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# Derivation of Hubble's Constant and the Quantization of the Gravitational Field

Michael Harney **mharney@signaldisplay.com**

**Abstract:** It is shown from Mach's principle and matter-wave theory that the speed of light is decreasing at a rate equal to Hubble's constant and from the matter-wave derivation a prediction is made for the mass of the photon as 5.81 x $10^{-69}$ kg. The mass-transformation equations of special relativity are also shown to be a result of Mach's principle. A formula for the quantization of the gravitational field is found by setting both matter wave and potential energy equations equal to each other through Hubble's constant.
**Keywords:** Hubble's constant, photon, Mach's principle, special relativity.

One of the implications of the Special Theory of Relativity is the concept of a massless photon. A massless photon seems to be required by Special Relativity as a photon of any mass would have to be infinite based on the mass-transformation equations where it's velocity is equal to c and the function is then undefined. The problem with a massless photon arises in describing the photon as a particle with momentum, spin, and its interaction with a gravitational field. All of these concepts are dealt with at an energy level and the familiar formula E = h*f where h is Planck's constant and f = frequency is used as a substitution when mass is required in the classical formulas, with energy being related through the photon's momentum, E = momentum*c.

It was shown in 1998 from the Super-Kamiokande experiment that neutrinos generated from cosmic rays in the upper atmosphere can oscillate between muon and electron types with a mass for the neutrino being inferred. There appears to now be interest in the physics community for finding a mass for formerly massless particles.

In the following analysis we will show a derivation for Hubble's constant from Mach's Principle and a derivation from matter wave equations that assume the photon has a mass, which we also predict from this equation to be 5.81 x $10^{-69}$ kg. Hubble's constant from both the matter wave and energy equations is quite close to the actual value, with the matter wave equation giving the closest match to the current estimated value of 70 km/sec/megaparsecs [1]. We also find a relationship by setting Hubble's constant equal in both matter wave and energy equations and this leads to the quantization of the gravitational field through Planck's constant.

First from Mach's Principle, let us assume an object in the Universe with rest mass $m_0$ and the entire mass of the Universe is acting on this object gravitationally, so that the gravitational potential energy of all objects in the Universe acting on this single rest mass is found by integrating Newton's force law over the radius of the Universe to obtain,

$$[G^*(\text{Mass of Universe})^*m_0 / (r = \text{radius of Universe})] = m_0{}^*c^2 , \quad (1)$$

Although (1) treats masses as if they are point sources and we are taking the entire mass of the Universe as a point source, it is assumed that this point source exists because of the distribution of matter through the Universe and the distance between any arbitrary object and the center of mass of the Universe is still on the order of magnitude of the radius of the Universe. From this relation of the gravitational potential energy of the mass of the Universe acting on one object we see immediately that the rest mass cancels from both sides and the speed of light is in fact dependent upon the gravitational constant, the mass of the Universe and the radius of the Universe. Of these three factors, the radius of the Universe is the one assumed most likely to change over time and this formula shows that the speed of light is changing very slowly relative to our own time frame. If we choose a density for the Universe of $5 \times 10^{-27}$ kg/m$^3$ [2], and assume a spherical shape then the mass of the Universe is calculated as the density*volume with radius of the Universe equal to $1.9 \times 10^{26}$ meters. The mass of the Universe is then found to be $1.44 \times 10^{53}$ kg. Substituting these values into equation (1), the speed of light c is calculated to be $2.25 \times 10^8$ m/sec, close to the actual value and within the same order of magnitude.

The conclusion from (1) is that the speed of light will decrease slowly as the Universe expands. In fact, if we rearrange equation (1) so that c is set in terms of r (radius) we have,

$$c = [G^*(\text{Mass of Universe})/r]^{1/2} \quad (2)$$

and differentiating c with respect to r gives us,

$$dc/dr = -1/2^*[(G^*(\text{Mass of Universe}))^{1/2}]^*r^{-3/2} \quad (3)$$

= Hubble's constant of
$0.59 \times 10^{-18}$ seconds$^{-1}$
or 18.2 km/sec/megaparsecs

and with the known expansion rate of the Universe being $3 \times 10^8$ m/sec = c (from Hubble's formula v = Hr and substituting the radius of the Universe to find v=c) we can find the rate of change in the speed of light using the chain rule as follows: dc/dt = dc/dr*dr/dt (dr/dt is the expansion rate of the Universe equal to c which we are allowing to be a variable as we assume it is changing) or,

$$dc/dt = -1/2^*[(G^*(\text{Mass of Universe}))^{1/2}]^*c^*r^{-3/2} , \quad (4)$$

which is a differential equation of the first order in terms of c with a solution of:

$$c(t) = k_1{}^*[e \wedge (-k_2{}^*t)] . \quad (5)$$

This shows the exponential decay of the speed of light with time variable t. The constant k1 is a constant of integration and decay-constant k2 has the following value:

$$k_2 = 1/2*[(G*(\text{Mass of Universe}))^{1/2}]*r^{-3/2} \quad (6)$$

$$= \text{Hubble's constant of } 0.59 \times 10^{-18} \text{ seconds}^{-1}$$
$$\text{or } 18.2 \text{ Km/sec/megaparsec}$$

