Ten prime-generating quadratic polynomials

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Abstract. In two of my previous papers I treated quadratic polynomials which have the property to produce many primes in a row: in one of them I listed forty-two such polynomials which generate more than twenty-three primes in a row and in another one I listed few generic formulas which may conduct to find such prime-producing quadratic polynomials. In this paper I will present ten such polynomials which I discovered and posted in OEIS, each accompanied by its first fifty terms and some comments about it.

I.

The polynomial 16*n^2 - 300*n + 1447.

Its first fifty terms:

1447, 1163, 911, 691, 503, 347, 223, 131, 71, 43, 47, 83, 151, 251, 383, 547, 743, 971, 1231, 1523, 1847, 2203, 2591, 3011, 3463, 3947, 4463, 5011, 5591, 6203, 6847, 7523, 8231, 8971, 9743, 10547, 11383, 12251, 13151, 14083, 15047, 16043, 17071, 18131, 19223, 20347, 21503, 22691, 23911, 25163, 26447.

Comments:

This polynomial generates 30 primes in a row starting from n = 0.
The polynomial 16*n^2 - 628*n + 6203 generates the same primes in reverse order.
I found in the same family of prime-generating polynomials (with the discriminant equal to -163*2^p, where p is even), the polynomials 4n^2 - 152n + 1607, generating 40 primes in row starting from n = 0 (20 distinct ones) and 4n^2 - 140n + 1877, generating 36 primes in row starting from n = 0 (18 distinct ones).
The following 5 (10 with their "reversal" polynomials) are the only ones I know from the family of Euler's polynomial n^2 + n + 41 (having their discriminant equal to a multiple of -163) that generate more than 30 distinct primes in a row starting from n = 0 (beside the Escott's polynomial n^2 - 79n + 1601):

(1) 4n^2 - 154n + 1523 (4n^2 - 158n + 1601);
(2) 9n^2 - 231n + 1523 (9n^2 - 471n + 6203);
(3) 16n^2 - 292n + 1373 (16n^2 - 668n + 7013);
(4) 25n^2 - 365n + 1373 (25n^2 - 1185n + 14083);
(5) 16n^2 - 300n + 1447 (16n^2 - 628n + 6203).

II.

The polynomial 2*n^2 - 108*n + 1259.

Its first fifty terms:

1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181, -191, -197, -199, -197, -191, -181, -167, -149, -127, -101, -71, -37, 1, 43, 89, 139, 193, 251, 313, 379, 449, 523, 601, 683, 769.

Comments:

This polynomial generates 92 primes (66 distinct ones) for n from 0 to 99 (in fact the next two terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for n = m - 35, which is m^2 - 69*m + 1231, but having six distinct primes more than this one.
The non-prime terms in the first 100 are: 1 (taken twice), 1369 = 37^2, 1849 = 43^2, 4033 = 37*109, 5633 = 43*131, 7739 = 71*109 and 8251 = 37*223.
For n = 2*m - 34 we obtain the polynomial 8*m^2 - 488*m + 7243, which generates 31 primes in a row starting from m = 0.
For n = 4*m - 34 we obtain the polynomial 32*m^2 - 976*m + 7243, which generates 31 primes in row starting from m = 0.

III.

The polynomial 2*n^2 - 212*n + 5419.

Its first fifty terms:

5419, 5209, 5003, 4801, 4603, 4409, 4219, 4033, 3851, 3673, 3499, 3329, 3163, 3001, 2843, 2689, 2539, 2393, 2251, 2113, 1979, 1849, 1723, 1601, 1483, 1369, 1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181.

Comments:

This polynomial generates 92 primes (57 distinct ones) for n from 0 to 99 (in fact the next seven terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for n = m - 35, which is m^2 - 69*m + 1231.
The non-prime terms in the first 100 are: 1, 1369 = 37^2, 1849 = 43^2, 4033 = 37*109 (all taken twice).
For n = 2*m + 54 we obtain the polynomial 8*m^2 + 8*m - 197, which generates 31 primes in a row starting from m = 0 (the polynomial 8*m^2 - 488*m + 7243 generates the same 31 primes, but in reverse order).

IV.

The polynomial 25*n^2 - 1185*n + 14083.

Its first fifty terms:

14083, 12923, 11813, 10753, 9743, 8783, 7873, 7013, 6203, 5443, 4733, 4073, 3463, 2903, 2393, 1933, 1523, 1163, 853, 593, 383, 223, 113, 53, 43, 83, 173, 313, 503, 743, 1033, 1373, 1763, 2203, 2693, 3233, 3823, 4463, 5153, 5893, 6683, 7523, 8413, 9353, 10343, 12473, 13613, 14803, 16043, 17333.

