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APPLICATIONS OF SEMIGENERALIZED α-CLOSED SETS

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Abstract. In this paper, we have study the some new properties of sgα- closed sets in topological spaces.

Keywords: Topological spaces, $sg\alpha$ -open sets.

1 INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Dimension theory plays an important role in the applications of General Topology to Real Analysis and Functional Analysis. Recently, as a generalization of closed sets, the notion of sgα-closed sets were introduced and studied by Rajesh and Krsteska [7]. In this paper, we have study the some new properties of sgα-closed sets in topological spaces.

2. PRELIMINARIES

Definition 2.1 *A subset A of a space* (X,τ) *is called semi-open [3](resp. α-open [4]) if A $\subset$ Cl(Int(A)) (resp. A $\subset$ Int(Cl(Int(A)))). The complement of a semi- open (resp. α-open) set is called semi-closed (resp. α-closed).*

The semi-closure [1] of a subset A of X, denoted by sCl(A), is defined to be the intersection of all semi-closed sets containing A in X. The α-closure of a subset is similarly defined.

Definition 2.2. *A subset A of a space X is called semi-generalized closed (briefly $sg\alpha$-closed)* [7] *if αCl(A) $\subset$ U whenever A $\subset$ U and U is semi-open in X. The complement of $sg\alpha$-closed set is called sgα-open.*

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The union (resp. intersection) of all sgα-open (resp. $sg\alpha$-closed) sets each contained in (resp. containing) a set A in a space X is called the $sg\alpha$-interior (resp. $sg\alpha$-closure) of A and is denoted by $sg\alpha$-Int(A) (resp. $sg\alpha$-Cl(A)) [7].

The family of all $sg\alpha$-open (resp. $sg\alpha$-closed) sets of (X,τ) is denoted by $sg\alpha\, O(X)$ (resp. $sg\alpha\, C(X)$). The family of all $sg\alpha$-open (resp. $sg\alpha$-closed) sets of (X,τ) containing a point $x \in X$ is denoted by $sg\alpha\, O(X, x)$ (resp. $sg\alpha\, C(X, x)$). It is well known that $sg\alpha\, O(X)$ forms a topology [7].

Definition 2.3. [2] *A family $\{A_\alpha : \alpha \in \Delta\}$ of subsets of a space X is said to be locally finite family if for each point x of X, there exists a neighborhood G of x such that the set $\{\alpha \in \Delta : G \cap A_\alpha = \Phi\}$ is finite.*

Lemma 2.4. [2] *If $\{A_\alpha : \alpha \in \Delta\}$ is a locally finite family of subsets of a space X, then the family $\{Cl(A_\alpha) : \alpha \in \Delta\}$ is a locally finite family of X and $Cl(\cup A_\alpha) = \cup Cl(A_\alpha)$.*

Definition 2.5. [2] *A family $\{A_\alpha : \alpha \in \Delta\}$ of subsets of a space X is said to be point-finite if for each point x of X, the set $\{\alpha \in \Delta : x \in A_\alpha\}$ is finite.*

Definition 2.6. [2] *An open cover $\{G_\alpha : \alpha \in \Delta\}$ of a space X is said to be shrinkable if there exists an open cover $\{H_\alpha : \alpha \in \Delta\}$ of X such that $Cl(H_\alpha) \subseteq G_\alpha$* for each $\alpha \in \Delta$.

Definition 2.7. [5] *The family $\{A_\alpha : \alpha \in \Delta\}$ and $\{B_\alpha : \alpha \in \Delta\}$ of subsets of a set X are said to be similar, if for each finite subset γ of Δ, the sets $\bigcap_{\alpha\in\gamma} A_\alpha$* and *$\bigcap_{\alpha\in\gamma} B_\alpha$ are either both empty or both non-empty.*

Theorem 2.8. [2] *Let X be any topological space. The following statements are equivalents:*

(i) *X is a normal space.*
(ii) *Each point-finite open cover of X is shrinkable.*
(iii) Each finite open cover of X has a locally finite closed refinement.

Theorem 2.9. [5] Let $\{U_\alpha : \alpha \in \Delta\}$ *be a locally finite family of open sets of a normal space X and $\{F_\alpha : \alpha \in \Delta\}$ a family of closed sets such that $F_\alpha \subseteq U_\alpha$* for each $\alpha \in \Delta$. *Then there exists a family $\{G_\alpha : \alpha \in \Delta\}$ of open sets such that $F_\alpha \subseteq G_\alpha \subseteq Cl(G_\alpha) \subseteq U_\alpha$ for each $\alpha \in \Delta$ and the families $\{F_\alpha : \alpha \in \Delta\}$* and *$\{Cl\, G_\alpha : \alpha \in \Delta\}$ are similar.*

Proposition 2.10. [7] *For subset A and $A_i (i \in I)$ of a space (X,τ), the following hold:*

(i) $A \subseteq sg\alpha\, Cl(A)$
(ii) *If $A \subseteq B$, then $sg\alpha\, Cl(A) \subseteq sg\alpha\, Cl(B)$.*
(iii) $sg\alpha\, Cl(\cap\{A_i : i \in I\}) \subseteq \cap\{sg\alpha\, Cl\, A_i : i \in I\}$.
(iv) $sg\alpha\, Cl(\cup\{A_i : i \in I\}) = \cup\{sg\alpha\, Cl\, A_i : i \in I\}$

Definition 2.11. *If A is a subset of a space X, then the* $sg\alpha$*-boundary of A is defined as* $sg\alpha\, Cl(A) \setminus sg\alpha\, Int(A)$ *and is denoted by* $sg\alpha\, Bd(A)$.

