ALGEBRA

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Abstract

This article raises some important points about algebra.

All of us are positive that 1 + 1 = 2 or 2 + 3 = 5. But, has anyone ever thought why this must always be so?

In the author's opinion, 1 + 1 = 2 or 2 + 3 = 5 are, at best, only approximations in the abstract. What exactly do these two equations mean? How can we interpret these two equations? We can interpret them (or misinterpret them) in at least several ways.

Firstly, we ask ourselves 1 (what?) + 1 (what?) = 2 (what?) and 2 (what?) + 3 (what?) = 5 (what?).

Secondly, we may interpret the above equations as 1 (pole) + 1 (pole) = 2 (poles) and 2 (poles) + 3 (poles) = 5 (poles), for example. Hence, we are saying 1 + 1 = 2 and 2 + 3 = 5.

Thirdly, we may construe that 1 + 1 is not necessarily equal to 2 or that 2 + 3 is not necessarily equal to 5. "How come?", you may wonder. Look at the following illustrations:-

a) Can't 1 (2 ft. long pole) + 1 (1 ft. long pole) = 3 (1 ft. long poles), i.e., 1 + 1 = 3, for example?
b) Can't 2 (2 ft. long poles) + 3 (1 ft. long poles) = 7 (1 ft. long poles), i.e., 2 + 3 = 7, for example?

Fourthly, can 1 (apple) + 1 (pear) = 2 (apples), or, 2 (apples) + 3 (pears) = 5 (pears), for example? Apparently, these two equations will hardly hold water now.

We can "justify" the equations 1 + 1 = 2 and 2 + 3 = 5 by ensuring that the numerals such as 1 and 2 in equation 1 + 1 = 2, and, 2, 3 and 5 in equation 2 + 3 = 5, share a "common property". For example, 1 (gram) + 1 (gram) = 2 (grams), and hence 1 + 1 = 2 ("gram" being the "common property"); 2 (2 ft. long poles) + 3 (2 ft. long poles) = 5 (2 ft. long poles), and hence, 2 + 3 = 5 ("2 ft. long poles" being the "common property").

But, can we be absolutely certain, e.g., that 1 (apple) + 1 (apple) = 2 (apples), "apple" here being the "common property"? In other words, could 1 (small apple) + 1 (small apple) = 2 (big apples)? This evidently could not be so.

Let us look at the following equation concerning apples:-

1 (small apple) + 1 (big apple) = 2 (big apples)

Is the above equation valid? This equation can or cannot hold water, depending on whether some "common property" exists or not. Let us look at the following illustrations:-

a) If, e.g., 1 (small apple) + 1 (big apple) = 3 (grams of apple) and 2 big apples = 6 (grams of apple) then, of course, this equation will not be valid (for 3 grams $\neq$ 6 grams).
b) Similarly, e.g., if 1 (small apple) + 1 (big apple) = 7 (cubic centimeters of apple) and 2 (big apples) = 14 (cubic centimeters of apple), this equation will not be justifiable (for 7 cubic centimeters $\neq$ 14 cubic centimeters).

This equation can be justified when the following conditions are present:-

a) If, e.g., 1 (small apple) + 1 (big apple) = 4 (grams of apple) and 2 (big apples) = 4 (grams of apple) , in which case the "4 grams of apple" is the "common property" belonging to each side of the equation, i.e., when we in effect have "4 grams of apple = 4 grams of apple" (here, common sense will tell us that the big apple on the left side of the equation is heavier than each of the two big apples on the other side of the equation).
b) Also, if, e.g., 1 (small apple) + 1 (big apple) = 7 (cubic centimeters of apple) and 2 (big apples) = 7 (cubic centimeters of apple), in which case the "7 cubic centimeters of apple" is the "common property" belonging to each side of the equation, i.e., when we in effect have "7 cubic centimeters of apple = 7 cubic centimeters of apple" (here, common sense will again tell us that the big apple on the left side of the equation is of greater volume than each of the two big apples on the other side of the equation.

In (a) above 1 (small apple) + 1 (big apple) = 2 (big apples) in terms of weight (grams), while in (b) above they are equal in terms of volume (cubic centimeters).

Thus, "common property" here should be a form of "precise description", e.g., precise measurements such as weight, length, et al.. For example, "4 grams" and "7 cubic centimeters" above are "precise properties". Things can only be considered equal if they display similar measurable, even discernable, properties or qualities such as weight, volume and length, and, perhaps, *physical features or appearance (*here, we should specify that things are equal because they are "equal" or similar in physical features or appearance - physical features or appearance tend to be subjective and difficult to judge, unlike the measurable properties such as weight and volume; physical features or appearance would give rise to sets, classes, types or species, hence, the algebra of sets or classes (set theory), for example).

Imprecise descriptions, which cannot be precisely measured or discerned, will not do. For example, 1 (unhappy man) + 1 (unhappy man) = 2 (unhappy men) may not be justifiable, for we cannot measure happiness or ascertain the degree of happiness in each of the men.

So, will, e.g., one plus one always equal two, or, two plus three always equal five, et al.? It depends on how we interpret these algebraic equations. There should be more precision (of interpretation) to algebraic equations. Otherwise, algebraic equations may not really make much sense if we look deeply into them, as illustrated above.

REFERENCES

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