The Scale Law

This article introduces a new physical law that I shall call The Scale Law, The Scaling Law, or The Scale Principle. This law, which is a Model Meta-law, is a model all the laws of physics must obey. The paper shows that the Scale Law can produce true descriptions of nature (exact laws) through two examples: (a) the Heisenberg uncertainty principle and (b) the black hole entropy. Further research I carried out showed that all normal laws of physics are special cases of this formulation. The Scale Law may be used (a) to find exact laws of physics. This is only possible if we know all the necessary information about the phenomenon we are trying to describe. I have illustrated this point in another paper where I derived the Lorentz transformations from the Scale Law [18]; and (b) to find numeric laws of physics. This case applies when we do not know all the information about the phenomenon we try to describe and therefore we may only find a numeric equation for the phenomenon. In order to illustrate the Scale Law more thoroughly, I introduced two numeric formulas for the proton radius. The first formula, which is likely to be numeric, suggests that generation 1, which makes up all the normal matter in the universe, exists because of the existence of the other two generations and the structure of the universe at the Planck scale. The second formula, which is likely to be more accurate and perhaps exact, matches the proton radius obtained by Lamb shift measurements in muonic hydrogen. Finally, based on a table of scale factors, the formulation predicts the size of the electron within a small range of possible values. This result is backed up by a separate prediction I made of the size of the electron based on an electron model with an infinite potential well [12]. Thus, the Scale Law opens a new window into the understanding of all levels of Nature.

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1. Introduction

This paper investigates four fundamental issues - the size of the proton, the Heisenberg uncertainty principle, the black hole entropy and the size of the electron.

The size of the proton has been measured by different methods (electronic hydrogen, elastic electron-proton scattering and muonic hydrogen) and the results have a relatively high degree of discrepancy. This discrepancy is known as the proton radius puzzle [1]. The first theoretical formula of the proton radius I discovered in 2012 predicts a value which is in agreement with the value obtained in the HERA experiment by electron-proton scattering. In 2010 Pohl et al. [2] measured the proton radius by measuring the Lamb shift of muonic hydrogen. The results indicated that the size of the proton was smaller than previously thought. The second theoretical formula for the proton radius I discovered predicts a value that is in agreement with the experiments carried out by Pohl et al and Antognini et al. [3].

Originally this article was entitled *The Quantum Scale Principle*. However, as I found later there are other relationships such Einstein's relativistic energy that obey the Scale Law. Therefore I changed the name of the principle to the *Scale Principle* and now to *The Scale Law.* Thus this principle turned out to be more general than I originally thought.

2. The Scale Law

In 2012 I formulated a theory known as Scale Law. I published the first version of this paper in May 2014. I shall assume that Model Meta-laws deal with dimensionless quantities and not with ratios of specific quantities we find in our universe. For example the representation of the Scale Law is

(1)

The Scale Law or the Scale Principle
$M_1 \, \Re \, s \, M_2$

M_1 = dimensionless Meta-quantity 1
M_2 = dimensionless Meta-quantity 2
s = dimensionless Meta-scale-factor
$\Re$ = Meta-relationship-type

Thus we arrive to the first property of this Meta-law:

Property
The Scale Law is a relationship between dimensionless Meta-quantities.

At the beginning of time, this Law and other Meta-laws generated all the normal laws of physics. Thus the Meta-quantities M_1 and M_2 became ratios (Q_1/Q_2 and Q_3/Q_4 respectively) as shown by the following transformation:

$M_1 \rightarrow (Q_1/Q_2)^n$
$\Re \rightarrow [<'\leq'='\geq'>]$
$M_1 \rightarrow (Q_3/Q_4)^m$
$s \rightarrow S$

And consequently Meta-equation (1) transformes into equation (2)

(2)

The Scale Law or the Scale Principle
$\left(\frac{Q_1}{Q_2}\right)^n [< \mid \le \mid = \mid \ge \mid >] S \left(\frac{Q_3}{Q_4}\right)^m$

Equation (2) is interpreted as follows: the ratios, Q_1/Q_2 , and Q_3/Q_4 , represent particular ratios, the scale factor S represents a specific scale factor, and the relationship $\Re$ represents a specific relationship (one out of five as shown).

