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A NOTE ON TRIGONOMETRIC FUNCTIONS AND INTEGRATION

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Abstract: The main purpose of this paper is to use the idea of finding the unlimited integration of the product of powers of the (sinz) and (cosz)-functions, and the product of powers of the (tanz) and (secz)-functions to derive tow trigonometrically identities.

Key words: Complex functions, integration, complex trigonometric functions.

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§1.INTRODUCTION

This paper is inspired and motivation by the work of [1].The theory of functions of complex variable , also called for brevity complex variables or complex analysis , is one of the most beautiful as well as useful branches of mathematics. Although originating in an atmosphere of mystery, suspicion and distrust ,as evidenced by the terms "imaginary" and "complex" present in the literature, it was finally placed on a sound foundation in the 19^{th} century through the efforts of Cauchy, Riemann, Weierstrass, Gauss and other great mathematicians. Today the subject is recognized as an essential part of the mathematical background of engineers, Physicists, mathematicians and other scientists. From the theoretical view –point this is because many mathematical concepts become clarified and unified when examined in the light of complex variable theory. From the applied viewpoint the theory is of tremendous value in the solution of problems of heat flow, potential theory , fluid mechanics, electromagnetic theory, aerodynamics, elasticity and many other field of science and engineering. Many authors doing in complex variables and gave some results about that [see (2),(3),(4)]. A. D. Sinder [5] proved that , if the complex –valued function f(t) is continuous on [a,b] and F′(t)=f(t) for all t in [a,b],then

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Murray R.Spiege [6]proved that, the integration of F(z)G(z) $\int_a^b f(t)dt = F(b) - F(a)$.
is equal to F(z)G(z)- $\int F(z)G(z)\,dz$.

Where F(z) and G(z) are complex variable functions . Ruel V. Churchill , James W. Brown and Roger F. Verhey [7] proved that $\int_\alpha^\beta z\,dz = \frac{1}{2}\left(\beta^2 - \alpha^2\right)$

Whenever the path of integrations is a contour.

From our we know study the integration of the trigonometric function (sinz cosz dz).

The solution of above integration we can find by substituting y=sin z to obtain that.

$$\frac{\sin^2 z}{2}+c_1$$

If we use the substituting y=cos z , we obtain $\frac{\cos^2 z}{2}+c_2$

It is clear from that the difference between these tow answers is constant number this implies to .

$\sin^2 z+\cos^2 z=c$,

where c is constant number and by substituting z=0 in the above equation we get.

$\sin^2 z+\cos^2 z=1$.

In our paper we shall is to use the idea of finding the unlimited integration of the product of powers of the (sin z) and (cos z)-functions, and the product of powers of the (tan z) and (sec z)-functions.

§2.THE MAIN RESULTS:

Theorem 2.1: For all complex values of z , the following

$$\sum_{j=0}^{m}(-1)^j\binom{m}{j}\frac{\sin^{2n+2j+2}z}{n+j+1}+\sum_{j=0}^{n}(-1)^j\binom{n}{j}\frac{\cos^{2m+2j+2}z}{m+j+1}=\frac{n!m!}{(n+m+1)!}$$

Will be true where n,m are positive integers.

Proof: We consider the following integration

$\int \sin^{2n+1} z \cos^{2m+1} z \, dz$

By writing this integration by the following

$\int \sin^{2n+1} z \cos^{2m} z \cos z \, dz$

$= \int \sin^{2n+1} z \, (1-\sin^2 z)^m \cos z \, dz$

By using the substituting y=sin z, we get

$\int y^{2n+1} \left(1-y^2\right)^m dy$

And by using Binomial formula we get

$\int \quad \sum_{j=0}^{m} (-1)^j \binom{m}{j} y^{2n+1} y^{2j} dy$

$= \int \quad \sum_{j=0}^{m} (-1)^j \binom{m}{j} y^{2n+2j+1} dy$

$= \sum_{j=0}^{m} (-1)^j \binom{m}{j} \int y^{2n+2j+1} dy$

$= \sum_{j=0}^{m} (-1)^j \binom{m}{j} \frac{\sin^{2n+2j+2} z}{2n+2j+2} + c_1$

$= \frac{1}{2} \sum_{j=0}^{m} (-1)^j \binom{m}{j} \frac{\sin^{2n+2j+2} z}{n+j+2} + c_1$