3. $sg\alpha$-NORMAL SPACE

Definition 3.1. *A topological space X is said to be* $sg\alpha$ *- normal [6] if whenever A and B are disjoint* $sg\alpha$*- closed sets in X, there exist disjoint* $sg\alpha$*-open sets U and V with* $A \subseteq U$ *and* $B \subseteq V$

Definition 3.2. *A family* $\{A_\alpha : \alpha \in \Delta\}$ *of subsets of a space X is said to be* $sg\alpha$*-locally finite family if for each point x of X, there exists an* $sg\alpha$*-open set G of X such that the set* $\{\alpha \in \Delta : G \cap A_\alpha = \phi\}$ *is finite*

Definition 3.3. *An open cover* $\{G_\alpha : \alpha \in \Delta\}$ *of a space X is said to be* $sg\alpha$*-shrinkable if there exists an* $sg\alpha$*-open cover* $\{H_\alpha : \alpha \in \Delta\}$ *of X such that* $sg\alpha\,Cl(H_\alpha) \subseteq G_\alpha$ for each $\alpha \in \Delta$.

Lemma3.4. *If* $\{A_\alpha : \alpha \in \Delta\}$ *is a locally finite family of subsets of a space X, then the family* $\{sg\alpha\,Cl(A_\alpha) : \alpha \in \Delta\}$ *is* $sg\alpha$*-locally finite family of X. Moreover,* $sg\alpha\,Cl(\bigcup A_\alpha) = \bigcup sg\alpha\,Cl(A_\alpha)$.

Proof. From Lemma 2.4, we obtain that the family $\{Cl(A_\alpha) : \alpha \in \Delta\}$ is a locally finite family of X whenever $\{A_\alpha : \alpha \in \Delta\}$ is locally finite. Since $sg\alpha\,Cl(A_\alpha) \subseteq Cl(A_\alpha)$ for every $\alpha \in \Delta$, the family $\{sg\alpha\,Cl(A_\alpha) : \alpha \in \Delta\}$ is a locally finite family of X. To prove that $sg\alpha\,Cl(\bigcup A_\alpha) = \bigcup sg\alpha\,Cl(A_\alpha)$, we have $\bigcup sg\alpha\,Cl(A_\alpha) \subseteq sg\alpha\,Cl(\bigcup A_\alpha)$ .Therefore, it is sufficient to prove that $sg\alpha\,Cl(\bigcup A_\alpha) \subseteq \bigcup sg\alpha\,Cl(A_\alpha)$. Suppose that $x \notin \bigcup sg\alpha\,Cl(A_\alpha)$, so $x \notin sg\alpha\,Cl(A_\alpha)$, for all $\alpha \in \Delta$. This means that there exists an $sg\alpha$-open set G such that $G \cap A_\alpha = \phi$ for all $\alpha \in \Delta$ and hence, $G \cap sg\alpha\,Cl(A_\alpha) = \phi$ for all $\alpha \in \Delta$. Since the family $\{A_\alpha : \alpha \in \Delta\}$ is locally finite, there exists an open set H which contains x and the set $\{\alpha \in \Delta : H \cap A_\alpha = \phi\}$ is finite. This means that there exists a finite subset M of Δ such that $H \cap A_\alpha = \phi$ for $\alpha \in M$. From above we obtain that x belongs to $X \setminus sg\alpha Cl(A_\alpha)$ for every $\alpha \in \Delta$. Since the family of $sg\alpha$- open sets forms a topology on X [7] , the set $V = \bigcap [X \setminus sg\alpha\,Cl(A_\alpha) : \alpha \in \Delta]$ is an $sg\alpha$-open set in X containing x. But $H \cap V$ is an $sg\alpha$-open set containing x and $(H \cap V) \cap A_\alpha = \phi$ for all $\alpha \in \Delta$. Therefore $(H \cap V) \cap (\bigcup A_\alpha) = \phi$. This implies that $x \in sg\alpha\,Cl(\bigcup A_\alpha)$ and this completes the proof.

Theorem 3.5. *Let X be any topological space. Then the following statements are equivalent:*

(1) *X is an* $sg\alpha$ - normalspace.

(2) *Each point – finite open cover of X is* $sg\alpha$ - shrinkable.

(3) *Each finite* $sg\alpha$ - open cover *of X has a locally finite* $sg\alpha$*-closed refinement*