Comments:
The polynomial generates 32 primes in row starting from n = 0.
The polynomial 25*n^2 - 365*n + 1373 generates the same primes in reverse order.
This family of prime-generating polynomials (with the discriminant equal to -4075 = -163*5^2) is interesting for generating primes of same form: the polynomial 25*n^2 – 395(n + 1601 generates 16 primes of the form 10*k + 1 (1601, 1231, 911, 641, 421, 251, 131, 61, 41, 71, 151, 281, 461, 691, 971, 1301) and the polynomial 25*n^2 + 25*n + 47 generates 16 primes of the form 10*k + 7 (47, 97, 197, 347, 547, 797, 1097, 1447, 1847, 2297, 2797, 3347, 3947, 4597, 5297, 6047).
Note:
All the polynomials of the form 25*n^2 + 5*n + 41, 25*n^2 + 15*n + 43,..., 25*n^2 + 5*(2k + 1)*n + p,..., 25*n^2 + 5*79*n + 1601, where p is a (prime) term of the Euler’s polynomial p = k^2 + k + 41, from k = 0 to k = 39, have their discriminant equal to -4075 = -163*5^2.

V.

The polynomial 16*n^2 - 292*n + 1373.

Its first fifty terms:

1373, 1097, 853, 641, 461, 313, 197, 113, 61, 41, 53, 97, 173, 281, 421, 593, 797, 1033, 1301, 1601, 1933, 2297, 2693, 3121, 3581, 4073, 4597, 5153, 5741, 6361, 7013, 7697, 8413, 9161, 9941, 10753, 11597, 12473, 13381, 14321, 15293, 16297, 17333, 18401, 20633, 21797, 22993, 24221, 25481, 26773.

Comments:

The polynomial generates 31 primes in row starting from n = 0.

The polynomial 16*n^2 - 668*n + 7013 generates the same primes in reverse order.

Note:

All the polynomials of the form p^2*n^2 ± p*n + 41, p^2*n^2 ± 3*p*n + 43, p^2*n^2 ± 5*p*n + 47, ..., p^2*n^2 ± (2k+1)*p*n + q, ..., p^2*n^2 ± 79*p*n + 1601, where q is a (prime) term of the Euler's polynomial q = k^2 + k + 41, from k = 0 to k = 39, have their discriminant equal to -163*p^2; the demonstration is easy: the discriminant is equal to b^2 – 4*a*c = (2*k + 1)^2*p^2 - 4*q*p^2 = - p^2 ((2*k + 1)^2 – 4*q) = - p^2*(4*k^2 + 4*k + 1 – 4*k^2 – 4*k - 164) = -163*p^2.

Observation:

Many of the polynomials formed this way have the capacity to generate many primes in row. Examples:

: 9*n^2 + 3*n + 41 generates 27 primes in row starting from n = 0 (and 40 primes for n = n - 13);

: 9*n^2 – 237*n + 1601 generates 27 primes in row starting from n = 0;

: 16*n^2 + 4*n + 41 generates, for n = n - 21 (that is 16*n^2 – 668*n + 7013) 31 primes in row.

VI.

The polynomial 4*n^2 - 284*n + 3449.

Its first fifty terms:

3449, 3169, 2897, 2633, 2377, 2129, 1889, 1657, 1433, 1217, 1009, 809, 617, 433, 257, 89, -71, -223, -367, -503, -631, -751, -863, -967, -1063, -1151, -1231, -1303, -1367, -1423, -1471, -1511, -1543, -1567, -1583, -1591, -1591, -1583, -1567, -1543, -1511, -1471, -1367, -1303, -1231, -1151, -1063, -967, -863, -751.

Comments:

The polynomial successively generates 35 primes or negative values of primes starting at n = 0.
This polynomial generates 95 primes in absolute value (60 distinct ones) for n from 0 to 99, equaling the record held by the Euler's polynomial for n = m - 35, which is m^2 - 69*m + 1231.
The non-prime terms (in absolute value) up to n = 99 are: 1591 = 37*43, 3737 = 37*101, 4033 = 37*109; 5633 = 43*131; 5977 = 43*139; 9017 = 71*127.
The polynomial 4*n^2 + 12*n - 1583 generates the same 35 primes in row starting from n = 0 in reverse order.

Note:

In the same family of prime-generating polynomials (with the discriminant equal to 199*2^p, where p is odd) there are the polynomial 32*n^2 - 944*n + 6763 (with its "reversed polynomial" 32*m^2 - 976*m + 7243, for m = 30 - n), generating 31 primes in row, and the polynomial 4*n^2 - 428*n + 5081 (with 4*m^2 + 188*m - 4159, for m = 30-n), generating 31 primes in row.

VII.

The polynomial n^2 + 3*n - 167.

Its first fifty terms:

-167, -163, -157, -149, -139, -127, -113, -97, -79, -59, -37, -13, 13, 41, 71, 103, 137, 173, 211, 251, 293, 337, 383, 431, 481, 533, 587, 643, 701, 761, 823, 887, 953, 1021, 1091, 1163, 1237, 1313, 1391, 1471, 1553, 1637, 1723, 1811, 1901, 1993, 2087, 2183, 2381, 2483.

Comments:

The polynomial generates 24 primes in absolute value (23 distinct ones) in row starting from n = 0 (and 42 primes in absolute value for n from 0 to 46).
The polynomial n^2 - 49*n + 431 generates the same primes in reverse order.

