# Can gravitons be effectively massive due to their *zitterbewegung* motion?

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**Abstract**: Following up on an earlier, De Broglie-Bohm approach within the framework of quantum gauge theory of gravity, and based on the Schrödinger-Dirac equation for gravitons, we argue that gravitons are effectively massive due to their localized circulatory motion. This motion is analogous to the proposed *zitterbewegung* (ZB) motion of electrons.

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## 1. Introduction

Following, D. Hestenes, the idea that the electron spin and magnetic moment are generated by a localized circulatory motion of the electron has been proposed independently by many physicists [1]. Schrödinger's *zitterbewegung* (ZB) model for such motion is especially noteworthy because it is grounded in an analysis of solutions to the Dirac equation [2-4]. Surely, if the ZB motion is a real physical phenomenon, then it tells us something fundamental about the nature of the electron. The role ascribed to the ZB motion in standard formulations of quantum mechanics, nonetheless, has been metaphorical at best.

The ZB of photons is studied via the momentum vector of the electromagnetic field. These studies show that ZB motion can occur only in the presence of virtual longitudinal and scalar photons [28-30]. The vector property of this motion is described by the polarization vectors of the electromagnetic field [28-30].

Various workers have attempted to derive General Relativity from a gauge-like principle, involving invariance of physics under transformations of the locally (i.e. in the tangent space at each point) acting Lorentz or Poincare group. ([5-7]). N. Wu [8−11] proposed a *Quantum Gauge Theory of*

*Gravity* (QGTG) based on the gravitational gauge group (G). In Wu's theory, the gravitational interaction is considered as a fundamental interaction in a flat Minkowski space-time and not as space-time geometry. A model of interacting massive gauge gravitons, and a proposed heavy gauge graviton resulting from shell decay of Higgs bosons, were developed recently by the author within the framework of QGTG [12-14]. In a recent paper, we also proposed the leading order approximation, a De Broglie-Bohm approach within the framework of QGTG [15].

Based on the Schrödinger-Dirac equation for gravitons, in this essay we argue that gravitons are effectively massive due to a motion analogous to the ZB motion.

**2. Fundamentals of QGTG**

According to N. Wu's theory, the infinitesimal transformations of the gravitational gauge group G can be written in the form [8]:

$$U_\varepsilon = 1 - i\varepsilon^\alpha P_\alpha, \ \alpha = 0,1,2,3, \qquad (1)$$

where $\varepsilon^\alpha$ are the infinitesimal parameters of the group, and $P_\alpha = -i\partial / \partial x^\alpha$ are the generators of the gauge group.

It is known that these generators commute each other [8]:

$$[P_\alpha, P_\beta] = 0. \qquad (2)$$

This property of the generators, however, does not imply that the gravitational gauge group is an Abelian group, since the elements of the gravitational group do not commute [8]:

$$[U_{\varepsilon_2}, U_{\varepsilon_2}] \neq 0. \qquad (3)$$

The gravitational gauge-covariant derivative is defined by:

$$D_\mu = \partial_\mu - igC_\mu(x), \qquad (4)$$

where $C_\mu(x)$ is the gravitational gauge field, and g is the gravitational gauge coupling constant. $C_\mu(x)$ is a Lorentz vector. Under gravitational gauge transformation, $C_\mu(x)$ transforms as:

$$C_\mu(x) \to C_\mu{}'(x) = \hat{U}_\varepsilon(x) C_\mu(x) \hat{U}_\varepsilon^{-1}(x) + \frac{i}{g}\hat{U}_\varepsilon(x)(\partial_\mu \hat{U}_\varepsilon^{-1}(x)), \qquad (5)$$

whereas $D_\mu$ transforms covariantly as:

$$D_\mu(x) \to D_\mu{}'(x) = \hat{U}_\varepsilon(x) D_\mu(x) \hat{U}_\varepsilon^{-1}(x). \qquad (6)$$

Gravitational gauge field $C_\mu(x)$ can be expanded in the form of linear combinations of generators of gravitational gauge group,

$$C_\mu(x) = C_\mu^\alpha(x) \cdot \hat{P}_\alpha, \qquad (7)$$

where $C_\mu^\alpha$ is the component field of the gravitational gauge field.

