COMMENT ON THE DIRAC EQUATION

John Shim

e-mail: jshim454@gmail.com

Abstract

This paper points out that the negative energy solutions of the Dirac equation are inconsistent with the observed characteristics of the positron. It also notes that the Dirac equation is not a quantum representation of the relativistic expression for the kinetic plus rest energy of a moving charge.

Keywords: Dirac equation, positron, anti-matter, energy sign ambiguity, negative energy.

The problem with using the positron as Dirac's anti-electron is that it does not have a negative energy as defined in the Dirac model. That is, the total energy of the particles generated in electron-positron annihilations is not the difference between the total kinetic and rest energies of the initial pair, which would be required if the positron's energy were negative, but the sum, indicating that the positron's energy is positive.

The negative energy solutions in the Dirac equation arise, of course, from its second order time derivative, as opposed to the first order in the Schrödinger equation. The sign ambiguity results from Dirac's use of the expression for the relativistic kinetic plus rest energy as the basis for his model rather than the classical expression for energy used by Schrödinger. So the same energetic sign problem exists in Einstein's energy expression,

although without the accompanying problems of the signs of time, charge, and probability densities that arise in the differential equation model.

There seem to me two issues that are relevant to this sign problem. The first is the actual significance of Einstein's total energy expression. As pointed out in [1], his derivation of the expression for the kinetic energy of a moving charge is consistent neither with relativity nor the classical model, so the source of the modification of the classical kinetic energy expression at high velocities is uncertain. Until one understands the actual reason for the modification, it seems to me that one cannot determine the true source of the squared energy term, and its significance.

The second issue is that Dirac's differential equation is not a quantum representation of Einstein's energy expression, any more than Schrödinger's is a representation of Newton's. Both Einstein's and Newton's energy expressions are in terms of the physical quantities of momentum and energy. In the quantum model, these are given by $\psi^*\partial\psi/\partial x$, and $\psi^*\partial\psi/\partial t$. The quantum representation of these quantities in Dirac's case would be $(\psi^*\partial\psi/\partial x)^2$ and $(\psi^*\partial\psi/\partial t)^2$, which of course would produce a non-linear equation. What Dirac in fact created, it seems to me, is something that is formally very similar to the Maxwell wave equation, but has no clear logical basis. As such, it gives no apparent information about the source and significance of the positron or other anti-particles, and introduces a sign ambiguity that has no clear physical meaning.

References

[1] J. Shim, On the Mass-Energy Relationship $E=mc^2$, Hadronic Journal, Vol. 36, No. 2 (2013)