Extended Ricci and holographic dark energy models in fractal cosmology

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We consider the fractal Friedmann-Robertson-Walker universe filled with dark fluid. By making use of this assumption, we discuss two types of dark energy models: Generalized Ricci and generalized holographic dark energies. We calculate the equation of state parameters, investigate some special limits of the results and discuss the physical implications via graphs. Also, we reconstruct the potential and the dynamics of the quintessence and k-essence(kinetic-quintessence) according to the results obtained for the fractal dark energy.

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I. INTRODUCTION

Recent astrophysical observations[1–5] (the type Ia supernovae surveys, large scale structure, cosmic microwave background anisotropy spectrum) show that our present universe has an accelerated expansion. The Universe expands faster than it should be! We cannot explain this cosmic acceleration with the help of the four fundamental interactions. In modern cosmology, it is accepted that our universe has a phase transition from decelerating to accelerating due to the presence of unknown enigmatic components. Investigating the nature of these contents has been one of the great challenges in modern physics. Planck-2013 observations of the cosmic microwave background indicates that the matter in our universe is dominated by dark energy (68.3 percent) and dark matter (26.8 percent)[5]. The remaining part (4.9 percent) is occupied by other ordinary matters.

There are several proposals to be candidates for dark part of the universe, but still the nature of dark universe is completely unknown[6]. The cosmological constant is the best instrument to identify this nature of the universe, but it causes some other difficulties like fine-tuning and cosmic-coincidence puzzle[7]. Actually, it represents the earliest and simplest theoretical candidate for dark energy, but it causes some other difficulties like fine-tuning and cosmic-coincidence puzzle[8]. The former cosmologists ask why the vacuum energy density is so small[9], and the latter ones say why vacuum energy and dark matter are nearly equal today[10]. According to the type Ia supernovae observations the time-varying dark energy models give a better fit compared with a cosmological constant[1, 10]. The value of the equation-of-state parameter ω gives three different phases of dark energy such as vacuum ($\omega = 1$), phantom ($\omega < 1$) and quintessence ($\omega > 1$). Also, many other candidates have been proposed to explain the nature of dark energy[11, 12]: tachyon, K-essence, quintom, dilaton, Chaplygin gas and Polytropic gas, however still the nature of dark universe is completely unknown[6]. A good review about the dark energy problem is given by Bamba et al. in 2012[13].

On the other hand, the modified gravity may also provide a good explanation for the dark matter[14, 15]. The method may resolve the coincidence problem, describe the phase transition of the universe, and be useful for high-energy physics problems[14]. In this work, we consider a time-dependent modification of general relativity to investigate the nature of dark universe.

The plan of the paper is the following. In the next section we introduce the fractal cosmology and give some important features of the theory. In the third section, we present the dark fluid content of a universe governed by fractal gravity. Mainly, we discuss generalized Ricci and generalized holographic dark energy models. In the fourth section, we establish a correspondence between our model and the quintessence and k-essence scalar field models. The final section is devoted to conclusions of the present work.

II. PRELIMINARIES: FRACTAL COSMOLOGY

The investigation of a consistent quantum theory of gravity is one of the important problems in modern theoretical physics. We know that the most of the theories of quantum gravity introduce our universe as a dimensional flow.

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Hence, one can discuss whether, and how these interesting features are connected to the ultraviolet(UV)-divergence problem. Calcagni[16] investigated quantum gravity in a fractal universe and discussed cosmology in that framework. Fractal cosmology is power-counting renormalizable, free from ultraviolet divergence, and a Lorentz invariant. In the present study, we discuss generalized Ricci and Holographic dark energies in the framework of fractal gravity proposed by Calcagni[17]. These models describes new types of evolution. With the help of this point of view, one can see that cosmological parameters depend on time in a strongly nonlinear manner[18].

Considering that matter is minimally coupled with gravity, we write the following total action[16, 17]

$$S = \frac{1}{2\kappa^2}\int d\varrho\sqrt{-g}[R - 2\Lambda - \omega\partial_\mu \upsilon\partial^\mu \upsilon] + S_m \tag{1}$$

where G, g, R, Λ and S_m are the gravitational constant, determinant of metric $g_{\mu\nu}$, Ricci scalar, cosmological constant and matter part of the total action, respectively. Also we have $\kappa^2 = 8\pi G$. Next, υ and ω are two quantities known as the fractal function and fractal parameter, respectively. It is important to mention here that $d\varrho(x)$ is Lebesgue-Stieltjes measure generalizing the standard four-dimensional measure d^4x. The dimension of ϱ is $[\varrho] = -4\alpha$, where α is a positive parameter. In the infrared (IR) and UV regimes, the values of α are $\alpha_{IR} = 1$ and $\alpha_{UV} = \frac{1}{2}$, respectively.

