A New Fundamental Factor in the Interpretation of Young's Double-Slit Experiment

F. BEN ADDA*

June 2, 2014

Abstract

In this paper, we reproduce the interference pattern using only space-time geodesics. We prove that fringes and bands can be reproduced by using fluctuating geodesics, which suggests that the interference pattern shown to occur with electrons, atoms, molecules and other elementary particles might be a natural manifestation of the space-time geodesics for the small scale world.

Pacs: 02.70.-c, 42.25.Hz

1 Introduction

It is known that the quantum theory does not determine the geometry of the space-time upon which it dictates the evolution of the wave function ([3]), and that the classical space-time is irrelevant for the small scale world. The double-slit diffraction plays a crucial role in our comprehension of the duality of matter. The Young's double-slit experiment ([23],[24]) was and remains the corner stone of a fundamental mystery in physics for more than 200 years: how can light be emitted and absorbed as corpuscular, and undergo interference pattern between source and detector as a wave?

In this paper, we will provide a new approach to answer the following question: is it possible to reproduce an interference pattern similar to the interference pattern observed in Young's double-slit experiment using only space-time geodesics? If a physical system in a given space-time is in free movement from one location to another following the path that requires the shortest time, then the geometry of the space-time dictates the behavior of the physical system in following the curve that minimizes the total time needed to travel between the two locations (the motion is constrained by the shape of the space time). Indeed, if the space is defined by a circular cylinder for example, then there are three possible geodesics: straight line segments parallel to the centerline of the cylinder, arcs of circle orthogonal to the centerline, and spiral helices obtained by combination of the two previous geodesics. Any free motion on the cylinder is controlled by the form of all possible geodesics that shape its geometry. Unfortunately the geodesics for the small scale world are still unknown. However, one can provide an example of geodesic that allows to produce Young's double-slit interference pattern, which induces a new understanding that might help to unlock a part of the mystery conveyed by this experiment.

*New York Institute of Technology. Email: fbenadda@nyit.edu

2 Prototype of Space-Time Geodesics

In a metric space-time, geodesics are defined to be the shortest path between two given positions, and a real understanding of the free motion of a physical system passes through our understanding of the space-time geodesics. In the case of a well defined space-time, geodesics can be found by minimizing locally the distance between two different positions using techniques of differential calculus. If the space-time is unknown, as for the small scale world (atomic world), our best understanding of the motion of a physical system is led by observations of experiments. However, some interpretations can be misled by the ignorance of the geodesics shape, and our understanding of the behavior of the physical system at small scale remains incomplete specially when our observation might alter the final state of the physical system, which is a paradox for the small scale world.

In this paper, a specific definition of geodesics that flare out from a given narrow slit and cover an angle θ will be postulated to reproduce some observed phenomena. Instead of observing the behavior of matter at small scale, we will observe paths that physical systems may take when they are free of movement in an homogenous space, and derive conclusions. More precisely using a specific geodesic (found in the simulation of an expanding space-time [4]), we will be able to reproduce the double-slit interference pattern as the one observed in Young's interference experiment.

2.1 Prototype Geodesics

Let us consider the following geodesic illustrated in two dimensions (Fig.0) defined by the graph of the function φ:

$$\varphi(x) = \sum_{i=0}^{N-1} g_i(x), \qquad \text{for} \quad N \geq 1 \tag{1}$$

where

$$g_i(x) = \begin{cases} \varphi_i(x) & \text{for} \quad x \in [2ir, 2(i+1)r] \\ 0 & \text{for} \quad x \notin [2ir, 2(i+1)r] \end{cases} \tag{2}$$

and

$$\begin{array}{rcl} \varphi_i : \ [2ir, 2(i+1)r] & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \varphi_i(x) \ = \ (-1)^i \sqrt{r^2 - \Big(x - (2i+1)r\Big)^2}, \end{array} \tag{3}$$

that verifies:

a) for $i = 0, \ldots, N-1$, the graph of φ_i represents the geodesic between two antipodal points on the circle of center $C_i = \Big((2i+1)r, 0\Big)$ and radius r;

b) for $i = 0, \ldots, N-1$, φ_i is continuous on the closed interval $[2ir, 2(i+1)r]$;

c) for $i = 0, \ldots, N-1$, φ_i is differentiable on the open interval $]2ir, 2(i+1)r[$;

d) for $i = 0, \ldots, N-1$, φ_i is not differentiable at the points $x_i = 2ir$ and $x_{i+1} = 2(i+1)r$.

