerator for both m_0 and m_i. These substitutions produce the following

$$m_0 = \frac{\cos(\phi) + \sin(2\phi)\sin(\phi)}{1 + sin^2(2\phi)}\ and\ m_i = \frac{\sin(\phi) - \cos(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}$$

$$\mathbf{M} = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left\{\left[\frac{\cos(\phi) + \sin(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}\right] + \left[\frac{\sin(\phi) - \cos(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}\right]\mathbf{i}\right\}$$

Appendix G

The objective of this Appendix is to determine the multiplicative product of **MR** where

$$\mathbf{M} = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left\{\left[\frac{\cos(\phi)+\sin(\phi)\sin(2\phi)}{1+sin^2(2\phi)}\right] + \left[\frac{\sin(\phi)-\cos(\phi)\sin(2\phi)}{1+sin^2(2\phi)}\right]\mathbf{i}\right\}$$

and

$$\mathbf{R} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i}\right)$$

These are respectively Equation 12.3 and Equation 12.1.2 from the main text.

To simplify the presentation, the author will make the following substitutions

$$Let\ A = \frac{\cos(\phi)+\sin(\phi)\sin(2\phi)}{1+sin^2(2\phi)}$$

$$Let\ B = \frac{\sin(\phi)-\cos(\phi)\sin(2\phi)}{1+sin^2(2\phi)}$$

$$Let\ K = \left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]$$

Therefore, the relationship for **M** becomes

$$\mathbf{M} = \pm K\{A + B\mathbf{i}\}$$

Next, determine **MR**.

$$\mathbf{MR} = \pm K\{A + B\mathbf{i}\}\left\{\pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i}\right)\right\}$$

The scalar portion is:

$$\pm\frac{KA}{\sqrt{2}}\left(\pm\frac{1}{\gamma} \pm b_i\right) \pm \frac{KB}{\sqrt{2}}\left(\pm b_i \pm \frac{1}{\gamma}\right) = \pm\frac{K}{\sqrt{2}}(A+B)\left(\pm\frac{1}{\gamma} \pm b_i\right)$$

The **i** portion is:

$$\pm\frac{KB}{\sqrt{2}}\left(\pm\frac{1}{\gamma} \pm b_i\right) \pm \frac{KA}{\sqrt{2}}\left(\pm b_i \pm \frac{1}{\gamma}\right) = \pm\frac{K}{\sqrt{2}}(A+B)\left(\pm\frac{1}{\gamma} \pm b_i\right)$$

The product **MR** is therefore:

$$\mathbf{MR} = \left[\pm\frac{K}{\sqrt{2}}(A+B)\left(\pm\frac{1}{\gamma} \pm b_i\right)\right] + \left[\pm\frac{K}{\sqrt{2}}(A+B)\left(\pm\frac{1}{\gamma} \pm b_i\right)\right]\mathbf{i}$$

The scalar term and the vector term are identical and can be factored out to produce:

$$\mathbf{MR} = \pm\frac{K}{\sqrt{2}}(A+B)\left(\pm\frac{1}{\gamma} \pm b_i\right)(1+\mathbf{i})$$

Substitution of K, A, and B produces

$$\mathbf{MR} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left(\frac{\cos(\phi)+\sin(\phi)\sin(2\phi)}{1+sin^2(2\phi)} + \frac{\sin(\phi)-\cos(\phi)\sin(2\phi)}{1+sin^2(2\phi)}\right)\left(\pm\frac{1}{\gamma} \pm b_i\right)(1+\mathbf{i})$$

The sine and cosine terms can be rearranged slightly to simplify the presentation:

$$\mathbf{MR} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left(\frac{\cos(\phi)+\sin(\phi)+[\sin(\phi)-\cos(\phi)]\sin(2\phi)}{1+sin^2(2\phi)}\right)\left(\pm\frac{1}{\gamma} \pm b_i\right)(1+\mathbf{i})$$

The author will simplify this as follows:

$$\mathbf{MR} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\mathrm{M_O}\left(\pm\frac{1}{\gamma} \pm b_i\right)(1+\mathbf{i})$$

Where $\mathrm{M_O}$ is as follows:

$$\mathrm{M_O} = \frac{\cos(\phi)+\sin(\phi)+[\sin(\phi)-\cos(\phi)]\sin(2\phi)}{1+sin^2(2\phi)} = A+B$$

Here the subscript O refers to the observer.

R is also represented as follows:

$$\mathbf{R} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i} \pm \sqrt{2}\mathbf{j} \pm \sqrt{2}\mathbf{k}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i} \pm \sqrt{2}\mathbf{j} \pm \sqrt{2}\mathbf{k}\right)$$

The sum of the **j** and **k** terms can have values of 0 or ±2. Therefore, the combinations are:

$$\mathbf{R} = \pm\frac{1}{\sqrt{2}}\left(\frac{1}{\gamma} + b_i\mathbf{i}\right) \pm \frac{1}{\sqrt{2}}\left(b_i + \frac{1}{\gamma}\mathbf{i}\right) + n_j\mathbf{j} + n_k\mathbf{k};\ n_j = 0, \pm 2;\ n_k = 0, \pm 2$$

It is only necessary here to multiply the (n_j**j** + n_k**k**) term by **M** and then add the result to the previous solution.

$$\mathbf{M} = \pm K\{A + B\mathbf{i}\}$$

$$\mathbf{M}(n_j\mathbf{j} + n_k\mathbf{k}) = \pm K\{A + B\mathbf{i}\}(n_j\mathbf{j} + n_k\mathbf{k});\ n_j = 0, \pm 2;\ n_k = 0, \pm 2$$

There is no scalar term for this multiplication.

There is no **i** portion for this multiplication.

The **j** portion is:

$$\pm K\left(An_j - Bn_k\right)\mathbf{j}$$

The **k** portion is:

$$\pm K\left(An_k + Bn_j\right)\mathbf{k}$$

The transform is therefore:

$$\mathbf{MR} = \pm K\left[\mathrm{M_O}\left(\pm\frac{1}{\gamma} \pm b_i\right)(1 + \mathbf{i}) + \left(An_j - Bn_k\right)\mathbf{j} + \left(An_k + Bn_j\right)\mathbf{k}\right];\ n_j = 0, \pm 2;\ n_k = 0, \pm 2$$

where:

$$K = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]$$

$$A = \frac{\cos(\phi) + \sin(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}$$

$$B = \frac{\sin(\phi) - \cos(\phi)\sin(2\phi)}{1 + sin^2(2\phi)}$$

$$\mathrm{M_O} = \frac{\cos(\phi) + \sin(\phi) + [\sin(\phi) - \cos(\phi)]\sin(2\phi)}{1 + sin^2(2\phi)} = A + B$$