The variation of the action (1) with respect to the Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2}$$

gives the Friedmann equations in a fractal universe as[17]

$$H^2 + \frac{k}{a^2} + H\frac{\dot\upsilon}{\upsilon} - \frac{\omega}{6}\dot\upsilon^2 = \frac{1}{3M_p^2}\rho + \frac{\Lambda}{3}, \tag{3}$$

$$\dot H + H^2 - H\frac{\dot\upsilon}{\upsilon} + \frac{\omega}{3}\dot\upsilon^2 - \frac{1}{2\upsilon}\partial_\mu\partial^\mu\upsilon = -\frac{1}{6M_p^2}(\rho + 3p) + \frac{\Lambda}{3}. \tag{4}$$

Here $a(t)$ is the cosmic scale factor, and it measures the expansion of the universe. The values $k = -1, 0, 1$ give the spatial curvature representing the open, flat, and closed universes, respectively. Also $H = \frac{\dot a}{a}$ is the Hubble parameter, M_p is the reduced Planck mass ($M_p^{-2} = 8\pi G$), ρ and p are the energy and pressure densities inside the universe.

The Friedmann equations (3) and (4) can be rewritten in the standard form[19, 20]

$$H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{1}{3M_p^2}\rho_t, \tag{5}$$

$$\dot H - \frac{k}{a^2} = -\frac{1}{2M_p^2}(\rho_t + p_t). \tag{6}$$

Here ρ_t and p_t are the total energy density and pressure defined as

$$\rho_t = \rho + \rho_f, \tag{7}$$

$$p_t = p + p_f, \tag{8}$$

and ρ_f and p_f are the energy and pressure due to the fractal contribution defined as below.

$$\rho_f = \omega M_p^2\dot\upsilon^2 - 3HM_p^2\frac{\dot\upsilon}{\upsilon}, \tag{9}$$

$$p_f = -HM_p^2\frac{\dot\upsilon}{\upsilon} - \frac{M_p^2}{\upsilon}\partial_\mu\partial^\mu\upsilon. \tag{10}$$

Note that if $\upsilon = 1$, from equations (9) and (10) we have $\rho_f = 0$ and $p_f = 0$, then equations (5) and (6) become the usual Friedmann equations in the Einstein gravity.

Next, the continuity equation in fractal cosmology is defined in the following form[17]

$$\dot{\rho}+(3H+\frac{\dot{\upsilon}}{\upsilon})(\rho+p)=0. \tag{11}$$

The gravitational constraint in a fractal spacetime is given by the following equation[17]

$$\dot{H}+3H^2+\frac{2k}{a^2}+\frac{1}{\upsilon}\partial_\mu\partial^\mu\upsilon+H\frac{\dot{\upsilon}}{\upsilon}+\omega(\upsilon\partial_\mu\partial^\mu\upsilon-\dot{\upsilon}^2)=0. \tag{12}$$

By considering a time-like fractal profile[17]

$$\upsilon=t^{-\beta}, \tag{13}$$

where $\beta=4(1-\alpha)$ is the fractal dimension; for the Friedmann-Robertson-Walker universe we find

$$\rho_f=-\omega M_p^2\beta^2t^{-2(\beta+1)}+3\beta HM_p^2t^{-1}, \tag{14}$$

$$p_f=\beta HM_p^2t^{-1}-\frac{\beta M_p^2}{t}\left(3H-\frac{1+\beta}{t}\right). \tag{15}$$

Here we have

$$\beta=0 \qquad \text{(in the IR regime)}, \tag{16}$$

$$\beta=2 \qquad \text{(in the UV regime)}. \tag{17}$$

Also, using $\upsilon=t^{-\beta}$ one can find the following flat Friedmann equations:

$$H^2-\beta Ht^{-1}+\frac{\omega\beta^2}{6t^{2(\beta+1)}}=\frac{1}{3M_p^2}\rho+\frac{\Lambda}{3}, \tag{18}$$

$$\dot{H}+H^2+\frac{H\beta}{t}-\frac{\omega\beta^2}{3t^{2(\beta+1)}}-\frac{\beta}{2t}(3H-\frac{1+\beta}{t})=-\frac{1}{6M_p^2}(\rho+3p)+\frac{\Lambda}{3}, \tag{19}$$

and the continuity equation

$$\dot{\rho}+(3H-\beta t^{-1})(\rho+p)=0. \tag{20}$$

On the other hand, the gravitational constraint in a fractal flat Friedmann-Robertson-Walker universe becomes

$$\dot{H}+3H^2+\left(2+\frac{3\omega}{t^{2\beta}}\right)\frac{\beta H}{t}-\frac{\beta(\beta+1)}{t^2}-\frac{\omega\beta(2\beta+1)}{t^{2\beta+2}}=0. \tag{21}$$