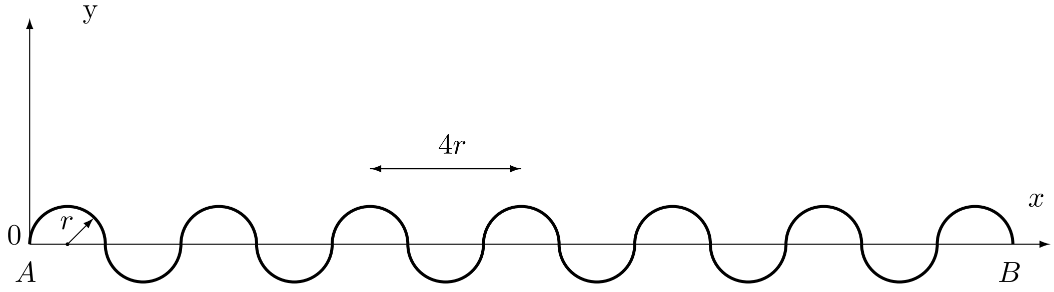

Figure 0: Geodesic in 2D for $N = 13$ and radius $r = 5\ mm$ between two points A and B.

The xy-plane in Fig.0 represents the plane of fluctuation of the geodesic and the x-axis is the geodesic axis that represents the geodesic overall direction. Since the geodesic given by (1) verifies the property a), then in two dimensions there exist two possible geodesics between two antipodal points of one circle of center $C_i = \Big((2i+1)r, 0\Big)$ and radius r for all i, and then between $A = (0,0)$ and $B = (26r, \varphi_i(26r))$ for example there exist 2^{13} geodesics that represent the path of least time between the two locations A and B, which means that any physical system that follows the path of least time in the plane will have 2^{13} possibilities and it is impossible to predict from which path the physical system will pass through. To reproduce the interference pattern in a given plane, we don't need a mathematical modeling of the geodesics in three dimensions, only the geodesics defined by (1) will be used within this paper.

2.2 Modeling Geodesics that cover an angle θ from a slit

Let us consider the function φ defined by (1). We denote the graph of φ by

$$G_\varphi = \{(x,y) \in \mathbb{R}^+ \times \mathbb{R} \ / \ y = \varphi(x) \ \} \tag{4}$$

and

$$G_{R_\theta(\varphi)} = \Big\{(x',y') \in \mathbb{R}^2 \ / \ \begin{pmatrix} x' \\ y' \end{pmatrix} = R_\theta \begin{pmatrix} x \\ y \end{pmatrix}, \text{with } \begin{pmatrix} x \\ y \end{pmatrix} \in G_\varphi \ \Big\} \tag{5}$$

and where R_θ is the rotation of center $(0,0)$ and angle θ given by

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{6}$$

To graph all geodesics starting from a slit S_1 and covering an angle θ between the slit's screen and a distant detector screen, we use the superimposition of graphs of geodesics given by the graph of the function (1). We take the center of the slit S_1 as the origin with coordinates $(0,0)$, and using a subdivision of θ into unit angle, we superimpose all the graphs $G_{R_\theta(\varphi)}$ for θ in $\{-\frac{\theta}{2}^\circ, .., -2^\circ, -1^\circ, 0, 1^\circ, 2^\circ.., +\frac{\theta}{2}^\circ\}$. If we add another slit S_2 of coordinates $(0,-d)$, then to represent all possible geodesics emerging from two distant slits to any distance in the right side of the slit's screen, we superimpose on the same graph the geodesics emerging from the slit S_1 with their translation from the slit S_2 using the translation $T_{\overrightarrow{v}}$ of vector $\overrightarrow{v} = \overrightarrow{S_1S_2}$. To simulate the movement of the slit S_2 on the y-axis, we vary d in the coordinates of the vector $\overrightarrow{v}$.

