A conjecture on the pairs of primes p, q, where q is equal to the sum of p and a primorial number

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Abstract. In a previous paper I stated a conjecture on primes involving the pairs of sexy primes, which are the two primes that differ from each other by six. In this paper I extend that conjecture on the pairs of primes [p, q], where q is of the form p + p(n)#, where p(n)# is a primorial number, which means the product of first n primes.

Conjecture:

If p and p + p(n)# are both primes, where p > p(n)#, n ≥ 2 and p(n)# is a primorial number (which means the product of first n primes), then the number m = p + p(n)#/2 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + p(n)#*r and r is positive integer.

Note:

For a list of primorial numbers, see the sequence A002110 in OEIS; the first ten primorial numbers are: 1, 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870.

Note:

Because p(0)# is, by convention or justified through the concept of "empty product", equal to 1, then p(1)# is equal to 2, p(2)# is equal to 6, p(3)# is equal to 30, p(4)# is equal to 210 and so on.

Comment:

The conjecture it will be then formulate:

: for p(2)# = 6:

If p and p + 6 are both primes, where p > 6, then the number m = p + 3 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 6*r and r is positive integer (this is the conjecture which I made in a previous paper, where I also verified it for the first fifteen pairs of sexy primes, with the difference that in the formulation from there x and y were "primes" not "primes or squares of primes");

: for p(3)# = 30:

If p and p + 30 are both primes, where p > 30, then the number m = p + 15 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 30*r and r is positive integer;

: for p(4)# = 210:

If p and p + 210 are both primes, where p > 210, then the number m = p + 105 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 210*r and r is positive integer;

: for p(5)# = 2310:

If p and p + 2310 are both primes, where p > 2310, then the number m = p + 1155 can be written at least in one way as m = x + y, where x and y are primes, y = x + 2310*r and r is positive integer.

Verifying the conjecture:
(for the first ten pairs of primes [p, p + 30], where p > 30)

: for [p, p + 30] = [31, 61] we have [x, y, r] = [23, 23, 0];
: for [p, p + 30] = [37, 67] we have [p, q, r] = [11, 41, 1];
: for [p, p + 30] = [41, 71] we have [p, q, r] = [13, 43, 1];
: for [p, p + 30] = [43, 73] we have [p, q, r] = [29, 29, 0];
: for [p, p + 30] = [53, 83] we have [p, q, r] = [19, 49, 1];
: for [p, p + 30] = [59, 89] we have [p, q, r] = [37, 37, 0] or [7, 67, 2];
: for [p, p + 30] = [67, 97] we have [p, q, r] = [41, 41, 0] or [11, 71, 2];
: for [p, p + 30] = [71, 101] we have [p, q, r] = [43, 43, 0] or [13, 73, 2];
: for [p, p + 30] = [73, 103] we have [p, q, r] = [29, 59, 1];
: for [p, p + 30] = [79, 109] we have [p, q, r] = [47, 47, 0].

Verifying the conjecture:
(for the first eight pairs of primes [p, p + 210], where p > 210)

: for [p, p + 210] = [13, 223] we have [x, y, r] = [59, 59, 0] or [29, 89, 2];
: for [p, p + 210] = [17, 227] we have [x, y, r] = [61, 61, 0];
: for [p, p + 210] = [19, 229] we have [x, y, r] = [17, 107, 3];
: for [p, p + 210] = [23, 233] we have [x, y, r] = [19, 109, 3] or [49, 79, 1];
: for [p, p + 210] = [29, 239] we have [x, y, r] = [67, 67, 0] or [7, 127, 4] or [37, 97, 2];
: for [p, p + 210] = [31, 241] we have [x, y, r] = [23, 113, 3] or [53, 83, 1];
: for [p, p + 210] = [41, 251] we have [x, y, r] = [73, 73, 0] or [43, 103, 2];
: for [p, p + 210] = [47, 257] we have [x, y, r] = [31, 121, 3].

Verifying the conjecture:
(for the first pair of primes [p, p + 2310], where p > 2310)

: for [p, p + 2310] = [23, 2333] we have [x, y, r] = [49, 1129, 36] or [109, 1069, 32] or [139, 1039, 30] or [169, 1009, 28] or [349, 829, 16] or [469, 709, 12] or [439, 739, 10].