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Two-dimensional Generalized Canonical Cosine-Cosine Transform

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Abstract: This paper is concerned with the definition of two-dimensional (2-D) generalized canonical cosine-cosine transform it is extended to the distribution of compact support by using kernel method. We have discussed inversion theorem for that transform. Lastly we have proved some properties of that transform.

Keywords: 2-D canonical transform, 2-D cosine-cosine transform, 2-D fractional Fourier transform, generalized function.

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1) Introduction : –

Now a days fractional Fourier transforms plays important role in information processing [5]. The fractional Fourier transform as an extension of the fourier transform. It has been used many applications such as optical system analysis, filter design, solving differential equations. Phase retrieval and pattern recognition etc. [8] [3],In fact the fractional Fourier transform is special case of the canonical transform. The canonical transform is defined as [9]

$$\{CTf(t)\}(s)=\frac{1}{\sqrt{2\pi ib}}\int_{-\infty}^{\infty}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t}f(t)\,dt \qquad b\neq 0 \quad \ldots\ldots\ldots(1)$$

$$=\sqrt{d}\,e^{\frac{i}{2}\left(cds^2\right)}f(d.s) \qquad b=0$$

And the constraint that ad-bc=1 must be satisfied. The canonical transform defined above in (1) are all one-dimensional [1-D] , in [1] [2], [13] they have generalized them from one–dimensional into the (2-D) cases, [7], [6], [11] ,[4],[14].The two-dimensional canonical cosine-cosine transform it is extended to the distribution of compact support by using kernel method [4] [10] .

The two-dimensional canonical cosine-cosine transform is defined as.

$$\{2DCCCT\,f(t,x)\}(s,w)=\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

When $b\neq 0$

Notation and terminology of this paper is as per [15], [16] The paper is organized as follows. Section 2 gives the definition of 2-D canonical cosine-cosine transform on the space of generalized function in section 3 inversion theorem is proved in section 4 some property are proved lastly the conclusion is stated.

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2) 2-D Generalized canonical cosine-cosine transform:

2.1) Definition:-2-D Canonical cosine-cosine transform:

$$\{2DCCCT\, f(t,x)\}(s,w)=\left\langle f(t,x), K_{C_1}(t,s)\; K_{C_2}(x,w)\right\rangle$$

$$\{2DCCCT\, f(t,x)\}(s,w)=$$

$$\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt \quad \text{where,}$$

$$K_{C_1}(t,x)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.\cos\left(\frac{s}{b}t\right), \qquad \text{when } b\neq 0$$

$$=\sqrt{d}\,e^{\frac{i}{2}(cds^2)}\delta(t-ds), \qquad \text{when b=0} \qquad \text{And}$$

$$K_{C_2}(x,w)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.\cos\left(\frac{w}{b}x\right), \qquad \text{when } b\neq 0$$

$$=\sqrt{d}\,e^{\frac{i}{2}(cdw^2)}\;\delta(x-dw), \qquad \text{when b=0}$$

$\therefore$ is the 2D generalized canonical cosine- cosine transform.

2.2) Definition:-2-D Canonical sine-sine transform:

$$\{2DCSST\, f(t,x)\}(s,w)=$$

$$(-1)\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt \quad b\neq 0,$$

As we have obtained few property of 2-D canonical sine-sine transform in [12], hence we established similar property of 2-D canonical cosine-cosine transform.

3) Inversion for 2-D dimensional canonical cosine–cosine transforms:

Any transform is used to solve differential equations, only if inverse of the transform is available we obtain inverse of 2-D canonical cosine-cosine transform in next theorem.

3.1) Inversion Theorem: If $\{2DCCCT\, f(t,x)\}(s,w)$ is canonical cosine- cosine transform of $f(t,x)$ then inverse of transform is given by

$$f(t,x)=e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-\frac{i}{2}\left(\frac{a}{b}\right)x^2}\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)\{2DCCCT\;f(t,x)\}(s,w)\,dsdw,$$