So after substituting mass and radius quantities already discussed from equation 1 we find that the decay constant for the speed of light is Hubble's constant (the value from equation 6 is about ¼ the current estimated value). As the speed of light with respect to time depends on Hubble's constant, and we use the speed of light to verify this constant (through Doppler redshift) it may be possible that these factors result in the ambiguity of the measured value of Hubble's constant. It does seem possible that the decay in the speed of light is controlled by the expansion rate of the Universe.

The integration constant k1 can be found in equation 5 by using the current value for the speed of light as $c(t) = 3 \times 10^8$ m/sec, using Hubble's constant for k2 and assuming an age for the Universe for the time variable (the hard part). After differentiating equation 5 we have,

$$dc/dt = -k1*k2*[e \wedge (-k_2*t)] \quad . \quad (7)$$

Two points are noted from equations 1 through 7 as follows: 1) the speed of light is decreasing exponentially over time, and 2) equation 7 implies a deceleration of light, which in turn suggests a photon mass (if equation 1 is rewritten as Newton's Force law on the rest mass of the photon, dc/dt is the acceleration in F = ma, and the same numerical value results). This suggests that the mass of the photon is based on a factor involved in the expansion of the Universe.

With such a small decay constant, equation 7 essentially resembles a linear slope with a value $dc/dt = 1.7 \times 10^{-10}$ m/sec$^2$. If we accept that Hubble's constant is at least twice this value then a closer value for dc/dt may be $3.4 \times 10^{-10}$ m/sec$^2$ or 1.0 cm/sec/year. When this number is applied to calculations of distance measurements between the earth and the moon using an earth-bound laser and mirrors placed on the moon during the Apollo era, it creates the effect of the moon receding from the earth at 1.3 cm/year. The data from the laser experiments show a rate of lunar recession of 3.82 cm/year [3], and theories that show tidal energy transfer between the Earth's oceans and it's crust provide a theoretical recession rate of 2.16 cm/year [4]. The difference between laser ranging data (3.82 cm/year) and tidal transfer theories (2.16 cm/year) is 1.66 cm/year and is comparable to the 1.3 cm/year estimated from the presently described theory of c-decay. Although dc/dt is hard to determine precisely without knowing the parameters of equation 6 better, the difference between laser ranging and theoretical models appear to allow for the existence of c-decay. Historical data has also been anaylzed on previous measurements of the speed of light to show it's decay[5].

Based on equation 5 and knowing the current value of $c = 3 \times 10^8$ m/sec, we can estimate that at t = 0 (early stages of the Universe), $c = 2.7 * 3 \times 10^8$ m/sec $= 8.1 \times 10^8$ m/sec. This higher value of c in the early Universe may explain the puzzling phenomena of superluminal objects traveling in many cases at twice the current speed of light. Although many explanations for the higher speed of these objects have focused on measurement techniques, some of these objects may be far enough away from us and have existed at an earlier time in the Universe when c was a much higher value.

If we accept that Mach's Principle is correct and that gravitational potential energy is equivalent to rest-mass energy, we may ask how this relationship and the decay of the speed of light affect the mass-transformation equation of special relativity. If we examine an equation similar to equation 1 where the gravitational potential energy of the Universe is acting on an accelerating object $m_0$ at velocity v,

$$[G*(\text{Mass of Universe})*m_0 / r] = (1/2)m_0*v^2$$

as v approaches c. Rewriting the gravitational potential energy on the left side as the total rest-mass energy of the Universe (which is also the limit to how much energy can be used to accelerate an object),

(Mass of Universe)$*c^2 = (1/2)m*v^2$ in the limit as v approaches c, m approaches the Mass of Universe and Mass of Universe/ $m_0 = v^2/(2c^2)$.