Whether nature produced the final ratios directly form the Scale Law (1) or through intermediate transformations, it is not known. However we shall normally write the Scale Law according to relationship (2) instead of using relationship (1) because this way the Law's shape is closer to the shape of the laws we are familiar with (the laws of physics).

Let us investigate relationship (2) more closely. The symbols shown there stand for

a) ***Quantities:***
(i) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Charge, etc), or
ii) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.
(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length).
The physical quantities can be variables (including differentials, derivatives, Laplacians, divergence, integrals, etc.), constants, dimensionless constants, any mathematical operation between the previous quantities, etc.

b) ***Relationship type:*** The relationship is one of five possibilities: **less than or equal to** inequation ($\le$), or **less than** inequation (<), or **equal to** - equation (=), or a **greater than or equal to** inequation ($\ge$), or a **greater than** inequation (>).

c) ***Scale factor**: S* is a dimensionless *scale factor*. This factor could be a real number, a complex number (strictly speaking real numbers are a particular case of complex numbers), a real function or a complex function. The scale factor could have more than one value in the same relationship. In other words a scale factor can be a quantum number. There must be one and only one scale factor per equation or inequation.

d) ***Exponents:*** *n* and *m* are integer exponents: 0, 1, 2, 3, …
Some examples are:

example 1: $n = 0$ and $m = 1$;
example 2: $n = 0$ and $m = 2$;
example 3: $n = 1$ and $m = 0$;
example 4: $n = 1$ and $m = 1$; (canonical form)
example 5: $n = 1$ and $m = 2$;
example 6: $n = 2$ and $m = 0$;
example 7: $n = 2$ and $m = 1$;

It is worthy to remark that:
i) The exponents, n and m, cannot be both zero in the same relationship.
ii) The number n is the exponent of both Q_1 and Q_2 while the number m is the exponent of both Q_3 and Q_4 regardless on how we express the equation or inequation. This means that the exponents will not change when we express the relationship in a mathematically equivalent form such as

$$\left(\frac{Q_4}{Q_3}\right)^m \left[< \middle| \leq \middle| = \middle| \geq \middle| >\right] S \left(\frac{Q_2}{Q_1}\right)^n \qquad (3)$$

iii) So far these integers are less than 3. However we leave the options open as we don't know whether we shall find higher exponents in the future.
iv) When the exponents, n and m, are both equal to *one,* then we say that the equation is in its ***canonical form***. Whenever we express a particular law of physics in the form of the Scale Law, we should use, if possible, its canonical form, provided we don't mix up the variables (defined in a).

The Scale Law (1) can also be written as

$$Q_1^n Q_4^m \left[< \middle| \leq \middle| = \middle| \geq \middle| >\right] S\, Q_2^n\, Q_3^m \qquad (4)$$

However we shall not use this form because is not as clear as expression (2).

The article entitled *"Where Do the Laws of Physics Come From?*" [4] provides additional essential information on the Scale Law.

Skeptical readers could consider this formulation as a method rather than a Meta-law if they feel more comfortable with this alternative interpretation. This will not change any of the conclusions presented here except the statements relating to the existence of Model Meta-laws.

3. The Scale Law as a Theory

The Scale Law in itself is a theory or formulation from which, in principle, we could correctly derive all the laws of physics provided we had all the required information about the phenomenon under study. Unfortunately this is not always the case.

The Scale Law is an exact theory that allows us to find other laws of physics. In order to discover other laws of physics (such as the Lorentz transformations) we need to know all the necessary information about the phenomenon we intend to discover or explain. However, when we do not have all the necessary information about the phenomenon, we may still use the Scale Law as a numerical method to finding numerical equations. We should keep in mind that, in principle, the Scale Law is NOT a numerical method but an exact law of nature: a Meta-law.

4. The Scale Law as a Numerical Method

We have to bear in mind that even if a given unknown phenomena obeys the S*cale Law* (expression 1), it is not guaranteed that we shall get the right variables and the correct scale factor when we apply this law, especially if we have no experimental data on the phenomenon we are trying to describe. For example, when we derive the Lorentz transformations through the Scale Law we knew, from the Michelson-Morley experiment, that the speed of light in vacuum was the same for all observers in relative uniform motion (non accelerating frames). However, when we don't know all the necessary information about the phenomenon we are studying, the Scale Law will produce a numeric relationship that will fit the data but that will not describe a physical law. Having no experimental data or no other data about the phenomenon we try to describe is a limitation in our knowledge and not a limitation of the principle introduced here (Scale Law). In the following sections I shall illustrate this law with four examples:

(1) The equation for the proton radius (Two numerical formulas are developed),
(2) The Heisenberg uncertainty principle (Exact description of nature),
(3) The black hole entropy (Exact description of nature), and
(4) The size of the electron (Numerical formula backed up by another theory of the author).