3 Interference of Geodesics in Phase

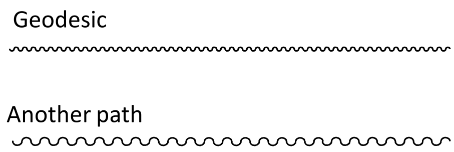

Figure 1: The first path is a geodesic defined by (1), between two arbitrary locations, with radius $r_0 = 0.25\ mm$. The second path is a perturbed geodesic defined by (1), between the same arbitrary locations, with radius $r = 0.625\ mm$.

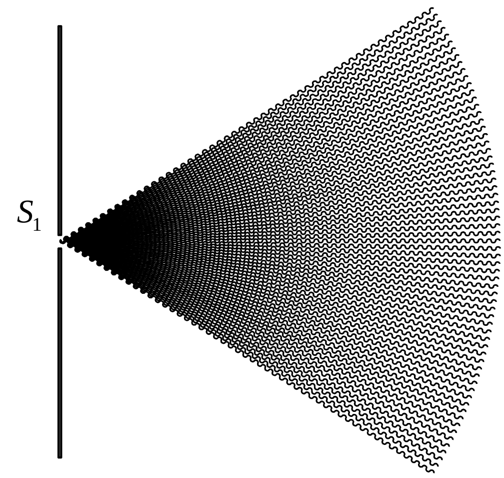

Figure 2: Illustration of 65 geodesics, using equation (1) for $r_0 = 0.25\ mm$, that cover an angle of $64°$ from a narrow slit, to represent all possible geodesics that diffract from the slit to any detector screen on the right side.

To simulate all possible geodesics that can be followed by a physical system through the region between two slits and any detector screen located on the right side of the slits screen, we superimpose two copies of all possible geodesics in phase that cover an angle $\theta = 64°$ to simulate Young's double-slit experiment. Let us consider the geodesics defined by (1) (see Fig.1), that cover the distance between two given positions $A = (0, 0)$ and $B = (x, \varphi_i(x))$ for $r_0 = 0.25\ mm$. For the small scale world, r can be any infinitesimal number, thus any simulation with these tiny numbers requires adequate tools for a clear observation. The process of simulation of geodesics emerging from the slits S_1 and S_2 is as follow:

(i) Using the graphs (5), and by considering the slit S_1 as the origin, we superimpose all the graphs $G_{R_\theta(\varphi)}$, for $r_0 = 0.25\ mm$, and for θ in $\{-32°, .., 0, .., +32°\}$, and we obtain the illustration given by Fig.2.

(ii) To represent geodesics emerging from a second slit S_2, at a distance d from S_1, that are in phase with those emerging from the slit S_1, we translate all the geodesics that emerge from the slit S_1 using the translation $T_{\overrightarrow{S_1S_2}}$ with $\overrightarrow{S_1S_2} = \begin{pmatrix} 0 \\ -d \end{pmatrix}$.

The representation, on the same graph, of all possible geodesics that emerge from the slit S_1 and all possible geodesics that emerge from the slit S_2 allows to simulate Young's double-slit experiment. Indeed, the superimposition of the two families of geodesics in phase exhibits interference pattern of fringes visible in the whole intersection region between geodesics (see the simulation in Fig.3 obtained for $d = 2\ mm$ and the simulation in Fig.4 obtained for $d = 4\ mm$). The bigger the distance between the two slits is, the larger the number of fringes is, and the closer the fringes are. Using the superimposition of all possible geodesics that can be followed by a physical system, we are able to reproduce the observed interference fringes in Young's interference experiment with the same properties. Moreover these fringes are observable not only on the detector screen, but in the whole region of geodesics intersection between the slits screen and the detector screen. When parts of geodesics exactly coincide they form a clear spot (the presence of the physical system is double), if not they form a dark spot of single paths.

