Few possible infinite sets of triplets of primes related in a certain way and an open problem

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Abstract. In this paper I make three conjectures about a type of triplets of primes related in a certain way, i.e. the triplets of primes [p, q, r], where 2*p^2 – 1 = q*r and I raise an open problem about the primes of the form q = (2*p^2 – 1)/r, where p, r are also primes.

Conjecture 1:

There exist an infinity of primes p such that 2*p^2 – 1 = q*r, where q and r are also primes.

Examples: such primes are: 5, 19, 23, 29, 31, 47, 53, 61, 67, 71, 79, 83, 97 (...).

Conjecture 2:

If p is prime and 2*p^2 – 1 = q*r, where q and r are also primes, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)*(r + n), such that p + m, q + n and r + n are also primes.

Examples:

: for p = 5, [q, r] = [7, 7]; for [m, n] = [24, 34], [p + n, q + n, r + n] = [29, 41, 41];

: for p = 19, [q, r] = [7, 103]; for [m, n] = [34, 34], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 23, [q, r] = [7, 151]; for [m, n] = [44, 40], [p + n, q + n, r + n] = [67, 47, 191];

: for p = 31, [q, r] = [17, 113]; for [m, n] = [22, 24], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 71, [q, r] = [17, 593]; for [m, n] = [26, 13], [p + n, q + n, r + n] = [97, 31, 607];

: for p = 83, [q, r] = [23, 599]; for [m, n] = [254, 210], [p + n, q + n, r + n] = [307, 233, 809]; also for [m, n] = [198, 258], [p + n, q + n, r + n] = [347, 281, 857];

: for p = 139, [q, r] = [17, 2273]; for [m, n] = [250, 110], [p + n, q + n, r + n] = [389, 127, 2383].

Conjecture 3:

If p is prime and 2*p^2 – 1 = q^2, where q is also prime, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)^2, such that p + m and q + n are also primes.

Example:

: for p = 5, q = 7; for [m, n] = [24, 34], [p + n, q + n] = [29, 41].

Open problem:

Which primes q can be written as q = (2*p^2 – 1)/r, where p, r are also primes?