Proof. (1) $\Rightarrow$(2): Let $\{U_\alpha : \alpha \in \Delta\}$ be a point – finite open cover of an $sg\alpha$-normal space *X*, we may assume α is well–ordered. We shall construct an $sg\alpha$- shrinkable family of $\{U_\alpha : \alpha \in \Delta\}$ by transfinite induction. Let $\mu \in \alpha$ and suppose that for each $\alpha < \mu$ we have $sg\alpha$-open set V_α such that $sg\alpha\,\mathrm{Cl}(V_\alpha) \subseteq U_\alpha$ and for each $v < \mu$, we have $(\bigcup_{\alpha \le v} V_\alpha) \cup (\bigcup_{\alpha > v} U_\alpha) = X$. Let $x \in X$, then since $\{U_\alpha : \alpha \in \Delta\}$ is a point-finite, there exists the largest element $\gamma \in \alpha$ such that $x \in U_\gamma$. If $\gamma \ge \mu$, then $x \in \bigcup_{\alpha \ge \mu} U_\alpha$ and if $\gamma < \mu$, then $x \in \bigcup_{\alpha \le \gamma} V_\alpha \subseteq \bigcup_{\alpha < \gamma} V_\alpha$. Hence, $(\bigcup_{\alpha < \mu} V_\alpha) \cup (\bigcup_{\alpha \ge \gamma} U_\alpha) = X$. Thus, U_α contains the complement of $(\bigcup_{\alpha < \mu} V_\alpha) \cup (\bigcup_{\alpha > \gamma} U_\alpha)$. Since X is $sg\alpha$-normal, there exists an $sg\alpha$-open set V_μ such that $X \setminus [(\bigcup_{\alpha < \mu} V_\alpha) \cup (\bigcup_{\alpha > \gamma} U_\alpha)] \subseteq V_\mu \subseteq sg\alpha\,\mathrm{Cl}(V_\mu) \subseteq U_\mu$. Thus, $sg\alpha\,\mathrm{Cl}(V_\mu) \subseteq U_\mu$ and $(\bigcup_{\alpha \le \mu} V_\alpha) \cup (\bigcup_{\alpha > \gamma} U_\alpha) = X$. Hence the construction of an $sg\alpha$- shrinkable family for $\{U_\alpha : \alpha \in \Delta\}$ is completed by transfinite induction. (2) $\Rightarrow$ (3) : Obvious . (3) $\Rightarrow$ (1) : Let X be a space such that each finite $sg\alpha$-open cover of X has a locally finite $sg\alpha$-closed refinement . Let A and B be two $sg\alpha$-closed sets in X. The $sg\alpha$-open cover $\{X \setminus A, X \setminus B\}$ of X has a locally finite $sg\alpha$-closed refinement υ. Let E be the union of members of υ disjoint from A and let F be union of members of υ disjoint from B. Then, by Lemma 3.4, E and F are $sg\alpha$-closed sets and $E \cup F = X$. Thus, if $U = X \setminus E$ and $V = X \setminus F$ then U and V are disjoint $sg\alpha$-open *sets* such that $A \subseteq U$ and $B \subseteq V$ Therefore, X is an $sg\alpha$-normal space .

Proposition 3.6. Let $\{U_\alpha\}_{\alpha\in\Delta}$ be a locally finite family of $sg\alpha$-open *sets* of an $sg\alpha$-*normal space X* and $\{F_\alpha\}_{\alpha\in\Delta}$ a family of $sg\alpha$-*closed sets* such that $F_\alpha \subseteq U_\alpha$ for each $\alpha \in \Delta$. Then there exists a family $\{G_\alpha\}_{\alpha\in\Delta}$ of $sg\alpha$-open *sets* such that $F_\alpha \subseteq G_\alpha \subseteq sg\alpha\,\mathrm{Cl}(U_\alpha) \subseteq U_\alpha$ and the families $\{F_\alpha\}_{\alpha\in\Delta}$ and $\{sg\alpha\,\mathrm{Cl}(G_\alpha)\}_{\alpha\in\Delta}$ are similar.

Proof . Let Δ be well-ordered with a least element. By transfinite induction, we shall construct a family $\{G_\alpha\}_{\alpha\in\Delta}$ of $sg\alpha$-open *sets* such that $F_\alpha \subseteq G_\alpha \subseteq sg\alpha\,\mathrm{Cl}(G_\alpha) \subseteq U_\alpha$ and for each element υ in Δ the family

$$\{K_\alpha^\upsilon\}_{\alpha\in\Delta} = \begin{cases} \mathrm{Cl}(G_\alpha) & \text{if } \alpha \le \upsilon \\ F_\alpha & \text{if } \alpha < \upsilon \end{cases}$$

is similar to $\{F_\alpha\}_{\alpha\in\Delta}$. Suppose that $\mu \in \alpha$ and that G_α are defined for $\alpha < \mu$ such that for each $\upsilon < \mu$ the family $\{K_\alpha^\upsilon\}_{\alpha\in\Delta}$ is similar $\{F_\alpha\}_{\alpha\in\Delta}$. Let $\{L_\alpha\}_{\alpha\in\Delta}$ be the family given by:

$$\{L_\alpha\}_{\alpha\in\Delta} = \begin{cases} \mathrm{Cl}(G_\alpha) & \text{if } \alpha < \upsilon \\ F_\alpha & \text{if } \alpha \ge \upsilon \end{cases}$$