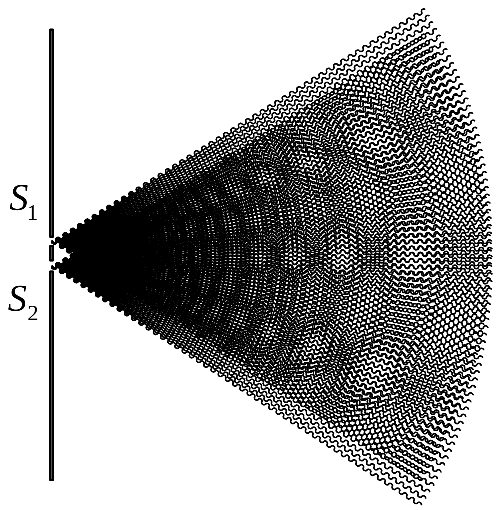

Figure 3: The superposition of family of geodesics of radius $r_0 = 0.25mm$ from the S_1 with the family of geodesics with the same radius from the slits S_2 generate fringes or bands in the geodesics intersection region, $d(S_1, S_2) = 2\ mm$.

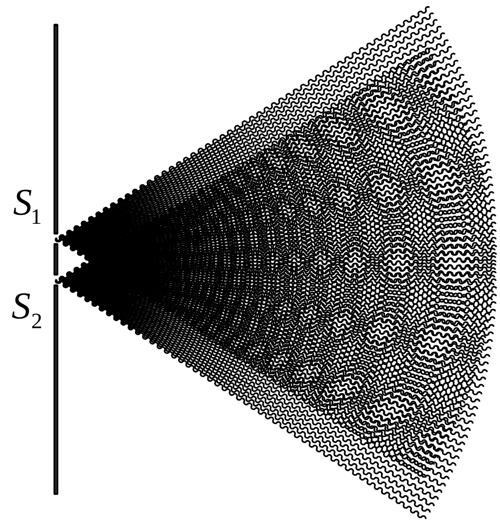

Figure 4: For the same families, the number of fringes increases when the distance between the slits S_1 and S_2 increases, $d(S_1, S_2) = 4\ mm$.

The more the geodesics tend to straight line geodesics, the less we can observe the fringes. The geodesic defined in (1) becomes a straight line as r tends to zero, and then the interference pattern disappears (see Fig.7).

4 Interference with Perturbation

A perturbation that modifies the path of a given physical system in free motion by modification of the amplitude of its geodesic fluctuation can be simulated by the use of a perturbed geodesic defined by (1) with a radius $r_1 = r_0 + \varepsilon$ (see Fig.1) for a perturbation ε that can be positive or negative. Experimental observations use light, and a photon or an electron cannot be detected without interaction with photons. Perturbation is used here in the sense that the physical system, by interaction with light, will follow another perturbed path that represents the new path of least time for the new physical system state after interaction. This new path of least time is obtained by modification of the amplitude of the geodesic fluctuation.

What happens to the observed interference pattern produced by the superimposition of geodesics from two slits (Fig.3 for example) if we add a perturbation to one of the two families of geodesics? Using the process (i), we can graph all geodesics with radius $r = r_0$ that emerge from the slit S_1, and using the process (ii) for $r = r_1$, we can represent the superimposition on the same graph of all possible geodesics that emerge from the slit S_1 with all perturbed geodesics that emerge from the slit S_2, respectively for $\varepsilon = 0.05\ mm$ and $\varepsilon = 0.375\ mm$. The result of the superimposition of the two families is given by the simulation obtained respectively in Fig.5 and Fig.6 (the simulation produced with $\varepsilon = -0.05\ mm$ and $\varepsilon = -0.375\ mm$ led to the same conclusion as for Fig.5 and Fig.6). In Fig.5 for $r_1 = 0.3\ mm$, degraded interference can be discerned with important disorder, meanwhile the interference pattern totally disappears in Fig.6 for $r_1 = 0.625\ mm$.

