# Two conjectures involving the sum of a prime and a factorial number

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**Abstract.** In this paper I state two conjectures about the sum of a prime and a factorial.

**Note:**

For a list of factorial numbers see the sequence A000142 in OEIS.

**Conjecture 1:**

For any odd prime p there exist at least one prime q such that p + n! = q, where n is a positive integer, n < p.

**Verifying the conjecture:**
(for the first five odd primes p)

: 3 + 2! = 5, so [p, q, n] = [3, 5, 2];
: 5 + 2! = 7 and 5 + 3! = 11 and 5 + 4! = 29, so [p, q, n] = [5, 7, 2] or [5, 11, 3] or [5, 29, 4];
: 7 + 3! = 13 and 7 + 4! = 31 and 7 + 5! = 127 and 7 + 6! = 727 so [p, q, n] = [7, 13, 3] or [ 7, 31, 4] or [7, 127, 5] or [7, 727, 6];
: 11 + 2! = 13 and 11 + 6! = 17 and 11 + 5! = 131 and 11 + 7! = 5051 and 11 + 10! = 3628811 so [p, q, n] = [11, 13, 2] or [11, 17, 6] or [11, 131, 10] or [11, 5051, 7] or [11, 3628811, 10];
: 13 + 3! = 19 and 13 + 4! = 37 and 13 + 6! = 733 so [p, q, n] = [13, 19, 3] or [13, 37, 4] or [13, 733, 6].

**Note:**

From the primes q, q ≥ 5, q ≤ 401, just three primes can't be written as p + n!, where p is a lesser odd prime and n is a positive integer, *i.e.* the primes 41, 101, 367 (but, interesting, 367 can be written as 7^3 + 5!); indeed:

: q can be written as p + 2! for q = 5, 7, 31, 43, 61, 73, 103, 109, 139, 151, 181, 193, 199, 229, 241, 271, 283, 313, 349 [...];
: q can be written as p + 3! for q = 11, 13, 17, 23, 29, 37, 47, 53, 59, 67, 79, 89, 107, 113, 157, 163, 173, 179, 197, 239, 251, 257, 263, 269, 277, 337, 359, 373, 379, 389 [...];

: q can be written as p + 4! for q = 71, 83, 97, 127, 131, 191, 223, 251, 281, 293, 307, 317, 331, 383, 397 [...];
: q can be written as p + 5! for q = 149, 167, 227, 233, 311, 347, 353, 401 [...].

**Conjecture 2:**

For any odd prime p, p ≥ 5, there exist an infinity of primes q of the form q = (p + n!)/n^k, where n and k are positive integers and n ≥ p.

**Examples:**

: for p = 5, we have:
  : q = 29 = (6! + 5)/5^2;
  : q = 1009 = (7! + 5)/5^1;
  : q = 1613 = (8! + 5)/5^2;
  : q = 72577 = (9! + 5)/5^1;
  [...]
: for p = 7, we have:
  : q = 103 = (7! + 7)/7^2;
  : q = 823 = (8! + 7)/7^2;
  [...]
: for p = 11, we have:
  : q = 329891 = (11! + 11)/11^2;
  [...]
: for p = 13, we have:
  : q = 2834329 = (13! + 13)/13^3;
  : q = 515847877 = (13! + 13)/13^2;
  [...].