Then $\{F_\alpha\}_{\alpha\in\Delta}$ and $\{L_\alpha\}_{\alpha\in\Delta}$. For suppose that $\alpha_1, \alpha_2, \ldots, \alpha_r \in \Delta$ and $\alpha_1, \alpha_2, \ldots, \alpha_j < \mu < \alpha_{j+1} < \ldots < \alpha_r$, then $\bigcap_{i=1}^{r}\{L_{\alpha i}\} = \bigcap_{i=1}^{r}\{K_{\alpha i}^{\alpha j}\}$. Therefore $\bigcap_{i=1}^{r}\{L_{\alpha i}\} = \phi$ if and only if $\bigcap_{i=1}^{r}\{F_{\alpha i}\} = \phi$ because $\{K_\alpha^\upsilon\}_{\alpha\in\Delta}$ is similar to $\{F_\alpha\}_{\alpha\in\Delta}$. Since $L_\alpha \subseteq G_\alpha$ for each α, the family $\{L_\alpha\}_{\alpha\in\Delta}$ is locally finite. Thus if Γ is the set of finite subsets of Δ and for each $\gamma \in \Gamma$, $E_\gamma = \bigcap_{\alpha\in\Delta} L_\alpha$, then $\{E_\gamma\}_{\gamma\in\Gamma}$ is locally finite family of $sg\alpha$-closed sets. Hence by Lemma 3.4, $E = \bigcup\{E_\gamma : E_\gamma \cap F_\mu\}$ is an sgα-closed set which is disjoint from F_μ. Therefore, there exists an sgα-open set G_μ such that $F_\mu \subseteq G_\mu \subseteq sg\alpha\,\mathrm{Cl}(G_\mu) \subseteq U_\mu$ and $sg\alpha\,\mathrm{Cl}(G_\mu \cap E) = \phi$. Now the sg$\alpha$-open set G_α is defined for $\alpha \le \mu$ and to complete the proof it remains to show that the family $\{K_\alpha^\upsilon\}_{\alpha\in\Delta}$ is similar to the family $\{F_\alpha\}_{\alpha\in\Delta}$. It is sufficient to show that the families $\{K_\alpha^\upsilon\}_{\alpha\in\Delta}$ and $\{L_\alpha\}_{\alpha\in\Delta}$ are similar. Suppose that $\alpha_1, \alpha_2, \ldots, \alpha_r \in \Delta$ and that $\bigcap_{i=1}^{r}\{L_{\alpha i}\} = \phi$, we have to show that $\bigcap_{i=1}^{r}\{K_{\alpha i}^{\alpha j}\} = \phi$ Suppose that $\alpha_1 < \alpha_2 < \ldots < \alpha_j < \mu < \alpha_{j+1} < \ldots < \alpha_r$ if $\alpha_j \ne \mu$ there is nothing to prove. If $\alpha_j = \mu$, then $L_{\alpha_1} \cap \ldots \cap L_{\alpha_{j-1}} \cap F_\mu \cap L_{\alpha_{j+1}} \cap \ldots \cap L_{\alpha_r} = \phi$. Hence by the construction $L_{\alpha_1} \cap \ldots \cap L_{\alpha_{j-1}} \cap sg\alpha\,\mathrm{Cl}(G_\mu \cap L_{\alpha_{j+1}}) \cap \ldots \cap L_{\alpha_r} = \phi$. Thus $\bigcap_{i=1}^{r}\{K_{\alpha i}^{\alpha j}\} = \phi$

4. $sg\alpha$ -COVERING DIMENSION

In this section, we introduce a type of a covering dimension by using $sg\alpha - open$ sets which we call the $sg\alpha$ – covering dimension function.

Definition 4.1. *The $sg\alpha-$ covering dimension of a topological space X is the least positive integer n such that every finite $sg\alpha - open$ cover of X has an $sg\alpha - open$ refinement of order not exceeding n or is ∞ if there is no such integer. We shall denote the $sg\alpha -$ covering dimension of a space X by $\dim_{sg\alpha} X$. If X is an empty set, then $\dim_{sg\alpha} X = -1$ and $\dim_{sg\alpha} X \le n$ if each finite $sg\alpha - open$ cover of X has an $sg\alpha - open$ refinement of order not exceeding n. Also we have $\dim_{sg\alpha} X = n$ if it is true that $\dim_{sg\alpha} X \le n$ but it is not true if $\dim_{sg\alpha} X \le n-1$. Finally, $\dim_{sg\alpha} X = \infty$ if for every integer n there exists a finite $sg\alpha - open$ cover which has no $sg\alpha - open$ refinement of order not exceeding n.*

Proposition 4.2. *If Y is an open and a closed subset of a space X, then $\dim_{sg\alpha} Y \le \dim_{sg\alpha} X$*

Proof. It is sufficient to prove that if $\dim_{sg\alpha} X = n$, then $\dim_{sg\alpha} Y \le n$. Let $\{U_1, U_2, \ldots U_k\}$ be an $sg\alpha-$open cover of the open set Y. Then U_i is $sg\alpha-$open in X for each i and since every open set is $sg\alpha-$open. Then the finite $sg\alpha-$open cover $\{U_1, U_2, \ldots U_k, X\setminus Y\}$ of X has an $sg\alpha-$open refinement $sg\alpha$ of order which not exceeding n .Let ε be all members of $sg\alpha$ except those members associated with X\Y , since every open set is $sg\alpha-$open , then each member of ε is $sg\alpha-$open in Y and also ε is a refinement of $\{U_1, U_2, \ldots U_k\}$ of order not exceeding n. This implies that $\dim_{sg\alpha} Y \le n$.

Now we give some characterizations of the $sg\alpha$- covering dimension in topological spaces.