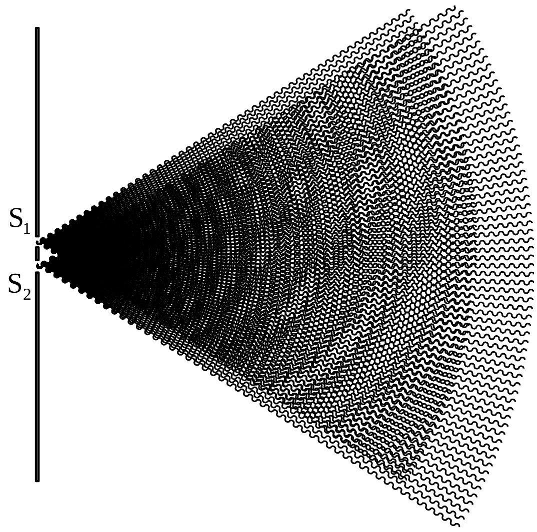

Figure 5: Important disorder in the interference pattern generated by the superposition of the family of geodesics of radius $r_0 = 0.25\ mm$ from the slit S_1 with the family of perturbed geodesics of radius $r_1 = 0.3\ mm$ from the slit S_2, $d(S_1, S_2) = 2\ mm$

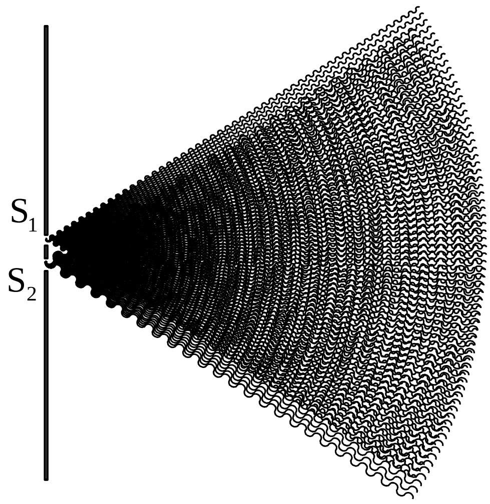

Figure 6: The interference pattern disappears when we superimpose the family of geodesics of radius $r_0 = 0.25mm$ from the slit S_1 with a family of perturbed geodesics of radius $r_1 = 0.625\ mm$) from the slit S_2, $d(S_1, S_2) = 2\ mm$

Perturbation of geodesics might explain the absence of interference pattern in the Young's double-slit experiment when detection is performed to locate from which slit the physical system passes through. The simulation given in Fig.5 provides a degraded interference pattern for perturbed geodesics with a perturbation $\varepsilon < r_0$ (some experiments were performed to demonstrate that degraded interference pattern could be obtained using particle detectors ([10],[15],[22])), meanwhile for a perturbation $\varepsilon \geq r_0$, the interference pattern completely disappears (see Fig.6), which suggests that observation must be conducted with minimal perturbation to minimize the effect of observation on altering the final state of the physical system. A more precise simulation of superimposition of geodesics shows that total disorder occurs for a perturbation starting from $\varepsilon = \frac{r_0}{2}$ as indicated in the following table:

Slit S_1	Slit S_2	$d(S_1, S_2)$	Interference
$r_0 = 1mm$	$r_1 = 1.1mm$	10mm	appearance of disorder
$r_0 = 1mm$	$r_1 = 1.2mm$	10mm	important disorder
$r_0 = 1mm$	$r_1 = 1.3mm$	10mm	more important disorder
$r_0 = 1mm$	$r_1 = 1.5mm$	10mm	total disorder

A more precise simulation can be elaborated to determine the minimal perturbation of paths that induces the total destruction of interferences.