It is important to mention here that in the infrared regime ($\beta=0$), equations (18) and (20) give the corresponding relations in the standard Einstein's general relativity (no gravitational constraint)[17]. In the ultraviolet regime ($\beta=2$) with no cosmological constant ($\Lambda=0$), the Friedmann equations (18) and (19) yield[17]

$$H^2-2Ht^{-1}-\frac{2\omega}{6t^6}=\frac{1}{3M_p^2}\rho, \tag{22}$$

$$\dot{H}+H^2-\frac{H}{t}+\frac{4\omega}{3t^6}+\frac{3}{t^2}=-\frac{1}{6M_p^2}(\rho+3p). \tag{23}$$

In this work, we consider a universe filled with the dark energy density ρ_D and the pressureless dark matter $p_m=0$. By making use of the equation (20), the energy equations for dark energy and dark matter in the ultraviolet regime reduce to

$$\dot{\rho}_D+(3H-2t^{-1})(1+\omega_D)\rho_D=0, \tag{24}$$

$$\dot{\rho}_m + (3H - 2t^{-1})\rho_m = 0. \tag{25}$$

Here $\omega_D = \frac{p_D}{\rho_D}$ defines the equation of state parameter of the dark energy. The gravitational constraint given by equation (21) in the ultraviolet regime yields

$$\dot{H} + 3H^2 + \left(2 + \frac{3\omega}{t^4}\right)\frac{2H}{t} - \frac{6}{t^2} - \frac{10\omega}{t^6} = 0. \tag{26}$$

Solution of these equations gives us the following results[17]:

$$H(t) = -2t^{-1} - \frac{22\omega}{13t^5}\frac{\Xi(\frac{15}{4};\frac{17}{4};\frac{3\omega}{2t^4})}{\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})}, \tag{27}$$

$$a^3(t) = t^{-6}\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4}). \tag{28}$$

Here Ξ (also denoted as ${}_1F_1$ or M) is Kummer's confluent hypergeometric function of the first kind:

$$\Xi(a;b;x) \equiv \frac{\Gamma(b)}{\Gamma(a)}\sum_{n=0}^{+\infty}\frac{\Gamma(a+n)}{\Gamma(b+n)}\frac{x^n}{n!}. \tag{29}$$

Next, the deceleration parameter q is calculated as

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{30}$$

III. DARK FLUIDS

The Ricci and holographic descriptions are two noteworthy candidates of dark energy that can explain the late-time accelerated expansion of the Universe. Thence, it would be a remarkable investigation to know which one is the most favored by current cosmic observational data[21]. In this section, we mainly focus on two special forms of energy density ρ_D: the generalized Ricci dark energy and the generalized holographic dark energy. With these extended formulations, the Ricci and holographic dark energies can be recovered or defined interchangeably.

- **Generalized Ricci dark energy**

The density of generalized Ricci dark energy[21] is defined by

$$\rho_D = 3Rf_1(\frac{H^2}{R}), \tag{31}$$

where $f_1(x) = \lambda x + (1-\lambda) > 0$, and λ is an arbitrary constant ($0 \leq \lambda \leq 1$). Here, the $\lambda = 0$ case leads to the original Ricci dark energy model. The Ricci scalar for the Friedmann-Robertson-Walker universe, in terms of the hubble parameter, is calculated as

$$R = -6(\frac{k}{a^2} + 2H^2 + \dot{H}). \tag{32}$$

In the flat ($k = 0$) Friedmann-Robertson-Walker universe we have $R = -12H^2 - 6\dot{H}$. Then, for the flat Friedmann-Robertson-Walker universe we find

$$\rho_D = 3H^2(13\lambda - 12) - 18(1-\lambda)\dot{H}. \tag{33}$$

By taking time derivative of equation (33) we obtain

$$\frac{\dot{\rho}_D}{\rho_D} = \frac{6(13\lambda - 12)H\dot{H} + 18(1-\lambda)\ddot{H}}{3(13\lambda - 12)H^2 - 18(1-\lambda)\dot{H}}. \tag{34}$$

Substituting the above result into equation (24) and using the solution (27), one can find the equation of state parameter ω_D of the generalized Ricci dark energy:

$$\omega_D = -1 - \frac{1}{(3H - 2t^{-1})}\frac{\dot{\rho}_D}{\rho_D} = -1 + \frac{\mathrm{I}_D}{\mathrm{II}_D}, \tag{35}$$

where

$$\begin{aligned} \mathrm{I}_D \;=\; & -2197t^4[t^4\omega(3+\lambda) + 3\omega^2(7\lambda - 6) + t^8(10\lambda - 9)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^3 \\ & +338\omega[t^4\omega(267 - 278\lambda) + \lambda t^8(24 - 25\lambda) + 6\omega^2(21 - 22\lambda)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4}) \\ & +104\omega^2[3\omega(53\lambda - 51) + t^4(199\lambda - 192)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4}) \\ & +48\omega^3(30 - 31\lambda)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^3, \end{aligned} \tag{36}$$

$$\begin{aligned} \mathrm{II}_D \;=\; & \Big\{169[\omega^2(13\lambda - 12) + t^8(16\lambda - 15) + t^4\omega(35\lambda - 33)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2 \\ & -52\omega[t^4\lambda + \omega(4\lambda - 3)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4}) \\ & +4\omega^2(6 - 5\lambda)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\Big\}\Big\{52t^4\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4}) + 33\omega\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\Big\}. \end{aligned} \tag{37}$$