Theorem4.3. ***If*** *X is a topological space, then the following statements about X are equivalent:*

(1) $\dim_{sg\alpha} X \le n$

(2) For any finite $sg\alpha$*-open cover* $\{U_1, U_2, \ldots, U_k\}$ *of X there is an* $sg\alpha-open$ cover $\{V_1, V_2, \ldots, V_k\}$ *of order not exceeding n such that* $V_i \subseteq U_i$ *for* $i = 1,2,\ldots,k$

(3) If $\{U_1, U_2, \ldots, U_{n+2}\}$ *is an* $sg\alpha$*- open cover of X , there is an* $sg\alpha$*- open cover* $\{V_1, V_2, \ldots, V_{n+2}\}$ *such that* $V_i \subseteq U_i$ *and* $\bigcap_{i=1}^{n+2} V_i = \phi$

Proof. (1) $\Rightarrow$ (2) : Suppose that $\dim_{sg\alpha} X \le n$ and the $sg\alpha-open$ cover $\{U_1, U_2, \ldots, U_k\}$ of X has an $\beta-open$ refinement β of order not exceeding n. If $W \in \beta$, then $W \subseteq U_i$ for some i. Let each $W \; in \; \beta$ be associated with one of the sets U_i containing it and let V_i be the union of those members of β thus associated with U_i , since every open set is $sg\alpha-open$, then V_i is $sg\alpha-open$ and $V_i \subseteq U_i$ and each point of X is in some member of β and hence in some V_i. Each point x of X is in at most n+1 members of $sg\alpha$, each of which is associated with a unique U_j and hence is in at most n+1 members of $\{V_i\}$. Thus $\{V_i\}$ is an $sg\alpha-open$ cover of X of order not exceeding n. (2) $\Rightarrow$ (3) : Obvious. (3) $\Rightarrow$ (2) : Let X be a space satisfying (3) and $\{U_1, U_2, \ldots, U_k\}$ a finite $sg\alpha-open$ cover of X , we can assume that k > n+1. Let $G_i = \mathrm{U_i} \; if \; \mathrm{i} \le \mathrm{n}+1$ and $\mathrm{G}_{\mathrm{n}+2} = \bigcup_{i=n+2}^{k} U_i$, then $\{G_1, G_2, \ldots, G_{n+2}\}$ is an $sg\alpha-open$ cover of X and so by hypothesis there is a $sg\alpha-open$ cover $\{H_1, H_2, \ldots, H_k\}$ such that $H_i \subseteq G_i$ and $\bigcap_{i=1}^{n+2} H_i = \phi$. Let $W_i = U_i \; if \; i \le n+1$ and let $W_i = U_i \bigcap G_{n+2}$ if $i > n+1$. Then $\delta = \{W_1, W_2, \ldots, W_k\}$ is an $sg\alpha-open$ cover of X each $W_i \subseteq U_i$ and $\bigcap_{i=1}^{n+2} W_i = \phi$. If there exists such set B of {1,2,…,k} with n+2 elements such that $\bigcap_{i=1}^{n+2} W_i \ne \phi$, let the members of δ be

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renumbered to give a family $P=\{P_1,P_2,\dots,P_k\}$ $\bigcap_{i=1}^{n+2} P_i \neq \phi$. By applying the above construction to P , we obtain the $sg\alpha-open$ cover $W'=\{W'_1,W'_2,\dots,W'_k\}$ such that $W' \subseteq P_i$ and $\bigcap_{i=1}^{n+2} W'_i=\phi$. Thus by a finite number of repetitions of this process we obtain an $sg\alpha-open$ cover $\{V_1,V_2,\dots,V_k\}$ of X , of order not exceeding n such that $V_i \subseteq U_i$, (2) $\Rightarrow$ (1) : Obvious.

Proposition 4.4. *In a topological space X if* $\dim_{sg\alpha} X=0$, *then X is a* $sg\alpha$ *-normal Space*.

Proof. Suppose that $\dim_{sg\alpha} X=0$ and let F and E be any two disjoint $sg\alpha$ -closed sets in X, then $\{X\setminus F, X\setminus E\}$ is an $sg\alpha-open$ cover of X ; hence , there exists a $sg\alpha-open$ refinement β of order not exceeding 0 . This means that all members of β are pair wise disjoint. Let G be the union of all member of β that are associated with X\E and H be the union of all members of β that are associated with X\F, hence , G and H are $sg\alpha-open$ sets such that $G\bigcup H=X, G\subseteq X\setminus E \; and \; H\subseteq X\setminus F$ and $G\bigcap H=\phi$. Thus G and H are disjoint $sg\alpha-open\ sets$ such that $F\subseteq G \; and \; E\subseteq H$. Hence X is a $sg\alpha-normal\ space$.

In $sg\alpha-normal\ spaces$, $sg\alpha-covering$ dimension can be defined in terms of the order of finite $sg\alpha-closed$ refinements of finite $sg\alpha-open$ cover.

Proposition4.5. If (X,τ) is a topological space, then the following statements about X are equivalent.

(1) $\dim_{sg\alpha} X \le n$.

(2) For any finite $sg\alpha-open$ cover $\{U_1,U_2,\dots,U_k\}$ of X there is a $sg\alpha-open$ cover $\{V_1,V_2,\dots,V_k\}$ such that $sg\alpha\, Cl(V_i)\subseteq U_i$ and the order for $\{sg\alpha\, Cl(V_1), sg\alpha\, Cl(V_2),\dots, sg\alpha\, Cl(V_k)\}$ does not exceed n

(3) For any finite $sg\alpha-open$ cover $\{U_1,U_2,\dots,U_k\}$ of X there is a $sg\alpha-closed$ cover $\{F_1,F_2,\dots,F_k\}$ such that $F_i \subseteq U_i$ and the order for $\{F_1,F_2,\dots,F_k\}$ does not exceed n.

(4) Every finite $sg\alpha-open$ cover of X has a finite $sg\alpha-closed$ refinement of order not exceeding n.