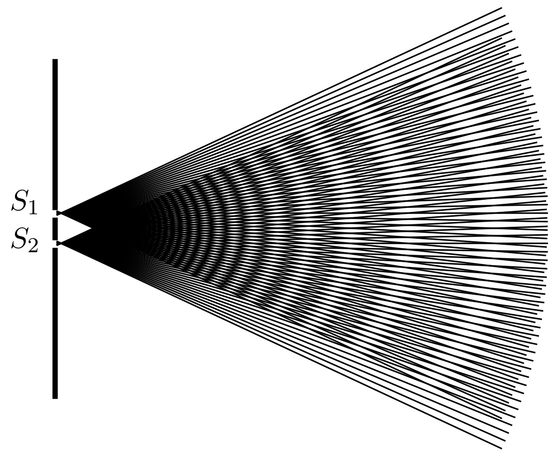

Figure 7: Straight line Geodesic in 2D. No appearance of fringes or bands in the geodesics intersection region for a line geodesics at distant slits. The angle of rotation of each geodesic is 1°, the distance of slits is $d(S_1, S_2) = 4\ mm$.

5 Interpretation of the Geodesic's Interference

All the simulations presented within this paper and represented by the 7 figures are highly reproducible. We did not involve any particular physical system (photon, proton, neutron, electron, and atoms) known to produce the interference pattern in Young's double-slit experiment ([1],[2],[5],[6],[7],[8],[9],[10],[11],[12],[13],[14],[15],[16],[17],[18],[19],[20],[21],[25],[26]). We just use a fundamental characteristic of the space-time to reproduce observation of interference pattern of fringes and bands similar to those observed in Young's double-slit experiment.

Producing the same phenomenon (interference of fringes) using graphs puts into question all the previous interpretations of the interference pattern, observed in Young's double-slit experiment, that relate this observed phenomenon to the wave nature of the observed physical system. Observing interference pattern that emerges with different kinds of physical systems as photon, proton, neutron, electron, and atoms that are well known as elementary particles, calls into consideration our understanding and interpretation of this phenomenon. The common factor for all these physical systems is the local characteristics of the space-time for the small scale world, and these characteristics (actually the geodesics) generate interference pattern of fringes and bands when they are superimposed. Meanwhile a perturbation of geodesics by observation in one of the two slits induces a degradation of fringes or their disappearance.

5.1 Physical Limit and Theoretical Limit

The geodesics defined by (1) used to produce Young's interference pattern for the double-slit experiment allow to distinguish the theoretical limit and the physical limit. Indeed:

1. for $r = 0$, the graph of the geodesic (1) is a straight line, meanwhile for $r > 0$ the graph of the geodesic (1) is the graph of a function with points of non differentiability; the smaller r is, the bigger the number of points of non differentiability is, which means that the smaller r is, the more the geodesics are unreachable (since the number of points with undefined tangent increases as the radius r decreases). For example, if $4r = 5.5 \times 10^{-7}m$ which is the distance between two successive maxima (that corresponds approximatively to the average of light wave length), then within 1.1 mm there are 4000 points of non differentiability (antipodal points).

2. at the scale for which two successive antipodal points of non differentiability of the geodesic (1) are indistinguishable under any possible physical observation, these geodesics are experimentally unreachable (example for $r = \frac{l_p}{2}$, where l_p is the Plank length) and theoretically they are piecewise reachable. However, they produce the interference pattern of Young's double-slit experiment for $r > 0$.

3. To obtain an infinity of geodesics between two arbitrary locations in three dimensions (see Fig.8), it is sufficient to rotate the graph of the geodesic (1) with respect to the x-axis and to use the existence of an infinity of geodesics between two antipodal points of one sphere to assert the existence of an infinity of geodesics in three dimensions between two arbitrary locations in space, where the fluctuation of the geodesic up and down in the xy-plane for example corresponds to a polarization in the y-direction. Any physical system assumed to follow paths of least time (between two given locations) defined by the geodesics illustrated in Fig.8, in an homogenous space-time will have an infinity of paths of least time defined by (1) in the space and it will be impossible to determine from which path the physical system passes through to travel from the two distant locations, which remains consistent with the quantum indeterminacy. The superimposition of this family of infinity of geodesics that covers an angle θ from two distant slits also produces the interference pattern observed in Young's double-slits experiment.