The Scale Law is an exact theory. However, if we are studying an unknown phenomena, it will be extremely difficult to discover a new law of physics unless we have all the necessary information about the phenomenon we intend to explain in mathematical terms. If we do not have all the necessary data the best we shall probably get will be a numerical equation, unless, of course, we are extremely lucky.

5. The Equation for the Proton Radius

The formulas presented in the following two subsections are the result of applying the Scale Law as a numeric method. This is so because we haven't developed an accurate model of the structure of the proton. However, these numerical equations and other exact equations allow us to build a table of scale factors which suggests that the electron could occupy a missing gap in the table (similar in concept to the periodic table of elements). Thus, this formulation leads to a possible range of values for the diameter of the electron (Section 8). The size of the electron is also calculated through another theory I developed and that is not based on the Scale Law [12]. The results from the two theories overlap. Furthermore, the value for the size of the electron predicted by the string theory is of the same order of magnitude as the value predicted here. This indicates that the range of values for the diameter of the electron found in this paper are likely to be correct.

5.1. First Formula for the Proton Radius

Let's assume that there is a fundamental relationship between the ***proton scale*** and the ***Planck scale*** and, in addition, let's assume that this relationship follows the Scale Law. Then we can write

$$\frac{r_p}{L_P} = S\left(\frac{M_P}{m_p}\right) \tag{5}$$

Where

$r_p =$ proton radius

$$L_P \equiv \sqrt{\frac{hG}{2\pi c^3}} = \sqrt{\frac{\hbar G}{c^3}} = \text{Planck distance} \tag{6}$$

$$M_P \equiv \sqrt{\frac{hc}{2\pi G}} = \sqrt{\frac{\hbar c}{G}} = \text{Planck mass} \tag{7}$$

$m_p =$ proton rest mass

$S =$ scale factor

$$\hbar \equiv \frac{h}{2\pi} = \text{reduced Planck's constant} \tag{8}$$

In order to find the *scale factor* we use a *scale table* (Table 1). The first row of this table shows the three known generations of particles (generations 1, 2, 3) and the Planck scale. The number below each generation is the exponent used in row 5 (last row). The second row indicates the nature of each column (either Distance or Mass). Row 3 shows the proton radius, proton mass, Planck mass and Planck distance. Row 4 shows the muon and the tau particle masses and row 5 shows the values of row 3 to the power of the exponent shown in row 1.

Generation 1 (electron/electron-neutrino/up quark/down quark) *(exponent = 2)*		**Generation 2** (muon/μ-neutrino /charm quark/strange quark) *(exponent =1)*	**Generation 3** (tauon/τ-neutrino/top quark/bottom quark) *(exponent = 1)*	**"Generation 4"** (Planck scale) *(exponent = 2)*	
Length	Mass	Mass	Mass	Mass	Length
r_p (proton radius)	m_p (proton mass)			M_P (Planck mass)	L_P (Planck length)
		m_μ (muon mass)	m_τ (tauon mass)		
r_p^2	m_p^2	m_μ	m_τ	M_P^2	L_P^2

TABLE 1: *Scale table. Each generation is composed of leptons, quarks and its antiparticles. The last row is interpreted as:* $r_p^2 \times m_p^2 \times m_\mu = m_\tau \times M_P^2 \times L_P^2$. *The Planck scale is considered as a "fourth generation" for calculations purposes only.*

We shall introduce the following postulates

i) *Generation postulate*
The Planck scale can be viewed as a *fourth generation* for the purpose of finding physical relationships.

ii) *Hierarchy postulate*
The Planck scale has the same hierarchy as generation 1 (the exponent is 2 for both).

iii) *Symmetry postulate*
There is a symmetry about the vertical axes that divides the *scale table* (Table 2) into two halves, and we interpret the last row (row 5) of this table as follows

$$r_p^2 \times m_p^2 \times m_\mu = m_\tau \times M_P^2 \times L_P^2 \tag{9}$$

This equation can be re-written to show that complies with the Scale Law (see eq. 1)

$$\frac{r_p}{L_P} = \left(\sqrt{\frac{m_\tau}{m_\mu}} \right) \frac{M_P}{m_p} \tag{10}$$