(5) If $\{U_1,U_2,\dots,U_k\}$ is an $sg\alpha-open$ cover of X, there is an $sg\alpha-closed$ cover $\{F_1,F_2,\dots,F_k\}$ such that $F_i \subseteq U_i$ and $\bigcap_{i=1}^{n+2} F_i=\phi$.

Proof . (1) $\Rightarrow$ (2): Suppose that $\dim_{sg\alpha} X\le n$, and let $\{U_1,U_2,\dots,U_k\}$ be $sg\alpha-open$ cover of X, then by Theorem 4.3 , there exists an $sg\alpha-open$ cover $\{W_1,W_2,\dots,W_k\}$ of order not exceeding n such that $W_i \subseteq U_i$. Since X is $sg\alpha-normal$, by Theorem 3.5, there exists an $sg\alpha-open$ cover $\{V_1,V_2,\dots,V_k\}$ such that $sg\alpha\, Cl(V_i)\subseteq W_i$ for each i . Then $\{V_1,V_2,\dots,V_k\}$

is an $sg\alpha-open$ $cover$ with the required properties. (2) $\Rightarrow$ (3) and (3) $\Rightarrow$ (4): Obvious. (4) $\Rightarrow$ (5): Let X be a space satisfying (4) and let $\Rightarrow \Pi = \{U_1, U_2, \ldots, U_{n+2}\}$ be $sg\alpha-open$ $cover$ of X. Then the cover Π has a finite $sg\alpha-closed$ refinement v of order not exceeding n. If $E \in v$, then $E \subseteq U_i$ for some i. Let each E in v be associated with one of the sets U_i containing it and let F_i be the union of those members of v which associated with U_i , then F_i is $sg\alpha-closed$, $F_i \subseteq U_i$ and $\{F_1, F_2, \ldots, F_{n+2}\}$ is a $sg\alpha-cover$ of X such that $\bigcap_{i=1}^{n+2} F_i = \phi$.(5) $\Rightarrow$(1) : Let X be a space satisfying (5) and let $\{U_1, U_2, \ldots, U_{n+2}\}$be an $sg\alpha-open$ $cover$ of X , by hypothesis there exists an $sg\alpha-closed$ cover $\{F_1, F_2, \ldots, F_{n+2}\}$ such that each $F_i \subseteq U_i$ and $\bigcap_{i=1}^{n+2} F_i = \phi$. By Proposition 3.6, there exist $sg\alpha-open$ $sets$ $\{V_1, V_2, \ldots, V_{n+2}\}$ such that $F_i \subseteq V_i \subseteq U_i$ *for each i and* $\{V_i\}$ is similar to $\{F_i\}$. Thus $\{V_1, V_2, \ldots, V_{n+2}\}$is an $sg\alpha-open$ $cover$ of X, each $V_i \subseteq U_i$ $\bigcap_{i=1}^{n+2} V_i = \phi$. Therefore by Theorem 4.3, $\dim_{sg\alpha} X \le n$ □

Proposition 4.6. If X is an $sg\alpha-normal$ $space$, then the following statements about X are equivalent:

(1) $\dim_{sg\alpha} X \le n$,

(2) *For any finite* $sg\alpha$*-closed sets* $\{F_1, F_2, \ldots F_{n+1}\}$ *and each family of* $sg\alpha$*-open sets* $\{U_1, U_2, \ldots U_{n+1}\}$ *such that* $F_i \subseteq U_i$ *there exists a family* $\{V_1, V_2, \ldots V_{n+1}\}$ *of* $sg\alpha$*-open sets such that* $F_i \subseteq V_i \subseteq sg\alpha\, Cl(V_i) \subseteq U_i$ *for each i, and* $\bigcap_{i=1}^{n+1} \alpha Bd V_i = \phi$

(3) *For each family* $sg\alpha$*-closed sets* $\{F_1, F_2, \ldots F_k\}$ *and each family of* $sg\alpha$*- open sets* $\{U_1, U_2, \ldots U_k\}$ *such that* $F_i \subseteq U_i$*, there exists a family* $\{V_1, V_2, \ldots V_k\}$ *and* $\{W_1, W_2, \ldots W_k\}$ *of* $sg\alpha$*-open sets such that* $F_i \subseteq V_i \subseteq sg\alpha\, Cl(V_i) \subseteq W_i \subseteq U_i$ *for each i and the order of the family* $\{sg\alpha\, Cl(W_1) \setminus V_1, sg\alpha\, Cl(W_2) \setminus V_2 \ldots\ldots sg\alpha\, Cl(W_k) \setminus V_k\}$ *does not exceed n – 1.*

(4) *For each family* $sg\alpha$*-closed sets* $\{F_1, F_2, \ldots F_k\}$ *and each family of* $sg\alpha$*-open sets* $\{U_1, U_2, \ldots U_k\}$ *such that* $F_i \subseteq U_i$*, there exists a family* $\{V_1, V_2, \ldots V_k\}$ *of* $sg\alpha$*-open sets* $F_i \subseteq V_i \subseteq sg\alpha\, Cl(V_i) \subseteq U_i$ *and the order of the family* $\{sg\alpha Bd(V_1), sg\alpha Bd(V_2), \ldots\ldots, sg\alpha Bd(V_k)$ *does not exceed n – 1.*