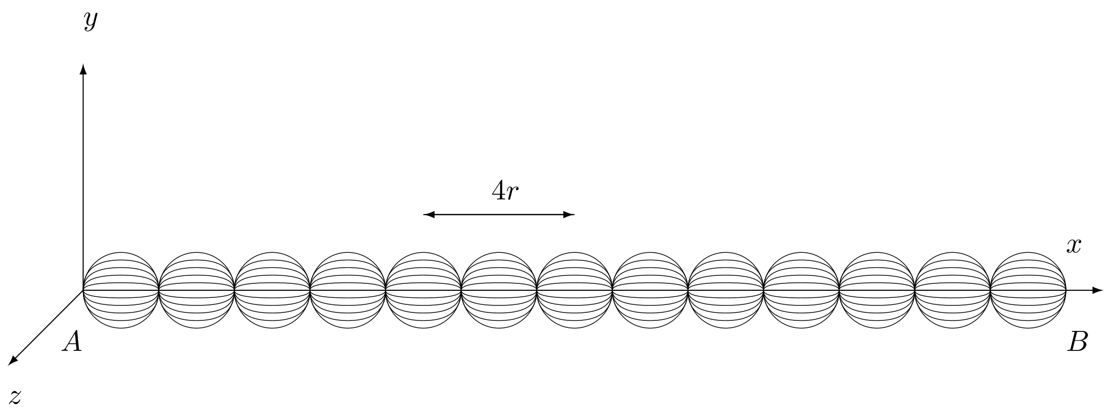

Figure 8: Geodesics in 3D for $N = 13$ and radius $r = 5$ mm between two points A and B.

5.2 Conclusion

For more than 200 years, the scientific community has thought that it was impossible to explain the interference pattern using defined paths, and that the interference pattern was a pure manifestation of the wave nature of the physical system. The example of geodesics used within this paper to reproduce the interference pattern conveys interesting ideas: (i) the interference pattern of Young's double-slit experiment might be a manifestation of the space-time geodesics since these geodesics clearly reproduce the interference pattern observed in Young's double-slit experiment, and explain why and how certain regions can be privileged for a physical system that follows these geodesics meanwhile other regions are almost forbidden for them, (ii) the perturbation of these geodesics provides an explanation of how observation affects the final stage of the physical system that follows these possible geodesics, (iii) the interpretation that relates the interference pattern observed in Young's double-slit experiment to the wave behavior of the physical system is not sustainable as the only interpretation, it could also be explained with paths and trajectories, and a rigorous interpretation cannot be considered as a reality if the space-time characteristics and properties are ignored (the interpretation of the interference pattern as water wave, or sound wave or any type of wave is an approximation of the observed phenomenon but it is not the only one, then it cannot be used as a determinant factor for the nature of the physical system), and (iv) reproducing the interference pattern by the geodesics of the space-time for the small scale word is a fundamental criteria for its consistency. Indeed, the straight line geodesic has to be excluded for the small scale world since it does not produce any interference pattern, meanwhile a geodesic with a tiny periodic fluctuation may lead to the geometry of the space-time for the small scale world.

Nevertheless, the diffraction of the geodesics from the slits to covers an angle θ was postulated in this work. To complete the interpretation of the interference pattern using geodesics, it is primordial to explain why the geodesics flare out (diffract) into the region beyond the slit that covers the angle θ. In the wave approach there is no clear difference between diffraction and interference, meanwhile, using geodesics the diffraction can be explained without interference and this will be the subject of the next paper. The main objective of this work is to prove that one can reproduce (and then model) the interference pattern observed in Young's double-slit experiment and explain it only with space-time geodesics (using path and trajectory), which suggests that this phenomenon might be a manifestation of the space-time characteristics for the small scale world.

The used geodesics (1) clearly reproduce interference pattern of fringes and bands, and systematic methods based on calculus of variation involving differential equation for the solutions of Fermat's principle are not suitable for these geodesics because of the existence of a big number of points of non differentiability for the small scale. Moreover this method does not distinguish the different small scales in its formalism, which means that the classical differential calculus is irrelevant to find the geodesics for the small scale world since the partial derivative is defined as a limit entity and we know that under the Planck length, all our tools have no physical sense. A new approach is needed to find the solutions of Fermat's principle of least time for the small scale world.