Solving equation (6) for r_p we obtain the equation for the proton radius in terms of the proton mass, the tau particle mass, the muon mass, the Planck mass and the Planck length

$$r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}} \right) \frac{M_P L_P}{m_p} \tag{11}$$

Hence

$$S \equiv \sqrt{\frac{m_\tau}{m_\mu}} = \sqrt{16.8167} = 4.100\,816\,992 \tag{12}$$

Where I used the CODATA value for the ratio $\frac{m_\tau}{m_\mu} = 16.8167$

We can simplify equation (11) by taking into account that the Planck mass times the speed of light in vacuum times the Planck length equals the reduced Planck's constant, mathematically

$$M_P\, c\, L_P = \frac{h}{2\pi} = \hbar \tag{13}$$

Thus equation (7) yields the implicit form for the proton radius

$$m_p c\, r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}} \right) \frac{h}{2\pi} = \left(\sqrt{\frac{m_\tau}{m_\mu}} \right) \hbar \tag{14}$$

Finally, solving equation (14) for r_p we obtain the first formula for the proton radius

$$r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}} \right) \frac{h}{2\pi\, m_p\, c} \qquad \text{(First formula)} \tag{15}$$

This is the formula for the proton radius in terms of the proton mass, the tau particle (tauon) mass and the muon mass. This formula produces the following value of the proton radius

$$r_p \cong 8.624\,383\,532 \times 10^{-16}\, m \cong 0.86244\, fm$$

In subsection 5.3 we shall see that this result is in agreement with the observed values obtained through the proton-electron scattering experiments.

5.2. Second Formula for the Proton Radius

Considering that formula (15) does not match the observed values in muonic hydrogen, we shall assume that the value of the scale factor is 4 instead of 4.100816992 (see equation 8). Thus scale table 1 will transform into scale table 2

Generation 1 *(exponent = 2)*		**Numbers** *(exponent =1)*	**Numbers** *(exponent = 1)*	**"Generation 4"** (Planck scale) *(exponent = 2)*	
Length	Mass	-	-	Mass	Length
r_p (proton radius)	m_p (proton mass)			M_P (Planck mass)	L_P (Planck length)
		1	16		
r_p^2	m_p^2	1	16	M_P^2	L_P^2

TABLE 2: *Scale table. The last row is interpreted as:* $r_p^2 \times m_p^2 \times 1 = 16 \times M_P^2 \times L_P^2$.

The numbers in column 3 and 4 are arbitrary provided that the ratio is 16 (The actual ratio must be very close to 16 to match the experimental data).

$$r_p^2 \times m_p^2 \times 1 = 16 \times M_P^2 \times L_P^2 \qquad (16)$$

$$r_p = \sqrt{16}\left(\frac{h}{2\pi\, m_p\, c}\right) \qquad (17)$$

This gives the second formula for the proton radius I introduced in 2014

$$r_p = \frac{2h}{\pi\, m_p\, c} \qquad \text{(Second formula)} \qquad (18)$$

According to this formula the value of the proton radius is

$$r_p \cong 8.412\,356\,415 \times 10^{-16} m \cong 0.84124\, fm$$

In the next subsection we shall see that this result is in agreement with the measurements from muonic hydrogen laser spectroscopy.

Later I shall use the implicit form of equation (18) which is

$$m_p\, c\, r_p = 4\hbar \qquad (19)$$

5.3. Comparison with the Experiment

Now we shall compare the theoretical values obtained in the previous two subsections (0.86244 fm and 0.84124 fm) with the experiments. This comparison is shown on Table 3

Experiment/Theory	**Proton radius** (fm)
I. Sick – electron-proton scattering – 2012 [5]	0.886 (8)
CODATA 2010 – (this value exceeds the 2006 value by 0.0007 fm) [6]	0.8775 (51)
Theoretical value (first formula)	0.86244
G.G. Simon – electron-proton scattering – 1980 [7]	0.862 (12)
HERA Collider – electron-proton scattering – 2007 [8][9]	0.86
R. Pohl et al. – lamb shift in muonic hydrogen – 2010 [2]	0.84184 (67)
Theoretical value (second formula)	0.84124
A. Antognini et al. – lamb shift in muonic hydrogen – 2013 [3]	0.84087 (39)

TABLE 3: *Experimental values of the proton radius (given in descending order from the top of the table).*

As Table 3 shows, there is some degree of discrepancy among the experimental values. The theoretical value of 0.86244 fm obtained with the first formula (see equation 15) is in agreement (within the experimental errors) with the experimental value of 0.862 fm measured by both Simon [7] and by the HERA collider [8][9].