Note:

We found in the same family of prime-generating polynomials (with the discriminant equal to 677) the polynomial 13*n^2 - 311*n + 1847 (13*n^2 - 469*n + 4217) generating 23 primes and two noncomposite numbers (in absolute value) in row starting from n = 0 (1847, 1549, 1277, 1031, 811, 617, 449, 307, 191, 101, 37, -1, -13, 1, 41, 107, 199, 317, 461, 631, 827, 1049, 1297, 1571, 1871).

Note:

Another interesting algorithm to produce prime-generating polynomials could be N = m*n^2 + (6*m + 1)*n + 8*m + 3, where m, 6*m + 1 and 8*m + 3 are primes. For m = 7 then n = t - 20 we get N = 7*t^2 - 237*t + 1999, which generates the following primes: 239, 163, 101, 53, 19, -1, -7, 1, 23, 59, 109, 173, 251 (we can see the same pattern: …, -1, -m, 1, …).

VIII.

The polynomial 81*n^2 – 2247*n + 15383.

Its first forty terms:

15383, 13217, 11213, 9371, 7691, 6173, 4817, 3623, 2591, 1721, 1013, 467, 83, -139, -199, -97, 167, 593, 1181, 1931, 2843, 3917, 5153, 6551, 8111, 9833, 11717, 13763, 15971, 18341, 20873, 23567, 26423, 29441, 32621, 35963, 39467, 43133, 46961, 50951.

Comments:

The polynomial generates 33 primes/negative values of primes in row starting from n = 0.
The polynomial 81*n^2 - 2937*n + 26423 generates the same primes in reverse order.

Note:

We found in the same family of prime-generating polynomials (with the discriminant equal to 64917 = 3^2*7213) the polynomial 27*n^2 – 753*n + 4649 (with its "reversed polynomial" 27*n^2 – 921*n + 7253), generating 32 primes in row and the polynomial 27*n^2 – 741*n + 4483 (27*n^2 – 1095*n + 10501), generating 35 primes in row, if we consider that 1 is prime (which seems to be constructive in the study of prime-generating polynomials, at least).

Note:

The polynomial 36*n^2 – 810*n + 2753, which is the known quadratic polynom that generates the most distinct primes in row (45), has the discriminant equal to 259668 = 2^2*3^2*7213.

IX.

The polynomial 4*n^2 + 12*n - 1583.

Its first forty terms:

-1583, -1567, -1543, -1511, -1471, -1423, -1367, -1303, -1231, -1151, -1063, -967, -863, -751, -631, -503, -367, -223, -71, 89, 257, 433, 617, 809, 1009, 1217, 1433, 1657, 1889, 2129, 2377, 2633, 2897, 3169, 3449, 3737, 4033, 4337, 4649, 4969.

Comments:

The polynomial generates 35 primes/negative values of primes in row starting from n = 0.
The polynomial 4*n^2 - 284*n + 3449 generates the same primes in reverse order.

Note:

Other related polynomials are:

: For n = 6*n + 6 than n = n - 11 we get 144*n^2 – 2808*n + 12097 which generates 16 primes in row starting from n = 0 (with the discriminant equal to 2^9*3^2*199);

: For n = 12*n + 12 than n = n - 15 we get 576*n^2 – 15984*n + 109297 which generates 17 primes in row starting from n = 0 (with the discriminant equal to 2^11*3^2*199).

Note:

So this polynomial opens at least two directions of study:

(1) polynomials of type 4*n^2 + 12*n - p, where p is prime (could be of the form 30*k + 23);

(2) polynomials with the discriminant equal to 2^n*3^m*199, where n is odd and m is even (an example of such a polynomial, with the discriminant equal to 2^5*3^4*199, is 36*n^2 – 1020*n + 3643 which generates 32 primes for values of n from 0 to 34).

X.

The polynomial 4*n^2 - 482*n + 14561.

Its first forty terms:

14561, 14083, 13613, 13151, 12697, 12251, 11813, 11383, 10961, 10547, 10141, 9743, 9353, 8971, 8597, 8231, 7873, 7523, 7181, 6847, 6521, 6203, 5893, 5591, 5297, 5011, 4733, 4463, 4201, 3947, 3701, 3463, 3233, 3011, 2797, 2591, 2393, 2203, 2021, 1847.

Comments:

This polynomial generates 88 distinct primes for n from 0 to 99, just two primes less than the record held by the polynomial discovered by N. Boston and M. L. Greenwood, that is 41*n^2 - 4641*n + 88007 (this polynomial is sometimes cited as 41*n^2 + 33*n - 43321, which is the same for the input values [-57, 42].

Note:

The non-prime terms in the first 100 are: 10961 = 97*113; 10547 = 53*199; 9353 = 47*199; 7181 = 43*167; 6847 = 41*167; 5893 = 71*83; 3233 = 53*61; 2021 = 43*47; 1681 = 41^2; 1763 = 41*43; 2491 = 47*53; 4331 = 61*71.

Note:

For n = m + 41 we obtain the polynomial 4*m^2 - 154*m + 1523, which generates 40 primes in a row starting from m = 0.