Thus, we would expect that as objects approach the speed of light, their mass approaches the incredibly large value of $1.44 \times 10^{53}$ kg. Although Special Relativity tells us that m goes to infinity as v approaches c, our current instruments probably can't tell the difference between infinity and $1.44 \times 10^{53}$ unless $m_0$ is very large to begin with. If $m_0$ is very large then the ratio Mass-Universe / $m_0$ may be more measurable. It is also interesting to note that although rest-mass is defined from special relativity and measured experimentally as $m_0*c^2$ , if we replace the right side of equation 1 with $(1/2)\, m_0*c^2$ , the classical kinetic energy formula for accelerating an object to the speed of light, the value obtained for c in equation 2 and Hubbles constant from equation 3 are closer to their measured values. Although Mach's principle is based on classical mechanics it is also defined as universal gravitational potential-energy being equivalent to the inertia of a local mass so the standard formula for rest-mass energy used in equation 1 keeps the formula in line with the definition.

Therefore, an explanation of special relativity based on Mach's Principle does change the mass-transformations but continues to keep the postulate that the speed of light is the upper speed limit of the Universe (because it happens to be the escape velocity of the Universe), even though this speed limit may be steadily decreasing.

From equations (3) and (6) for Hubble's constant, it is seen that Hubble's constant must not be constant but a function of R as well, and by taking the derivative dH/dr and multiplying by (dr/dt = c) one arrives at a rate of change for H of $dH/dt = 1.4 \times 10^{-36}$ seconds$^{-2}$.

Next, let's revisit matter waves as described by DeBroglie in 1924. From the knowledge of light having particle-like properties, DeBroglie drew the conclusion that matter should have wave-like properties and assigned the formula (again using the energy relation of E = h*f and incorporating momentum):

Matter-wavelength of particle = h / (mass-particle * velocity-particle) , (8)

where h = Planck's constant of 6.62 x $10^{-34}$ Joules*sec.

It has been shown in the Bohr model of the atom that the stability of electronic orbits is obtained by requiring an integral number of matter-wavelengths of the electron to equal the circumference of each orbit. This ensures stability of each electronic orbit in the Bohr model of the atom, and we will apply this same principle to the matter-wavelength of a photon and the diameter of the Universe.

We then set the matter-wavelength of a photon equal to the diameter of the Universe,

Diameter of Universe = h / (mass of photon * c) , (9)

From equation 9, we can then state,

2 x 1.9 x $10^{26}$ meters = h / (mass-photon * speed of light) , (10)

For the speed of light we use c = 3 x $10^{8}$ m/sec and we find the mass of the photon as

Mass-photon = 5.81 x $10^{-69}$ kg or 3.23 x $10^{-33}$ eV.

The assumption that we made in (9), that the matter-wavelength of a photon is equal to the diameter of the Universe, can be verified by solving Schrodinger's equation for the probability density function of a particle in a two-dimensional box (assuming the particle is a photon and the box is the Universe – the two dimensional derivation is approximate to our three-dimensional universe but the results are amazingly close to what we would expect). For the two-dimensional box the solution to Schrodinger's equation for the quantized energies of the particle is:

$$E = [(n_x)^2 + (n_y)^2]*h^2/(8mL^2) \quad . \qquad (10b)$$

Where h = Planck's constant and $n_x$ and $n_y$ are the quantum numbers that give the multiple number of wavelengths that are possible inside the box from the Schrodinger solution (the quantum numbers also specify allowable energies of the particle, or in our case the allowable rest-mass energies of the photon). Because we have assumed the photon has a rest-mass we set the rest-energy of the photon equal to the ground state of (10b) which corresponds to $n_x$ and $n_y$ = 1,

$$h^2/(8*(\text{mass of photon})*L^2) = (\text{mass of photon})*c^2 \quad . \qquad (10c)$$

Now we take L which is the length of the box and set it equal to 2r, where r is the radius of the Universe (1.9 x $10^{26}$ meters) and we find:

$$\text{Mass of photon} = [h^2/(8*(2r)^2c^2)]^{1/2} = 2 \times 10^{-69} \text{ kg} \quad . \qquad (10d)$$

When we compare the results of 10a with 10d, the photon rest-mass is basically the same and the assumption that the photon matter-wavelength is equal to the diameter of the Universe (which is the solution to Schrodinger's equation for the photon trapped in the Universe, equation 10b), produces a result from quantized energy levels that matches 10a. From this assumption we see that the rest-mass energy of the photon (which from equation 1 is also equal to it's gravitational potential energy) is equivalent to it's zero-point energy in the quantum well of the Universe.