Time evolution of the equation of state parameter ω_D, equation (35), for the cases of $\lambda = 0$ and $\lambda = 1$ are plotted in Figs. 1 and 2, respectively. Figure 1 shows that at the late times ($t \to \infty$), the the equation of state parameter of the original Ricci dark energy yields $\omega_D = -1.15$. On the other hand, ω_D at early times behaves like phantom model ($\omega_D < -1$) of dark energy. Next, Figure 2 shows that at the late times ($t \to \infty$), the the equation of state parameter of the generalized Ricci dark energy, for $\lambda = 1$, yields $\omega_D = -1.25$. Besides, at early times ω_D behaves like quintessence model ($\omega_D > -1$) of dark energy.

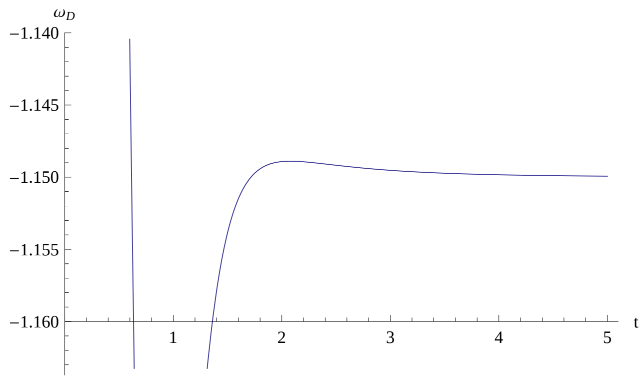

FIG. 1: Time evolution of equation of state parameter ω_D of the original Ricci dark energy ($\lambda = 0$) in fractal cosmology, equation (35), for $\omega = +1$.

By making use of these results we can calculate the energy density ρ_D and the pressure density p_D. Hence, the generalized Ricci dark energy density in exact form is found as

$$\begin{aligned} \rho_D = \frac{12}{169t^{10}\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})} & \Big\{169[\omega^2(13\lambda - 12) + t^8(16\lambda - 15) + t^4\omega(35\lambda - 33)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2 \\ & -52(t^4\lambda - 3\omega + 4\omega\lambda)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Big\}. \end{aligned} \tag{38}$$

Time evolution of the energy density ρ_D in fractal cosmology, equation (38), for the cases $\lambda = 0$ and $\lambda = 1$ are plotted in Figure 3.

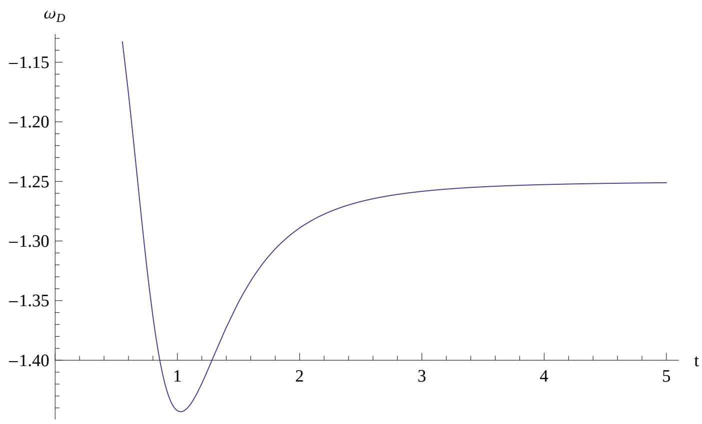

FIG. 2: Time evolution of equation of state parameter ω_D of the generalized Ricci dark energy in fractal cosmology, equation (35), for $\omega = +1$ and $\lambda = 1$.

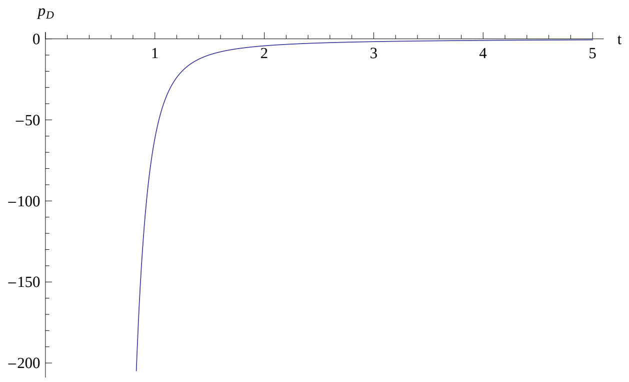

FIG. 3: For $\omega = +1$, time evolution of the original ($\lambda = 0$) Ricci dark energy density.