On the other hand, the theoretical value of 0.84124 fm obtained through the second formula (see equation 18) is in agreement (within the experimental errors) with the experimental values of 0.84184 fm and 0.84087 fm measured by R. Pohl [3] and A. Antognini [4], respectively.

6. The Heisenberg Uncertainty Principle

I shall show that the Heisenberg uncertainty principle is a special case of the Scale Law. I must clarify that this analysis is not the derivation of the uncertainty principle. To find the scale factor in this special case we need to do a much deeper investigation (exactly what Heisenberg did). However the point here is not to prove that the Heisenberg principle's scale factor can be easily found through the Scale Law, but to prove that the uncertainty principle is a special case of the more general formulation presented here.

Length *(Exponent = 1)*	**Momentum** *(Exponent = 1)*	**Momentum** (Planck Scale) *(Exponent = 1)*	**Length** (Planck Scale) *(Exponent = 1)*
Δx	Δp	$M_P c$	L_P

TABLE 4: *We don't label the columns as Generations because we are not dealing with particles*.

From Table 4 we establish the relationship

$$\Delta x\, \Delta p = S\, M_P c\, L_P \tag{20}$$

which can be re-written in the form of the Scale Law

$$\frac{\Delta p}{M_P c} = S\, \frac{L_P}{\Delta x} \tag{21}$$

Considering equation (13) we can write

$$M_P c\, L_P = \hbar \tag{22}$$

$$\Delta x\, \Delta p = S\, \hbar \tag{23}$$

or

$$\Delta p\;\; \Delta x = S\, \hbar \tag{24}$$

Substituting the equal sign with a greater than or equal to (≥) and the scale factor, *S*, with ½ , we get the *Heisenberg uncertainty principle*. Although we have not found the exact expression of the principle, we have proved that the uncertainty principle is a special case of the Scale Law.

7. Black Hole Entropy

Following a similar analysis as the one I carried out in the previous section I show that the black hole entropy is a special case of the Scale Law. As before, to find the exact scale factor in this special case we need to do a much more detailed investigation (either as S. Hawking did or as the author did in another paper [10]). Let us consider the following scale table

Length *(Exponent = 2)*	**Constant** *(Exponent = 1)*	**Entropy** *(Exponent = 1)*	**Length** (Planck Scale) *(Exponent = 2)*
R	k_B	S_{BH}	L_P
R^2	k_B	S_{BH}	L_P^2

TABLE 5: *Scale Table. We don't label the columns as Generations because we are not dealing with particles*.

Where

R = black hole radius
k_B = Boltzmann's constant
S_{BH} = Berkenstein-Hawking's black hole entropy

According to the Scale Law we write

$$R^2 k_B = S\, S_{BH}\, L_P^2 \tag{25}$$

Solving this equation for the entropy yields

$$S_{BH} = \frac{1}{S}\frac{k_B R^2}{L_P^2} \tag{26}$$

Because we already know the equation for the black hole entropy we know that the scale factor is 1/π, therefore we write

$$S_{BH} = \frac{k_B \pi R^2}{L_P^2} \tag{27}$$

If we multiply the second side by 4/4 we shall get the area of a sphere of radius *R* in the numerator, thus

$$S_{BH} = \frac{k_B 4\pi R^2}{4 L_P^2} \qquad (28)$$

Considering that the Planck length is given by

$$L_P \equiv \sqrt{\frac{\hbar G}{c^3}} \qquad (29)$$

Substituting the Planck length in equation (28) with the value given by equation (29) gives

$$S_{BH} = \frac{k_B c^3 4\pi R^2}{4\hbar G} \qquad (30)$$

Substituting the area of the event horizon, $4\pi R^2$, with A_H yields

$$S_{BH} = \frac{k_B c^3}{4\hbar G} A_H \quad \text{(Berkenstein-Hawking's formula for the black hole entropy)} \qquad (31)$$

This is the Berkenstein-Hawking formula for the black hole entropy. Thus I have proved that black hole entropy is a special case of the Scale Law.