Although component field $C_\mu^\alpha$ resembles a second-rank tensor, this is not a tensor field. The index $\alpha$ is not an ordinary Lorentz index but a gauge group index. Since gravitational gauge field $C_\mu^\alpha$ has only one Lorentz index, it is a kind of vector field.

The strength of the gravitational gauge field is defined by the second-order Lorenz tensor:

$$F_{\mu\nu} = \frac{1}{-ig}\left[D_\mu, D_\nu\right], \qquad (8)$$

or:

$$F_{\mu\nu} = \partial_\mu C_\nu(x) - \partial_\nu C_\mu(x) - igC_\mu(x)C_\nu(x) + igC_\nu(x)C_\mu(x), \qquad (9)$$

$F_{\mu\nu}$ is a vector in group space; therefore, it can be expanded in group space as:

$$F_{\mu\nu}(x) = F_{\mu\nu}^\alpha(x) \cdot \hat{P}_\alpha. \qquad (10)$$

The explicit form of the strength of the component field:

$$F_{\mu\nu}^\alpha = (\partial_\mu C_\nu^\alpha) - (\partial_\nu C_\mu^\alpha) - gC_\mu^\beta(\partial_\beta C_\nu^\alpha) + gC_\nu^\beta(\partial_\beta C_\mu^\alpha). \qquad (11)$$

The strength of the gravitational gauge field transforms covariantly under gravitational gauge transformation. In analogy with traditional gauge field theory, the kinematical term for the gravitational gauge field can be written as:

$$\Im_0 = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}g_{\alpha\beta}F^{\alpha}_{\mu\nu}F^{\beta}_{\rho\sigma}. \qquad (12)$$

It can be demonstrated that this Lagrangian is not invariant under gravitational gauge transformation. It transforms covariantly as follows:

$$\Im_0 \to \Im_0{}' = (\hat{U}_{\varepsilon}\Im_0). \qquad (13)$$

To resume the gravitational gauge symmetry of the action, we introduce an essential factor in the form of:

$$e^{I(C)} = e^{g\eta^{\mu}_{1\alpha}C^{\alpha}_{\mu}}, \ \ I(C) = g\eta^{\mu}_{1\alpha}C^{\alpha}_{\mu}. \qquad (14)$$

The full Lagrangian $\Im$ is then given by:

$$\Im = e^{I(C)}\Im_0, \ . \qquad (15)$$

The action S for the gravitational gauge field is defined by:

$$S = \int d^4x\Im. \qquad (16)$$

It can be proven that this action has local gravitational gauge symmetry [8]. According to the gauge principle, global symmetry gives out a conserved current:

$$T^{\mu}_{i\alpha} = e^{I(C)}\left(-\frac{\partial\Im_0}{\partial(\partial_{\mu}C^{\alpha}_{\nu})}\partial_{\alpha}C^{\alpha}_{\nu} + \eta^{\mu}_{1\alpha}\Im_0\right). \ (17)$$

We call quantity $T^{\mu}_{i\alpha}$ inertial energy-momentum tensor [8].

The Euler-Lagrange equations for $C^{\alpha}_{\mu}$ gauge fields are:

$$\partial_{\mu}\frac{\partial\Im}{\partial(\partial_{\mu}C^{\alpha}_{\nu})} = \frac{\partial\Im}{\partial C^{\alpha}_{\nu}}. \qquad (18)$$

These forms are identical with those that occur in quantum field theory [8]. By inserting equation (15) into (18), we get:

$$\begin{aligned}\partial_{\mu}\frac{\partial\Im_0}{\partial(\partial_{\mu}C^{\alpha}_{\nu})} &= \frac{\partial\Im_0}{\partial C^{\alpha}_{\nu}} + g\eta^{\nu}_{1\alpha}\Im_0 \\ &-g\partial_{\mu}(\eta^{\mu}_{1\beta}C^{\alpha}_{\mu})\frac{\partial\Im_0}{\partial(\partial_{\mu}C^{\alpha}_{\nu})}\end{aligned} \ . \qquad (19)$$