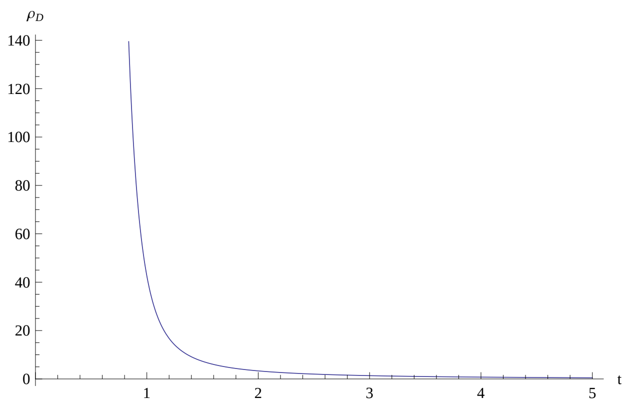

FIG. 4: For $\omega = +1$, time evolution of the generalized Ricci dark energy ($\lambda = 1$).

Next, the pressure density p_D is calculated as

$$p_D = -\frac{12\Omega_1}{\Omega_2}, \tag{39}$$

where

$$\Omega_1 = 2197[3\omega^3(13\lambda - 12) + 3t^8\omega(63\lambda - 58) + t^{12}(74\lambda - 69) + t^4\omega^2(178\lambda - 165)]\Xi(\frac{11}{4}; \frac{13}{4}; \frac{3\omega}{2t^4})^3$$

$$+338\omega[9t^4\omega(15\lambda-16)+3\omega^2(23\lambda-24)+t^8(119\lambda-123)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})$$
$$-52\omega^2(4t^4+3\omega)(103\lambda-102)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^2+24\omega^3(67\lambda-66)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^3, \quad (40)$$

$$\Omega_2=169t^{10}\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\left\{13(4t^4+3\omega)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})-6\omega\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\right\}. \quad (41)$$

Now, in the Figures 5 and 6, we plot time evolution of the pressure density p_D in fractal cosmology, equation (39), for the cases $\lambda=0$ and $\lambda=1$.

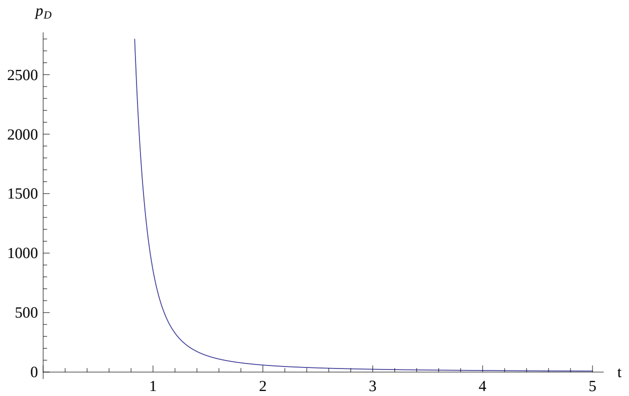

FIG. 5: For $\omega=+1$, time evolution of the pressure density when ($\lambda=0$).

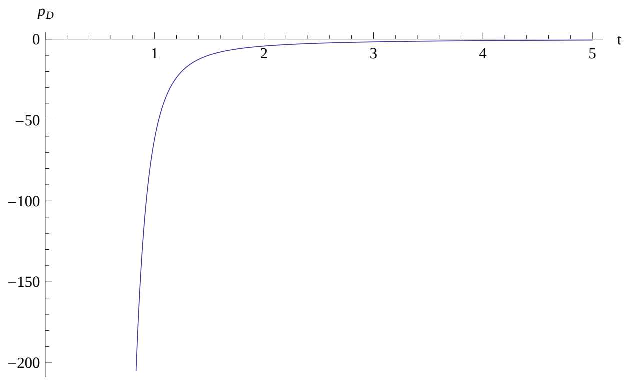

FIG. 6: For $\omega=+1$, time evolution of the pressure density when ($\lambda=1$).

On the other hand, for the deceleration parameter, we have[22]

$$q=-1-\frac{\dot{H}}{H^2}=\frac{\mathrm{I}_q}{\mathrm{II}_q}, \quad (42)$$

where

$$\mathrm{I}_q=16\omega^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^2-169(3t^4+\omega)(t^4+2\omega)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})-52\omega(2t^4+\omega)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4}) \quad (43)$$

$$\mathrm{II}_q=2\left[13t^4\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})+11\omega\Xi(\frac{15}{4};\frac{17}{4};\frac{3\omega}{2t^4})\right]^2. \quad (44)$$

- **Generalized holographic dark energy**

The density of generalized holographic dark energy[21] is given by

$$\breve{\rho}_D = 3H^2 f_2(\frac{R}{H^2}). \tag{45}$$

Here $f_2(x) = \xi x + (1-\xi) > 0$ and ξ is an arbitrary constant with $0 \le \xi \le 1$. For $\xi = 0$, we recover energy density of the original holographic dark energy whereas $\xi = 1$ leads to energy density of the original Ricci dark energy.