References

[1] Arndt M. et al., *Wave-particle duality of C60 molecules*, Nature, 401, 680682, 1999.

[2] Bach R. et al., *Controlled double-slit electron diffraction*, New Journal of Physics, 15, 033018, 2013.

[3] Bacry H., *Localizability and space in quantum Physics*, Lecture note in physics, Vol. 308, Springer Verlag, 1988.

[4] Ben Adda F., Porchon H., *Light Fractal Geodesics in an Expanding Space-Time*, arXiv:1302.1490v1, 2013.

[5] Carnal O., Mlynek J., *Young's double-slit experiment with atoms: a simple atom interferometer*, Phys. Rev. Lett., 66, 2689-2692, 1991.

[6] Crease R. P., *The most beautiful experiment*, Physics World, 15, 19-20, 2002.

[7] Donati O. et al., *An Experiment on Electron Interference*, American Journal of Physics, 41:639-644, 1973.

[8] Eibenberger S. et al., *Matter-wave interference with particles selected from a molecular library with masses exceeding 10000 amu*, Physical Chemistry Chemical Physics, 15, 14696-14700, 2013.

[9] Gilson G., *Demonstrations of Two-Slit Electron Interference*, Amer. J. Phys., 57, 680, 1989.

[10] Greenberger D.M., Yasin A., *Simultaneous wave and particle knowledge in a neutron interferometer*, Physics Letters A, 128, 391-4, 1988.

[11] Hackermüller L. et al., *Wave nature of biomolecules and fluorofullerenes*, Phys. Rev. Lett., 91, 090408 , 2003.

[12] Jönsson C., *Electron diffraction at multiple slits*, American Journal of Physics, 42:4-11, 1974.

[13] Matteucci G., *Electron wavelike behavior: A historical and experimental introduction*, Amer. J. Phys., 58, 1143-1147, 1990.

[14] Merli P. G. et al., *On the statistical aspect of electron interference phenomena*, American Journal of Physics, 44, 306-307, 1976.

[15] Mittelstaedt P.et al., *Unsharp particle-wave duality in a photon split-beam experiment*, Foundations of Physics, 17, (9): 891-903, 1987.

[16] Nairz O. et al., *Diffraction of Complex Molecules by Structures Made of Light*, Phys. Rev. Lett., 87, 160401, 2001.

[17] Nairz O. et al., *Quantum interference experiments with large molecules*, American Journal of Physics, 71:319-325, 2003.

[18] Steeds J., *The double-slit experiment with single electrons*, Physics World, 16, 20, 2003.

[19] Taylor G. I., *Interference fringes with feeble light*, Proceedings of the Cambridge Philosophical Society, 15, 114-115, 1909.

[20] Tonomura A., *The double-slit experiment with single electrons*, Physics World, 16, 20-21, 2003.

[21] Tonomura A. et al., *Demonstration of singleelectron buildup of an interference pattern*, Amer. J. Phys., 57, 117-120, 1989.

[22] Wootters W. K., Zurek W. H., *Complementarity in the double-slit experiment: Quantum nonseparability and a quantitative statement of Bohr's principle*, Phys. Rev. D, 19 473-484, 1979.

[23] Young T., *The Bakerian Lecture: On the Theory of Light and Colours*, Philosophical Transactions of the Royal Society of London, 92: 12-48, 1802.

[24] Young T., *A Course of Lectures on Natural Philosophy and the Mechanical Arts*, London :Printed for J. Johnson, Vol. 1, Lecture 39, pp. 463-464, 1807.

[25] Zeilinger A. *Experiment and the foundations of quantum physics*, Reviews of Modern Physics, 71, (2): S288. 1999.

[26] Zeilinger A. et al., *Single-and double-slit diffraction of neutrons*, Rev. Mod. Phys., 60, 1067-1073, 1988.