Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture

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Abstract. In this paper I make few conjectures about a way to write an odd prime p, id est p = q – r + 1, where q and r are also primes; two of these conjectures can be regarded as generalizations of the twin primes conjecture, which states that there exist an infinity of pairs of twin primes.

Conjecture 1

(Which can be regarded as a generalization of the twin primes conjecture)

Any odd prime p can be written in an infinity of distinct ways like p = q – r + 1, where q and r are also primes; in other words, there exist an infinity of pairs of primes (q, r) such that q – r = p – 1, for any odd prime p (it can be seen that for p = 3 the conjecture states the same thing with the twin primes conjecture).

Conjecture 2

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 11 – 5 + 1; 17 – 11 + 1; 23 – 17 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k - 1 (sexy primes are the primes that differ by each other by six).

Conjecture 3

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 13 – 7 + 1; 19 – 13 + 1; 37 – 31 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k + 1.

Conjecture 4

Any prime p of the form p = 6*k - 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h - 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 5
(Which can be regarded as a generalization of the twin primes conjecture)

There exist an infinity of pairs of primes (p, q), where p is of the form 6*k – 1 and q is of the form 6*h + 1, such that q – p + 1 = 3^n, for any n non-null positive integer (it can be seen that for n = 1 the conjecture states the same thing with the twin primes conjecture).

Example: for n = 2 we have the pairs of primes (p, q): (11, 19); (23, 31) etc.; for n = 3 we have the pairs of primes (5, 31); (11, 37) etc.

Conjecture 6

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1.

Example: the number 49 = 7^2 can be written as 61 – 13 + 1; 67 – 19 + 1; 79 – 31 + 1 etc.

Conjecture 7

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h - 1 and r is a prime of the form q = 6*i - 1.

Example: the number 49 = 7^2 can be written as 53 – 5 + 1; 59 – 11 + 1; 71 – 23 + 1 etc.