$$\breve{\rho}_D = 3H^2(1-13\xi) - 18\xi\dot{H}. \tag{46}$$

If we write $\lambda = 1-\xi$ in the energy density for the generalized Ricci dark energy, the relation is reduced to the one for the generalized holographic dark energy:

$$\breve{\rho}_D = \lim_{\lambda\to 1-\xi} \rho_D. \tag{47}$$

Thence, we obtain

$$\breve{\omega}_D = \lim_{\lambda\to 1-\xi} \omega_D = -1 - \frac{1}{(3H-2t^{-1})}\frac{\dot{\breve{\rho}}_D}{\breve{\rho}_D} = -1 + \frac{\breve{\mathrm{I}}_D}{\breve{\mathrm{II}}_D}, \tag{48}$$

where

$$\begin{aligned}\breve{\mathrm{I}}_D &= 2197t^4[t^4\omega(4-\xi) + 3\omega^2(1-7\xi) + t^8(1-10\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^3\\ &-338\omega[t^4\omega(278\xi-11) + (1-\xi)t^8(25\xi-1) + 6\omega^2(22\xi-1)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\\ &-104\omega^2[3\omega(2-53\xi) + t^4(7-199\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\\ &-48\omega^3(31\xi-1)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^3,\end{aligned} \tag{49}$$

$$\begin{aligned}\breve{\mathrm{II}}_D &= \Big\{169[\omega^2(1-13\xi) + t^8(1-16\xi) + t^4\omega(2-35\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\\ &-52\omega[t^4(1-\xi) + \omega(1-4\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\\ &+4\omega^2(1+5\xi)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\Big\}\Big\{52t^4\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4}) + 33\omega\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\Big\}.\end{aligned} \tag{50}$$

Thus, we find

$$\begin{aligned}\breve{\rho}_D = \frac{12}{169t^{10}\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})}&\Big\{169[\omega^2(1-13\xi) + t^8(1-16\xi) + t^4\omega(2-35\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\\ &-52(t^4 - \xi t^4 - 3\omega + 4\omega - 4\omega\xi)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Big\},\end{aligned} \tag{51}$$

$$\breve{p}_D = -\frac{12\breve{\Omega}_1}{\Omega_2}, \tag{52}$$

where

$$\begin{aligned}\breve{\Omega}_1 &= 2197[3\omega^3(1-13\xi) + 3t^8\omega(5-63\xi) + t^{12}(5-74\xi) + t^4\omega^2(13-178\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^3\\ &-338\omega[9t^4\omega(1+15\xi) + 3\omega^2(1+23\xi) + t^8(4+119\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})\\ &-52\omega^2(4t^4+3\omega)(1-103\xi)\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^2 + 24\omega^3(1-67\xi)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^3.\end{aligned} \tag{53}$$

Moreover, for $\breve{\rho}_D + \breve{p}_D$ we find

$$\breve{\rho}_D + \breve{p}_D = \frac{12\Delta}{\Omega_2}, \tag{54}$$

where

$$\Delta = 2197t^4[\omega t^4(\xi-4)+3\omega^2(7\xi-1)+t^8(10\xi-1)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^3$$
$$+338\omega[6\omega^2(22\xi-1)+7t^8(25\xi-1)+t^4\omega(287\xi-11)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})^2\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})$$
$$+104\omega^2[t^4(7-199\xi)+3\omega(2-53\xi)]\Xi(\frac{11}{4};\frac{13}{4};\frac{3\omega}{2t^4})\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^2+48\omega^3(31\xi-1)\Xi(\frac{11}{4};\frac{17}{4};\frac{3\omega}{2t^4})^3, \quad (55)$$

In this relation, by taking $\xi = 1$ we can easily write $\rho_D + p_D$ for the original Ricci dark energy. In Figure 5, under $\xi = 1$ limit, we plot $\breve{\rho}_D + \breve{p}_D$ for the generalized holographic dark energy (or $\rho_D + p_D$ for the original Ricci dark energy) in the ultraviolet regime. Figure 5 implies that we have the null energy condition, i.e. $\rho + p \geq 0$, so the scenario can be realized by an ordinary scalar field, for which $\rho + p = \dot{\phi}^2 \geq 0$.