If we now take equation 9 and rearrange the c term with the diameter term ( = 2r):

$$c = h / (\text{mass of photon} * 2r) \;, \qquad (11)$$

where r = radius of Universe. Then we can take the derivative, dc/dr as we performed for the energy relationship in equation 3,

$$dc/dr = - h / (2 * \text{mass-photon} * r^2) \;, \qquad (12)$$

$$= \text{Hubble's constant of } 1.57 \times 10^{-18} \text{ seconds}^{-1}$$
or 48.4 km/sec/megaparsecs

We then use the chain rule again, dc/dt = dc/dr * dr/dt and use dr/dt = c = 3 x $10^{8}$ m/sec to form the first-order differential equation,

$$dc/dt = - k2*dr/dt = - k2*c, \qquad (13)$$
$$\text{with } k2 = h / (2 * \text{mass of photon} * r^2) \;, \qquad (14)$$

and using values of R = 1.9 x $10^{26}$ meters, mass-photon = 5.81 x $10^{-69}$ kg, h = Planck's constant we find,

$$k2 = \text{Hubble's constant of } 1.57 \times 10^{-18} \text{ seconds}^{-1}$$
or 48.4 km/sec/megaparsecs

This is also found by solving for the mass of the photon in (10) and substituting into (14) such that, k2 = c/r, or the formula for Hubble's constant when v = c. Notice that by taking the mass of the photon multiplied by c we have derived a momentum for the photon that is different than the familiar formula, p = E/c = h*f/c = h / wavelength. It is assumed that total momentum of the photon is the vector sum of the mass-momentum formula and the traditional formula p = h / wavelength. With most wavelengths (in fact, for all physical processes currently known that generate electromagnetic waves), the momentum due to h/wavelength is much higher than the contribution from mass-photon*c by a factor of at least $10^{30}$.

Compare the value of Hubble's constant calculated in (14) with that of (6). The value found by using matter-wavelength relationships is a little more than twice the value from energy relationships.

The solution to (13) is similar to that of (5), as both are first-order differentials:

$$c(t) = k_1*[e \wedge (-k_2*t)] , \quad (15)$$

with k1 being a constant of integration and k2 as determined from (14).

As k2 from (14) is about twice the value of k2 determined from energy relationships in (6) (and both are sufficiently large), k1 from (15) will also be close to the value of k1 in (5) (determined by using a known value of c, current time since expansion t, and k2 in (15) – estimating t = 1/k2). Therefore, dc/dt as determined by taking the derivative of (15),

$$dc/dt = -k1*k2*[e \wedge (-k_2*t)] , \quad (16)$$

Again, from equations (12) and (14) for Hubble's constant we see that it is not constant but is a function of R. Along the same lines as we did with the potential energy derivation, we take the derivative dH/dr and multiply by (dr/dt = c) to arrive at dH/dt = $4.98 \times 10^{-36}$ seconds$^{-2}$. This is about one-third the rate of change in H estimated by the potential-energy derivation.

We can also set (14) equal to approximately twice (6):

$$k2 = h / (2 * \text{mass of photon} * r^2) = 2 * 1/2*[(G*\text{Mass of Universe})^{1/2}]*r^{-3/2} . \quad (17)$$

From (17) we can solve for G, the gravitational constant,

$$G = [h / (2 * \text{mass of photon} * r^{1/2})]^2 * (\text{Mass of Universe})^{-1} = 1.2 \times 10^{-10} . \quad (18)$$

Compared to the traditional value for G of $6.67 \times 10^{-11}$, the results of (18) are about twice this value.

It can also be shown by manipulation of (17) that,

$$G * \text{Mass of Universe} / r = [h / (2 * \text{mass of photon})]^2 , \quad (19)$$

Or that the potential energy of the mass of the Universe on any object is a function of Planck's constant and the mass of the photon. And based on (19), we can quantize gravitational potential energy levels as follows:

$$G * \text{Mass of Universe} / (n^2 * r) = [h / (2 * n * \text{mass of photon})]^2 , \quad (20)$$

where n goes from 1 to infinity.

The quantization of the gravitational field may explain the pattern of galactic clusters where there are open spaces that are completely void of galaxies and then dense clusters of galaxies. This quantization becomes more apparent at the larger scale of galaxies where the potential energies are higher (n is closer to 1) and the quantization appears more continuous at smaller scales such as that of our solar system (where n approaches infinity).

The derivation of Hubble's constant has been shown by using concepts of Mach's Principle and Special Relativity (gravitational potential energy = rest-mass energy), and from Quantum Mechanics (matter-wavelength formula) and by assuming in the derivation a change in the speed of light as the radius of the Universe increases. A very close match to the measured Hubble's constant was found in both cases and a decay rate for the speed of light was determined to be the same by both cases. By setting the equations for Hubble's constant in both energy and matter-wave forms equal to each other, a relationship between gravitational potential energy and Planck's constant is found, resulting in the quantization of the gravitational field.

It is hoped that the developments in this paper will inspire investigations into the speed of light decay and measurement of gravitational potential energy at quantized levels. The predicted mass of the photon as suggested herein is also cause for more investigation, as current upper limits on the photon's mass are still above the predicted value by a factor of about $10^{10}$.

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