Suppose that the gravitational gauge field $C^{\alpha}_{\mu}$ is very weak in vacuum, i.e. $gC^{\alpha}_{\mu} \approx 0$. In leading order approximation, by substituting equation (14) to equations (19), we obtain:

$$\partial_{\mu}\frac{\partial\Im_0}{\partial(\partial_{\mu}C_{\alpha\nu})} = -\partial_{\mu}F^{0\mu\nu}_{\alpha} = \frac{\partial\Im_0}{\partial C_{\alpha\nu}} = 0. \qquad (20)$$

The gravitons' equations of motion thus become:

$$\partial_{\mu}F^{0\mu\nu}_{\alpha} = 0. \qquad (21)$$

We define:

$$F^{\alpha}_{ij} = -\varepsilon_{ijk}B^{\alpha}_{k}, \ \ F^{\alpha}_{0i} = E^{\alpha}_{i}. \qquad (22)$$

Equation (21) then takes the form:

$$\nabla B^{\alpha} = 0, \qquad (23)$$

$$\frac{\partial}{\partial t}E^{\alpha} - \nabla\times B^{\alpha} = 0. \qquad (24)$$

Taking definitions (22) into account, we derive the following equations:

$$\nabla E^{\alpha} = 0, \qquad (25)$$

$$\frac{\partial}{\partial t}B^{\alpha} + \nabla\times E^{\alpha} = 0. \qquad (26)$$

But for their superscript α, equations (23-26) would be the ordinary Maxwell equations. In conventional quantum field theory the gravitational field in vacuum is extremely weak. The gravitational wave in vacuum, therefore, is composed of four independent vector waves.

Although the gravitational gauge field is a vector field, its component fields $C^{\alpha}_{\mu}$ have one Lorentz index μ and one group index α. Both indexes have the same behavior under Lorentz transformation – a behavior that makes the gravitational gauge field to resemble a tensor field. We thus call this gravitational gauge field a pseudo-tensor field. The spin of the gravitational gauge field, determined by its behavior under Lorentz transformation, is 2.

In conventional quantum field theory, a spin-1 field is a vector field, and a vector field is a spin-1 field. In QGTG, this correspondence is violated. This is because, unlike in conventional gauge field theory where the spin of a field is independent of the group index, in QGTG the group index contributes to the spin of a field.

**3. Can gravitons be effectively massive due to a motion analogous to ZB motion?**

In a recent paper, we proposed a De Broglie-Bohm approach to the QGTG to explain the propagation of gravitons [15].

Alternatively, the propagation of gravitons can be described by the quantity $E^{\alpha} - iB^{\alpha}$ [15], where $E^{\alpha}$ and $B^{\alpha}$ are the gravitational electric and the gravitational magnetic field [17-27], respectively.

This description is formulated in the same way in which the Schrödinger wave function $\psi = \mathrm{R}e^{iS/\hbar}$ describes the motion of material particles in de Broglie-Bohm theory [21-22].

With this choice, the physical meaning of the graviton wave function is acquired *ab initio*. Indeed, there are impressive similarities between the present formulation of QGTG and de Broglie-Bohm theory [15].

We consider the complex valued quantity:

$$\psi^{\alpha} = \frac{1}{\sqrt{2}}(E^{\alpha} - iB^{\alpha})\,[15]. \qquad (27)$$

Introducing $\psi^{\alpha}$ in the field equations of vacuum graviton (23-26), these become:

$$\nabla \cdot \psi^{\alpha} = 0, \qquad (28)$$

$$\frac{\partial \psi^{\alpha}}{\partial t} = i\nabla \times \psi^{\alpha}. \qquad (29)$$

Relation (28) can be regarded as a constraint on the field $\psi^{\alpha}$. Relation (29) can take another form by introducing the following set of hermitian matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$
$$\lambda_3 = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \qquad (30)$$

These matrices satisfy the commutation relation:

$$[\lambda_i, \lambda_j] = -\varepsilon_{ijk}\lambda_k. \qquad (31)$$

Equation (29) can then be written as:

$$i\frac{\partial \psi^{\alpha}}{\partial t} = H\psi^{\alpha}, \qquad (32)$$

or, carrying off the components,

$$i\frac{\partial \psi_i^{\alpha}}{\partial t} = H_{ij}\psi_j^{\alpha} = -(\lambda_k)_{ij}\nabla^k \psi_j^{\alpha}\,[15]. \qquad (33)$$

Equation (33) has the form of a Schrödinger-Dirac equation.