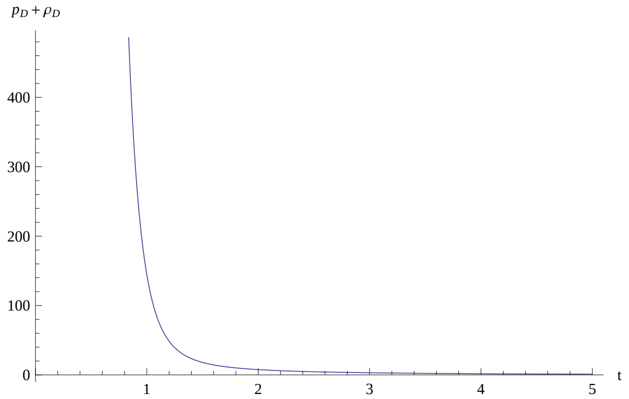

FIG. 7: For $\omega = +1$, time evolution of $\breve{\rho}_D + \breve{p}_D$ for the generalized ($\xi = 1$) holographic dark energy (or $\rho_D + p_D$ for the original ($\lambda = 0$) Ricci dark energy) in the ultraviolet regime.

IV. RECONSTRUCTION OF FRACTAL SCALAR FIELDS

In this part of the study, we reconstruct the potential and the dynamics of two scalar fields according the evolutionary form of the fractal dark fluids. Mainly, we compare the generalized Ricci dark energy density with the energy density of corresponding scalar field. It is worth to discuss this correspondence[30–35], because the scalar fields give an effective description of the dark universe. On this purpose, we consider quintessence and k-essence(kinetic quintessence) scalar fields.

- **Fractal quintessence**

The quintessence is described by the following action[8]:

$$S_Q = -\frac{1}{2}\int d^4x\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)]. \quad (56)$$

Taking the variation of this action with respect to the metric tensor $g^{\mu\nu}$ yields the energy-momentum tensor of the quintessence field.

$$T^Q_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\lambda\delta}\partial_\lambda\phi\partial_\delta\phi - g_{\mu\nu}V(\phi). \quad (57)$$

Therefore, for the quintessence field energy and pressure densities are found as[23, 24]

$$\rho_Q = \frac{\dot{\phi}^2}{2} + V(\phi), \quad (58)$$

$$p_Q = \frac{\dot{\phi}^2}{2} - V(\phi), \quad (59)$$

and the equation-of-state parameter of the quintessence field is obtained as

$$\omega_Q = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{60}$$

In this relation, it is seen that, when we have $\omega_Q < -\frac{1}{3}$, the universe accelerates for $\dot{\phi}^2 < V(\phi)$[23, 25]. Now, we can investigate the correspondence between the fractal dark fluid and the quintessence dark energy model. Comparing equation (60) with the equation-of-state parameter given in equation (35) gives $\omega_Q = \omega_D$, and equating (31) and (58) implies that we can take $\rho_Q = \rho_D$. From this point of view we obtain

$$\begin{aligned} \dot{\phi}^2 &= (1+\omega_D)\rho_D \\ &= \frac{\mathrm{I}_D}{\mathrm{II}_D}\left[3H^2(13\lambda - 12) - 18(1-\lambda)\dot{H}\right], \end{aligned} \tag{61}$$

and

$$\begin{aligned} V(\phi) &= \frac{1}{2}(1-\omega_D)\rho_D \\ &= \frac{1}{2}\left(2 - \frac{\mathrm{I}_D}{\mathrm{II}_D}\right)\left[3H^2(13\lambda - 12) - 18(1-\lambda)\dot{H}\right]. \end{aligned} \tag{62}$$

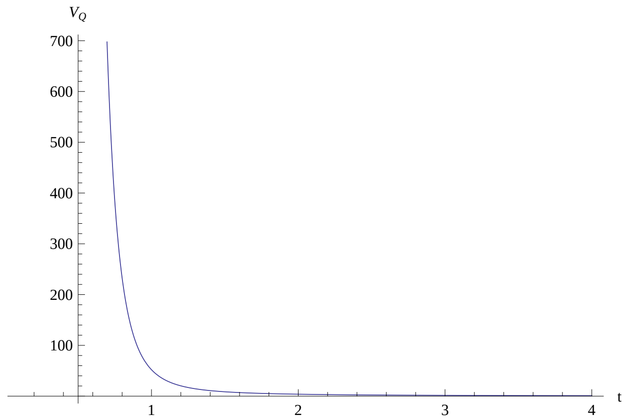

FIG. 8: For $\omega = +1$, time evolution of quintessence potential for the generalized ($\lambda = 1$) Ricci dark energy.

- **Fractal k-essence**

The general action for the k-essence scalar field model as a function of ϕ and $\chi = \frac{1}{2}\dot{\phi}^2$ is given by the following equation[26, 27]:

$$S_K = \int d^4x\sqrt{-g}p(\phi,\chi), \tag{63}$$

where the Lagrangian density $p(\phi,\chi)$ describes the density of pressure. The idea of the k-essence scalar field is used to discuss the late time acceleration of the universe, and motivated from the Born-Infeld action of string theory[25, 28, 29]. In the k-essence gravity, the energy and pressure density of the scalar field ϕ can be defined, respectively, as[23]:

$$\rho_K = f(\phi)[3\chi^2 - \chi], \tag{64}$$

$$p_K = f(\phi)[\chi^2 - \chi]. \tag{65}$$

Therefore, the equation-of-state parameter of k-essence scalar field is written as