Proof. (1)$\Rightarrow$ (2): Suppose that $\dim_{sg\alpha} X \le n$,and let $\{F_1, F_2, \ldots F_{n+1}\}$ be $sg\alpha$-closed sets and $\{U_1, U_2, \ldots U_{n+1}\}$ $sg\alpha$-open sets such that $F_i \subseteq U_i$. Since $\dim_{sg\alpha} X \le n$, the $sg\alpha$-open cover of X consisting of sets of the form $\bigcap_{i=1}^{n+2} H_i$, where $H_i = U_i$ or $H_i = X \setminus F_i$ for each i , has a finite $sg\alpha$-open refinement $\{W_1, W_2, \ldots W_q\}$ of order not exceeding n, Since X is $sg\alpha$-

normal, there is an $sg\alpha$-closed cover $\{K_1, K_{2,}.....K_q\}$ such that $K_r \subseteq W_r$ for $r = 1,2,,,,,,,q$. Let N_r denoted the set i such that $F_i \cap W_r \neq \emptyset$. For $r = 1,2,,,,,,,q$, we can find $sg\alpha$-open sets V_{ir} for r in N_r such that $K_r \subseteq V_{ir} \subseteq sg\alpha\, Cl(V_{ir}) \subseteq W_r$ and $sg\alpha\,\mathrm{Cl}(V_{ir}) \subseteq W_{jr}$ *if* $i < j$ .Now for each $i = 1,2,..,n+1$, let $V_r = \bigcup_r \{V_{ir} : i \in N_r\}$. Then V_i is $sg\alpha$-open and $F_i \subseteq V_i$ for if $x \in F_i$ and $x \in K_r$; then $i \in N_r$ so that $x \in V_{ir} \subseteq V_i$. Further more if $i \in N_r$ so that $F_i \cap W_r \neq \emptyset$ then W_r is not contained in $X \setminus F_i$ so that $W_r \subseteq U_i$. Thus, if $i \in N_r$, then $V_{ir} \subseteq U_i$ and since $sg\alpha\,\mathrm{Cl}(V_i) = \bigcup_r \{sg\alpha\,\mathrm{Cl}(V_{ir}) : i \in N_r\}$, it follows that $sg\alpha\,\mathrm{Cl}(V_i) \subseteq U_i$. Finally suppose that $x \in \bigcap_{i=1}^{n+2} sg\alpha\,\mathrm{Bd}(V_i)$ and since $sg\alpha Bd(V_i) \subseteq \bigcup_r \{sg\alpha Bd(V_{ir}) : i \in N_r\}$, it follows that for each i there exists i_r such that $x \in sg\alpha\,\mathrm{Bd}(V_{ir})$ and if $i \neq j$, then $r_i \neq r_j$ for if $r_i = r_j = r$, then $x \in sg\alpha\,\mathrm{Cl}(V_{ir})$ and $x \in sg\alpha\,\mathrm{Cl}(V_{jr})$ but $x \in V_{ir}$ and $x \in V_{jr}$ which are absurd, since either $sg\alpha\,\mathrm{Cl}(V_{ir}) \subseteq V_{jr}$ or $sg\alpha\,\mathrm{Cl}(V_{jr}) \subseteq V_{ir}$. For each $i, x \notin V_{irj}$ so that $x \notin K_{rj.}$ But $\{K_r\}$ is a $sg\alpha$-cover of X and so there exists r_0 different form each of the r_i such that $x \in K_{r_o} \subseteq W_{r_o}$. Since $x \in V_{irj}$, it follows that $x \in W_{r_i}$ for $i = 1, 2, \ldots, n+1$ so that $x \bigcap_{i=0}^{n+1} V_n$. Since the order of $\{W_r\}$ does not exceed n, this is absurd. Hence $\bigcap_{i=1}^{n+1} sg\alpha\,\mathrm{Bd}(V_i) = \emptyset$.