8. The Size of the Electron

To illustrate the significance of the formulation presented here I shall draw a scale table with the Heisenberg uncertainty principle (HUP) in column 1, the Planck's momentum-length relationship (Planck's spatial relationship) in column 2, the implicit formula for the proton radius given by equation (19) in column 3 and the formula for the proton radius given by equation (15) in column 4. The table is shown below

(see next page)

Heisenberg uncertainty principle	**Planck Momentum-Length equation**	**Proton radius equation (muonic hydrogen)**	**Proton radius equation (proton-electron scattering)**
$\Delta p \Delta x \geq \frac{\hbar}{2}$	$M_P c L_P = \hbar$	$m_p c r_p = 4\hbar$	$m_p c r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}}\right)\hbar$
Inequation	Equation	Equation	Equation
Δp	$M_P c$	$m_p c$	$m_p c$
Δx	L_P	r_p	r_p
$scale\ factor = 0.5$	$scale\ factor = 1$	$scale\ factor = 4$	$scale\ factor \approx 4.1008$

TABLE 6: *Table of scale factors. Comparison of the Planck Scale, the HUP and the proton radius equations.*

Table 6 reveals the *natural mechanism of scale factors*. A scale factor of 1 corresponds to the Planck's space relationship (momentum-length equation - there is also a similar temporal relationship given by the product of the Planck energy and the Planck time). A scale factor of 4 (or approximately 4) corresponds to the proton; and a scale factor of $S_U \cong 3.387 \times 10^{38}$ corresponds to the universe [11].

$$T = \frac{h^2}{2\pi^2 c\, G\, m_e\, m_p^2} \quad \text{(formula for the age of the universe)} \qquad (32)$$

This equation can be re-written as

$$m_e c^2\, T = \left(\frac{2}{\alpha_G}\right)\hbar \qquad (33)$$

Where

$$\alpha_G = \left(\frac{m_p}{M_P}\right)^2 \quad \text{(gravitational coupling constant for the proton)} \qquad (34)$$

The universal scale factor S_U is

$$S_U = \frac{2}{\alpha_G} \cong 3.387 \times 10^{38} \quad \text{(scale factor for the universe)} \qquad (35)$$

Both the table of scale factors given above and the periodic table of elements have something in common: they allow us to make predictions. While the periodic table was used to predict the properties of new elements, the table of scale factors (Table 6) can be used to predict the properties of fundamental particles such as the size of the electron. This prediction emerges from Table 6 as we see that there are no particles with scale factors between 1 and 4.

This suggests that the table is missing one or more columns between column 2 (Planck column) and 3 (proton column). We assume that one of these missing columns correspond to a fundamental particle such as the electron. If this assumption is correct, then the *electron scale factor,* S_e, must be greater than 1 and less than 4 (or less than 4 approximately). According to this prediction the equation for the electron radius will be given by

$$M_P\, c\, r_e = S_e\, \hbar \qquad (36)$$

$$r_e = S_e \frac{\hbar}{M_P\, c} \qquad \text{(formula for the radius of the electron)} \qquad (37)$$

And the diameter of the electron, d_e, will then be

$$d_e = 2S_e \frac{\hbar}{M_P\, c} \qquad \text{(formula for the diameter of the electron)} \qquad (38)$$

$$d_e = S_e\, 2L_P$$

Now we calculate the *range* for the electron scale factor including the middle of the range:

a) Lower limit of the range

If S=1 then the minimum value of the range for the diameter of the electron is

$$d_{e-\min}(1) = 1 \times 2 \frac{\hbar}{M_P\, c} = 2L_P = 3.232\,399 \times 10^{-35}\, m \qquad (39)$$

b) Middle of the range

If S=2 then the middle of the range is

$$d_{e-mid}(2) = 2 \times 2 \frac{\hbar}{M_P\, c} = 4L_P = 6.464\,797 \times 10^{-35}\, m \qquad (40)$$

c) Upper limit of the range

If S=4 then the maximum value of the range for the diameter of the electron is

$$d_{e-\max}(4) = 4 \times 2 \frac{\hbar}{M_P c} = 8 L_P = 1.292\,959 \times 10^{-34} m \tag{41}$$