$$\omega_K = \frac{\chi - 1}{3\chi - 1}. \tag{66}$$

This relation shows that the k-essence scalar field behaves like the phantom dark energy when the value of parameter χ lies in the interval $\frac{1}{3} < \chi < \frac{1}{2}$[25]. Equating the relation of the equation-of-state parameter of k-essence scalar field with equation (35) gives

$$\chi = \frac{\omega_D - 1}{3\omega_D - 1}. \quad (67)$$

In addition to this result, equating (31) and (64) we have

$$f(\phi) = \frac{3H^2(13\lambda - 12) - 18(1-\lambda)\dot{H}}{3\chi^2 - \chi}. \quad (68)$$

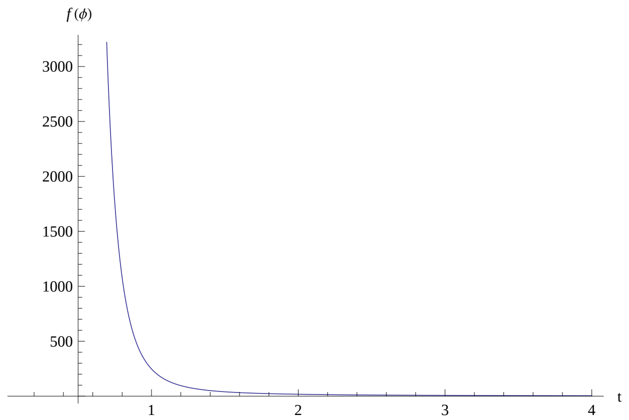

FIG. 9: For $\omega = +1$, time evolution of k-essence potential for the generalized ($\lambda = 1$) Ricci dark energy.

V. CONCLUSIONS

The motivation of fractal gravity is the following: (i) the most of quantum gravity theories define our universe as a dimensional flow and (ii) we wonder whether and how the features of dark universe are connected to the UV-divergence problem. Fractal features of quantum gravity demonstrates new types of evolution. Besides, it is known that the time-varying dark energy models give a better fit compared with the cosmological constant. The holographic dark energy is one of the most interesting dark energy models. From this point of view, we investigated the extended versions of Ricci and holographic dark energy models in fractal theory of gravity. If one write $\lambda = 1 - \xi$ in the energy density for the generalized Ricci dark energy, the definition is reduced to the one for the generalized holographic dark energy. Next, when we assume $\xi = 1$ or $\lambda = 0$, we recover the energy density of original Ricci dark energy, and conversely the case of taking $\xi = 0$ or $\lambda = 1$ gives the original holographic dark energy. We performed the required calculations to find the equation of state parameters of these dark energy models, and plotted them. The analysis was performed under the limits $\Lambda = 0$ (no cosmological constant), and $\beta = 2$ (ultraviolet regime). We got phantom and quintessence types of dark energies.

It is known that the scalar field models of dark energy can be used as an effective theory to investigate dark contents of the universe. Thence, the reconstruction of the scalar fields based on some dark energy models give very important results. This point motivated us to reconstruct the quintessence and k-essence models of dark energy. We implement a connection between generalized fractal Ricci dark energy and two scalar field models. It is important to mention here that these correspondences are very important to understand how various candidates of dark energy are mutually related to each other. Such scalar fields have very exciting feature of understanding the phantom crossing while the reconstructed scalar potential has interesting physical implications on cosmology.

Appendix: the fractal dark energy in terms of redshift z

The fractal field equations for homogenous, isotropic and flat Friedmann-Robertson-Walker in the ultraviolet regime can also be rewritten in the following form:

$$H^2 = \frac{1}{3M_p^2}\rho_t, \tag{69}$$

and

$$\dot{H} = -\frac{1}{2M_p^2}(\rho_t + p_t). \tag{70}$$

From equation (25), we find the energy density of matter as

$$\rho_m = \rho_{m_0}(1+z)^3 t^2, \tag{71}$$

where ρ_{m_0} is an integration constant which gives the present value of the dark energy density and

$$z = \frac{1}{a} - 1 = t^2 \Xi^{-1}(\frac{11}{4}; \frac{13}{4}; \frac{3\omega}{2t^4}) - 1 \tag{72}$$

is the redshift. Furthermore, we also get

$$H = -\frac{\dot{z}}{z+1}, \tag{73}$$

and

$$\dot{H} = -\frac{\ddot{z}(z+1) - \dot{z}^2}{(z+1)^2}. \tag{74}$$

So from equations (33), (73) and (74), we can found the expression for generalized Ricci dark energy density ρ_D as

$$\rho_D = 18(1-\lambda)(1-\lambda)\ddot{z}(z+1) + 3(7\lambda - 6)\frac{\dot{z}^2}{(z+1)^6}. \tag{75}$$

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