Proof: The two dimensional canonical cosine- cosine transform if $f(t,x)$ is given by

$$\{2DCCCTf(t,x\}(s,w) =$$

$$\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.f(t,x)dxdt$$

$$f(s) = \{2DCCCT\, f(t,x)\}(s,w)$$

$$\therefore f(s) = \frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)dxdt$$

$$f(s).\sqrt{2\pi ib}\sqrt{2\pi ib}\; e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)dxdt$$

$$\therefore C_1(s) = f(s)\sqrt{2\pi ib}\sqrt{2\pi ib}\; e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2} \quad \text{And} \quad g(t,x) = e^{-\frac{i}{2}\left(\frac{d}{b}\right)t^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)x^2}f(t,x)$$

$$C_1(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(t,x).\cos\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)dxdt$$

$$\therefore \frac{s}{b} = \eta \quad \text{and} \quad \frac{w}{b} = \xi, \qquad \frac{d}{s} = d\eta \quad \text{and} \quad \frac{dw}{b} = d\xi$$

$$\therefore C_1(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(t,x)\cos(\eta t).\cos(\xi x)\,d\eta\,d\xi$$

By using inversion for cosine transform.

$$\therefore g(t,x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_1(s).\cos(\eta t).\cos(\xi x)d\eta.d\xi$$

$$g(t,x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(s)\sqrt{2\pi ib}\sqrt{2\pi ib}\; e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos(\eta t).\cos(\xi x)d\eta.d\xi$$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.f(t,x)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(s)\sqrt{2\pi ib}\sqrt{2\pi ib}\,e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)\frac{ds}{b}\frac{dw}{b}$$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.f(t,x)=\sqrt{2\pi ib}\sqrt{2\pi ib}\frac{1}{b}\frac{1}{b}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)f(s)\;dsdw$$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.f(t,x)=\frac{\sqrt{2\pi ib}}{b}\frac{\sqrt{2\pi ib}}{b}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)f(s)\;dsdw$$

$$f(t,x)=\;e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-\frac{i}{2}\left(\frac{a}{b}\right)x^2}\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\cos\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)\{2DCCCT\,f(t,x)\}(s,w)dsdw$$

4) Property of two-dimensional canonical cosine–cosine transforms

4.1) Separability: If $f(t,x)=f_1(t).f_2(x)$

then $\{2DCCCT\,f(t,x)\}(s,w)=\{CCT\,f_1(t)\}(s).\{CCT\,f_2(x)\}(w)$

4.**2) Linearity:** If C_1, C_2 are constant and f_1, f_2 are functions of $t\,\&\,x$ then

$$\{2DCCT\left(C_1f_1(t,x)+C_2f_2(t,x)\right)\}(s,w)=C_1\{2DCCCTf_1(t,x)\}(s,w)+C_2\{2DCCCT\,f_2(t,x)\}(s,w)$$

4.3) Scaling Property: If $\{2DCCCT\,f(t,x)\}(s,w)$ is the canonical cosine- cosine transform of $f(t,x)$ then

$$\{2DCCCTf(pt,qx)\}(s,w)=$$

$$\frac{1}{p.q}\exp\left[\frac{i}{2}\left(\frac{d}{b}\right)\left((p^2-1)\left(\frac{s}{p}\right)^2+(q^2-1)\left(\frac{w}{q}\right)^2\right)\right]\{2DCCCT\,f(u,v)\}\left(\frac{s}{p},\frac{w}{q}\right)$$

4.4) Addition property: If $\{2DCCCT\,f(t,x)\}(s,w)$ and $\{2DCCCT\,g(t,x)\}(s,w)$ are canonical cosine-cosine transform of $f(t,x)$ and $g(t,x)$ then

$$\{2DCCCT\left[f(t,x)+g(t,x)\right]\}(s,w)=\{2DCCCT\,f(t,x)\}(s,w)+\{2DCCCT\,g(t,x)\}(s,w)$$

4.5) Shifting property: If $\{2DCCCT\, f(t,x)\}(s,w)$ is canonical cosine-cosine transform of $f(t,x)$ then

$\{2DCCCT\, f(t-p,x-q)\}(s,w)=$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)(p^2+q^2)}\left[\cos\left(\frac{s}{b}p\right)\cos\left(\frac{w}{b}q\right)\left\{2DCCCT\, f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)-i\sin\left(\frac{s}{b}p\right)\cos\left(\frac{w}{b}q\right)\left\{2DCSCT\, f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)\right.$$

$$\left.-i\cos\left(\frac{s}{b}p\right)\sin\left(\frac{w}{b}q\right)\left\{2DCCST\, f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)-\sin\left(\frac{s}{b}p\right)\sin\left(\frac{w}{b}q\right)\left\{2DCSST\, f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)\right]$$

5) Conclusion: - In this paper two-dimensional canonical cosine-cosine is Generalized in the form the distributional sense, we have inversion theorem for this transform is proved some property of generalized 2-D canonical cosine–cosine transform are discussed.

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