Following J.P. Vigierr [28], the graviton's velocity operator in the Heisenberg picture is defined by:

$$\frac{dU}{dt} = (i\hbar)^{-1}[U, H]. \qquad (34)$$

Kobe [28] has shown that, for U(t), equation (34) yields the value:

$$U(t) = U_{\parallel}(0) + U_{\perp}(0)$$
$$+U_{\perp}(0)\cos\omega t + \lambda U'_{\perp}(0)\sin\omega t, \qquad (35)$$

where $\omega = pc/\hbar$ is the angular frequency of the corresponding classical gravitational wave in the leading order approximation of QGTG.

The Heisenberg equation for the displacement operator:

$$\frac{dx}{dt} = (i\hbar)[x, H] = U(t) \quad (36)$$

yields by integration:

$$x(t) = x(0) + V_{||}(0)t + \int_0^t dt' \exp\{-it'\hbar' H\} U_{\perp}(0) \quad (37)$$

The first term on the right hand side of equation (37) is the initial position of the graviton. Since the graviton moves with a constant longitudinal velocity operator, $U(0)_{||}$, in the direction of its constant momentum, $\hat{p}$, the second term of equation (37) is the subsequent displacement of the graviton. The third term thus yields the time dependence of the displacement $X(t)$ due to the graviton's motion (analogous to the ZB motion). After integration of equation (37) we derive:

$$x(t) = x_1'(0) + U_{||}(0) + U_{\perp}(0)\omega^{-1}\sin(\omega t) - U_{\perp}'(0)\lambda\omega^{-1}\cos(\omega t) \quad (38)$$

- a formula tied with a constant displacement $x'(0) - x(0) = U_{\perp}'\lambda\omega^{-1}$. The last two terms of (38) evidently imply a spatial extension $x_{\perp}(t)$ resulting from a motion analogous to the ZB motion.

The displacement operator $x'(0) - x(0)$ precesses about the displacement $U_{||}(t)t$ with an angular frequency $\omega$, associated to an amplitude $c/\omega$. This suggests that the orbital angular momentum of the ZB motion corresponds to the graviton's spin operator ***S,*** associated with an effective 'relativistic graviton mass', $m_0 = E/c^2 = \hbar\omega/c^2$. The graviton's spin operator S is defined by:

$$S = \frac{1}{2}\{x_{\perp}(t) \times m_0 U_{\perp}(t) + H.c.\} = \lambda\hbar\hat{p} \quad (39)$$

The graviton can thus be considered as a particle of mass $m_0 = \hbar\omega_0/c^2$, moving around its direction of propagation $\hat{p}$, in a circle of radius $c/\omega$. The corresponding orbital momentum is $\lambda\hbar$ and its speed on this circle of radius $c/\omega$ is the distance $2\Pi(c/\omega)$, travelled in one period divided by $T = 2\pi/\omega$. This trembling motion of the graviton reduces the mass of a particle to the frequency of this motion.

The longitudinal component of the velocity associated with a constant energy $E = pc$ is constant. This longitudinal component is given by:

$$U_{||}(0) = p[p \cdot pU(0)] = i^2 pHE^{-2}. \quad (40)$$

Equation (40) is the corresponding graviton operator in the direction of the momentum, i.e. $c^2 pH_g^{-1} = c^2 pH_g E_g^{-2}$, corresponding to the gravitons Dirac energy

$$E_g = \left[\left((pc)^2 + m_g c\right)^2\right]^{\frac{1}{2}}. \quad (41)$$

## 4. Conclusion

Adopting a De Broglie-Bohm approach in Quantum Gauge Theory of Gravity QGTG, and based on the Schrödinger-Dirac equation for gravitons, we find that gravitons are effectively massive due to their localized circulatory motion which reduces the mass of the particle to the frequency of this motion. This motion is analogous to the ZB motion of electrons.

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