By same method we can find $\int \sin^{2n+1} z \cos^{2m+1} z \, dz$

By writing this integration by the following

$\int \quad \cos^{2m+1} z \sin^{2n} z \sin z \, dz$

Using the substituting y=cos z, for obtain

$$\int y^{2n+1}\left(1-y^{2}\right)^{m}dy$$

And by using Binomial formula we get

$$-\int \quad \sum_{j=0}^{n}(-1)^{j}\binom{n}{j}y^{2m+1}\,y^{2j}dy$$

$$=-\int \quad \sum_{j=0}^{n}(-1)^{j}\binom{n}{j}y^{2m+2j+1}dy$$

$$=-\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\int y^{2m+2j+1}dy$$

$$=-\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\frac{\cos^{2m+2j+2}z}{2m+2j+2}+c_2$$

$$=-\frac{1}{2}\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\frac{\cos^{2m+2j+2}z}{m+j+2}+c_2$$

But the difference between tow answers is constant number, this implies to

$$\frac{1}{2}\sum_{j=0}^{m}(-1)^{j}\binom{m}{j}\frac{\sin^{2n+2j+2}z}{n+j+1}+\frac{1}{2}\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\frac{\cos^{2m+2j+2}z}{m+j+1}=c(n,m)$$

And by choose $z=0$,we get

$$C(n,m)=\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\frac{1}{m+j+1}$$

We assume that $x=m+1$,which implies

$$\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}\frac{1}{x+j}=\frac{n!}{x(x+1)\ldots(x+n)}$$

Thus $c(n,m)=\frac{n!}{(m+1)(m+2)\ldots(m+n+1)}=\frac{n!m!}{(m+n+1)!}$

By same method in the proof of Theorem(2.1)we can prove the following theorem

Theorem2.2: For all complex values of z , the following

$$\sum_{j=0}^{m}(-1)^{j}\binom{m}{j}\frac{\tan^{2n+2j+2}z}{n+j+1}+\sum_{j=0}^{n}(-1)^{n+j+1}\binom{n}{j}\frac{\sec^{2m+2j+2}z}{m+j+1}$$

$= (-1)^{n+1} \frac{n!m!}{(n+m+1)!}$, will be true where n, m are positive integers

Proof: We consider the following integration

$$\int \tan^{2n+1} z \sec^{2m+2} z \, dz$$

We take y=tan z and y=sec z, so it easy to complete our prove by same method in Theorem(2.1).

We closed our paper by the following corollaries after depended on some fundamental relations of exponential and trigonometric functions.

Corollary 2.3: For all complex values of z, the following

$$\sum_{j=0}^{m}(-1)^j \binom{m}{j} \frac{(e^{iz}-e^{-iz})^{2n+2j+2}}{(n+j+1)(2i)^{2n+2j+2}} + \sum_{j=0}^{n}(-1)^j \binom{n}{j} \frac{(e^{iz}+e^{-iz})^{2m+2j+2}}{(m+j+1)(2)^{2m+2j+2}} = \frac{n!m!}{(n+m+1)!}$$

Will be true where n, m are positive integers

Corollary 2.4: For all complex values of z, the following

$$\sum_{j=0}^{m}(-1)^j \binom{m}{j} \frac{(e^{iz}-e^{-iz})^{2n+2j+2}}{i(n+j+1)(e^{iz}+e^{-iz})} + \sum_{j=0}^{n}(-1)^{n+j+1} \binom{n}{j} \frac{2^{2m+2j+2}}{(m+j+1)(e^{iz}+e^{-iz})} = (-1)^{n+1} \frac{n!m!}{(n+m+1)!}$$

Will be true where n , m are positive integers

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