$(2) \Rightarrow (3)$: Let (2) hold , let $F_1, F_2, \ldots\ldots F_k$ be $sg\alpha$ -closed sets and let $U_1, U_{2,}.....U_K$ be $sg\alpha$-open sets such that $F_i \subseteq U_i$. We can assume that $k > n+1$; otherwise , there is nothing to prove. Let the subset $\{1,2,\ldots\ldots k\}$ containing $n+1$ elements be enumerated as $C_1, C_2, \ldots, C_q$, where $q = kc_{n+1}$. By using (2), we can find sgα-open sets $V_{i,r}$ for i in C_i such that $F_i \subseteq V_{i,1} \subseteq sg\alpha\, Cl(V_{i,1}) \subseteq U_i$ and $\bigcap_{i=1}^{n+1} sg\alpha\,\mathrm{Bd}(V_{i,r}) = \emptyset$. We have a finite family $\{sg\alpha\,\mathrm{Bd}(V_{i,r}) : i \in C_i\}$ of $sg\alpha$-closed sets of the $sg\alpha$-normal space X and $sg\alpha Bd(V_{i,1}) \subseteq U_i$ for each i in C_1. Thus, by Proposition 3.6, for each i in C_1, there exists an $sg\alpha$-open set G_i such that $sg\alpha\,\mathrm{Bd}(V_{i,1}) \subseteq G_i \subseteq sg\alpha\, Cl(G_i) \subseteq U_i$ and $\{sg\alpha\,\mathrm{Cl}(G_i)\}_{i\in C_1}$ is similar to $\{sg\alpha\,\mathrm{Bd}(V_{i,r})\}_{i\in C_1}$, so that in particular $\bigcap_{i\in C_i} sg\alpha\,\mathrm{Cl}(G_i) = \emptyset$. Let $W_{i,1} = V_{i,1} \cup G_i$ if $i \in C_1$, then $sg\alpha\, Cl(V_{i,1}) \subseteq W_{i,1} \subseteq sg\alpha\, Cl\,(W_{i,1}) \subseteq U_i$ and since $(sg\alpha\, Cl(W_{i,1}) \setminus V_{i,1}) \subseteq sg\alpha\, Cl(G_i)$, we have $\bigcap_{i\in C_1} (sg\alpha\,\mathrm{Cl}(W_{i,1} \setminus V_{i,1}) = \emptyset$. If $i \notin C_1$, let $V_{i,1}$ be an $sg\alpha$-open set such that $F_i \subseteq V_{i,1} \subseteq sg\alpha\, Cl(V_{i,1}) \subseteq U_i$ and let $W_{i,1} = U_i$. Then for $i = 1, 2, \ldots, k$ we have sgα-open sets $V_{i,1}$ and $W_{i,1}$ such that $F_i \subseteq V_{i,1} \subseteq sg\alpha\, Cl(V_{i,1}) \subseteq U_i$ and $\bigcap_{i\in c}(sg\alpha\,\mathrm{Cl}(W_{i,1} \setminus V_{i,1}) = \emptyset$. Suppose that $1 < m \le q$ and for $i = 1,2,\ldots.k$ we find $sg\alpha$-open sets $V_{i,m-1}$ and $W_{i,m-1}$ such that $F_i \subseteq V_{i,m-1} \subseteq sg\alpha Cl(V_{i,m-1}) \subseteq W_{i,m-1} \subseteq U_i$ and $\bigcap_{i\in c}(sg\alpha Cl(W_{i,m-1}) \setminus V_{i,m-1}) = \emptyset$ if $1 \le j \le m-1$. By the above argument we can find $sg\alpha$-open sets $V_{i,m}$ and $W_{i,m}$ such that $sg\alpha\, Cl(V_{i,m-1}) \subseteq$

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$V_{i,m} \subseteq sg\alpha\, Cl(V_{i,m}) \subseteq W_{i,m} \subseteq W_{i,m-1}$ and $\bigcap_{i \in cm} (sg\alpha Cl(W_{i,m}) \setminus V_{i,m}) = \phi$ Since $sg\alpha Cl(W_{i,m}) \setminus V_{i,m} \subseteq (sg\alpha Cl(W_{i,m-1}) \setminus V_{i,m-1})$. We have $\bigcap_{i \in c_j} (sg\alpha Cl(W_{i,m}) \setminus V_{i,m}) = \phi$ if $j \leq m$. Thus by induction for $i = 1,2,....k$, we can find $sg\alpha$-open sets V_i and $W_i = (V_{i,q}$ and $W_{i,q}$ respectively). Such that $F_i \subseteq V_i \subseteq sg\alpha\, Cl(V_i) \subseteq W_i \subseteq U_i$ and $\bigcap_{i \in cj} (sg\alpha Cl(W_i) \setminus V_i) = \phi$, for $j = 1,2,....k$. Thus the order of the family $\{sg\alpha Bd(W_1 \setminus V_1),...,$ $sg\alpha Bd(W_k \setminus V_k)\}$ does not exceed $n-1$. (3)$\Rightarrow$(4): Obvious. (4)$\Rightarrow$(1): Let (4) hold and let $\{U_1, U_{2,}.....U_{n+2}\}$ be an $sg\alpha$-open cover of X. Since X is $sg\alpha$-normal, there exists an $sg\alpha$-closed cover $\{F_1, F_{2,}.....F_{n+1}\}$ of X such that $F_i \subseteq U_i$ for each i. By hypothesis there exists a family of $sg\alpha$-open sets $\{V_1, V_{2,}.....V_{n+1}\}$ such that $F_i \subseteq V_i \subseteq sg\alpha\, Cl(V_i) \subseteq U_i$ for each i, and the family $\{sg\alpha Bd(V_1), sg\alpha Bd(V_2),....\ \ ..,sg\alpha Bd(V_{n+2})$ has order not exceeding $n-1$. Let $L_j = sg\alpha Cl(V_j) \setminus \bigcup_{i<j} V_i$ for $j = 1,2,..\ \ .,n+2$. For each j, L_j is an $sg\alpha$-closed, and $\{L_1, L_{2,}.....L_{n+2}\}$ is an $sg\alpha$-closed cover of X, for if $x \in X$, there exists j such that $x \in V_j$ and $x \notin V_i$ for $i < j$ so that $x \in L_j$. Now $L_j = sg\alpha Cl(V_j) \bigcap_{i<j} (X \setminus V_j)$ so that $\bigcap_{j=1}^{n+2} L_j = \bigcap_{j=1}^{n+2} sg\alpha\, Cl(V_j) \bigcap_{i=1}^{n+1} (X \setminus V_j) \subseteq \bigcap_{j=1}^{n+1} sg\alpha Bd(V_j) = \phi$. Thus $\{L_1, L_2,,,,,,, L_{n+2}\}$, is an $sg\alpha$-closed cover of X, $L_i \subseteq sg\alpha\, Cl(V_j) \subseteq U_j$ and $\bigcap_{j=1}^{n+2} L_i = \phi$ Hence by proposition 4.5, $\dim_{sg\alpha} X \leq n$.

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