Using a quantum mechanical method [12] based on an infinite potential well I found that the diameter of the electron is not bigger than 10 times the Planck length. This corresponds to a scale factor smaller or equal than 5. In summary, the electron size must satisfy the following two conditions:

Condition 1

Based on the prediction from the scale table (Table 10), the electron diameter should be

$$2L_P < d_e < 8L_P \tag{42}$$

Condition 2

Based on the quantum model of the electron size [12], the electron diameter should be

$$d_e \le (1+\sqrt{5})\pi\, L_P \tag{43}$$

$$d_e \le 10.166\, L_P \tag{44}$$

As a consequence the following range of diameters will satisfy both conditions

$$2L_P < d_e < 8L_P \tag{45}$$

Now we shall complete Table 6 by adding one more column for the electron (at least it will be more complete than before). This column will be inserted between the Planck column and the proton column. Table 7 shows the addition.

This table suggests nature uses the following mechanism

Scale Factor Mechanism

We start with the Planck's space relationship (Planck momentum-length relationship) where we make two substitutions: we substitute L_P with r_e and the Planck scale factor of 1 with a number greater than 1 and less than 4 (the electron scale factor). This will give us the electron radius equation. Then we continue from the electron radius equation where we make two substitutions: we substitute M_P with m_p and the electron scale factor with 4 (the proton scale factor). This will yield the proton radius equation.

It is worthy to remark that even if experiments determine that the scale factor for the proton is not exactly 4, but slightly higher or slightly lower than 4, they won't change the prediction we made from the table because there will still be a gap that must be filled up.

Heisenberg uncertainty principle	Planck Momentum-Length equation	(Gap) Electron radius equation	Proton radius equation (muonic hydrogen)	Proton radius equation (proton-electron scattering)
$\Delta p \Delta x \geq \frac{\hbar}{2}$	$M_P c L_P = \hbar$	$M_P c r_e = S_e \hbar$	$m_p c r_p = 4\hbar$	$m_p c r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}}\right)\hbar$
Inequation	Equation	Equation	Equation	Equation
Δp	$M_P c$	$M_P c$	$m_p c$	$m_p c$
Δx	L_P	r_e	r_p	r_p
$scale\ factor = 0.5$	$scale\ factor = 1$	$1 < scale\ factor\ (S_e) < 4$	$scale\ factor = 4$	$scale\ factor \approx 4.1008$

TABLE 7: *Table of scale factors. A column was added for the electron. This table suggests that the diameter of the electron (and possibly the diameter for any other fundamental particle) is greater than* $2L_P$ *and smaller than* $8L_P$.

Despite the fact that we have not found the exact scale factor for the electron, we have found a very small range of possible values. Because I have carried out the analysis with two different methods I strongly believe that the result (above range of diameters) is correct.

9. Conclusions

In summary, the relationships introduced in this paper are in excellent agreement with the experiments. The first formula for the proton radius (equation 15), which matches the size of the proton measured by proton-electron scattering, suggests that there is a fundamental relationship between all generations of matter and the Planck scale. It seems that the Planck scale is a "fourth generation" that dictates the properties of the matter we are made of. Furthermore, this equation suggests that, in order to exist, the generation that makes up all the normal matter in the universe would need the other two generations. It would seem that the Big Bang was unable to create stable matter without creating two additional unstable generations along the way. The answer to the question as to why Nature has chosen to have more than one generation of matter would be that the laws of physics require them.

The second formula for the proton radius (equation 18), on the other hand, matches the size of the proton measured by the experiment based on lamb shift in muonic hydrogen. Detailed quantum mechanical calculations carried out were so far unable to solve the puzzle. If the size of the proton depends on the method used to measure it, then, both equations of the proton radius presented here could be correct.

The Scale Law is significant because of several reasons

i. unveils a "hidden" resemblance all (or most) laws of physics share, regardless of the nature of the phenomenon they describe [4, Table 3],
ii. allows us to make predictions such as the one we did for the size of the electron (Section 8),
iii. allows us to unveil exact laws of physics as we did when we derived the equations for the Lorentz transformations [14]. (See also [4, 13, 14, 15, 16, 17, 18])
iv. provides a rational explanation about the origin of the laws of physics and the fine structure constant [4].

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