An interesting formula for generating primes and five conjectures about a certain type of pairs of primes

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I just enunciate a formula which often leads to primes and products of very few primes and I state five conjectures about the pairs of primes of the form [(q^2 - p^2 – 2*r)/2,(q^2 – p^2 + 2*r)/2], where p, q, r are odd primes.

Conjecture 1:

For any r prime, r ≥ 5, there exist an infinity of pairs of primes (p, q) such that the numbers (q^2 - p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Conjecture 2:

For any pair of primes (p, r), p ≥ 5, r ≥ 5, there exist an infinity of primes q such that the numbers (q^2 - p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Note:

The numbers m = (q^2 - p^2 – 2*r)/2 and n = (q^2 – p^2 + 2*r)/2, where p, q, r are odd primes, seems to be often primes and generally products of very few primes.

Examples:

: For (p, q, r) = (13, 11, 5) we have (m, n) = (19, 29);
: For (p, q, r) = (17, 11, 5) we have (m, n) = (79, 89);
: For (p, q, r) = (37, 11, 5) we have (m, n) = (1039, 1049);

: For (p, q, r) = (13, 11, 7) we have (m, n) = (17, 31);
: For (p, q, r) = (19, 11, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (23, 11, 7) we have (m, n) = (197, 211);

: For (p, q, r) = (17, 13, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (19, 13, 7) we have (m, n) = (89, 103);
: For (p, q, r) = (37, 13, 7) we have (m, n) = (593, 607);

: For (p, q, r) = (19, 17, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (23, 17, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (29, 17, 7) we have (m, n) = (269, 283);

: For (p, q, r) = (11, 7, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (13, 7, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (17, 7, 7) we have (m, n) = (113, 127);

: For (p, q, r) = (19, 11, 11) we have (m, n) = (109, 131);
: For (p, q, r) = (31, 11, 11) we have (m, n) = (409, 431);
: For (p, q, r) = (61, 11, 11) we have (m, n) = (1789, 1811).

Conjecture 3:

For any p prime, p ≥ 5, there exist an infinity of primes q such that the numbers (q^2 - p^2 – 2*p)/2 and (q^2 – p^2 + 2*p)/2 are both primes.

Conjecture 4:

If x, y and r are odd primes such that y = x + 2*r, where r ≥ 5, then there exist p and q also primes such that x = (q^2 - p^2 – 2*r)/2 and y = (q^2 – p^2 + 2*r)/2.

Examples:

: For (x, y, r) = (17, 31, 7), we have (p, q) = (11, 13);
: For (x, y, r) = (29, 43, 7), we have (p, q) = (17, 19);
: For (x, y, r) = (53, 67, 7), we have (p, q) = (13, 17).

Conjecture 5:

For any p prime, p ≥ 7, there exist a pair of smaller primes (q, r) such that the numbers x = (p^2 - q^2 – 2*r)/2 and y = (p^2 – q^2 + 2*r)/2 are both primes.

Examples:

: For p = 7, (q, r) = (5, 5) and (x, y) = (7, 17);
: For p = 11, (q, r) = (5, 5) and (x, y) = (43, 53) and also
(q, r) = (7, 7) and (x, y) = (43, 53) and also
(q, r) = (7, 5) and (x, y) = (31, 41);
: For p = 13, (q, r) = (7, 7) and (x, y) = (53, 67) and also
(q, r) = (11, 5) and (x, y) = (19, 29) and also
(q, r) = (11, 7) and (x, y) = (17, 31).