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DERIVATIONS ON SEMIPRIME NEAR-RINGS

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Abstract: The main purpose of this paper is to study and investigate some results concerning derivation d on semiprime near-ring N , we obtain d is commuting (resp.centralizing on N) on N.

Key words: Semiprime near-ring,commuting, centralizing, derivation.

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§1.INTRODUCTION

This paper is inspired by the work of A.Boua and L.Oukhtite [17],the study of derivations of near- rings was initiated by H. E. Bell and G. Mason in 1987[1] , but thus for only a few papers on this subject in near- rings have been published (see [2],[3] ,[4] and [5]) . Bell and Kappe [6] proved that , if d is a derivation of a semiprime ring R which is either an endomorphism or anti- endomorphism, then d = o . They also showed that if d is a derivation of a prime ring R which h acts as a homomorphism on U , where U is a non-zero right ideal then d =o on R these results were proved for near-rings in [2] , where if d(xy) = d(x)d(y) or d(xy) = d(y)d(x) for all $x,y\in U$, U be a non-empty subset of N and d be a derivation of N , then d is said to acts as a homomorphism or anti-homomorphism on U .respectively. R is said to be prime if xRy = {o}for $x,y\in R$ implies x = o or y = o , and semiprime if xRx = {o} for $x\in R$ implies x = o. Chung and Luh [7] proved that every semicommuting automorphism of a prime ring is commuting provided that R has either characteristic different from 3 or non- zero center and thus they proved the commutativity of prime rings having nontrivial semicommuting automorphism except in the indicated cases . Kaya and Koc [8] proved that every semicentralizing (hence every semicommuting) automorphism of a prime ring is in fact commuting . Bell and Mason [9] investigated

SCP-mappings and Daif 2-derivation in near –rings , the mapping d is called strong commutativity preserving (SCP) on S if [d(x), d(y)] = [x,y] for all x,y ∈ S , where S is a subset of N . Deng , Serif and Nurean [10] proved ,let N be a prime near-ring , if N admits a Daif 2 –derivation d , then N is a commutative ring . Wang [11] proved , let n be a positive integer , N an n!- torsion free prime near-ring and d a derivation such that $d^n(N) = \{o\}$. Then d(z) = {o} , where Z is the center of N. Hirano , Kaya and Tominaga [12] proved , let U be a non-zero ideal of a prime ring R , d be non-trivial derivation of R ($d \neq o_R$)if d is centralizing (resp - skew-centralizing) on U , then R is commutative , where d an (additive group) endomorphism of R. Recently , Mehsin[13] proved , let N be a semiprime near-ring, if N admits a Daif 2- derivation d , then d is commuting on N.Also,Mehsin[14]proved, let N be a semiprime near-ring , U a non-zero semigroup ideal of N and d a non-zero (1,β)-derivation of N such that d(U)x = {o}for all x∈N and β(N) = N , then d is semicentralizing(resp. semicommuting) on N.In this paper , we shall study when a semiprime near-rings admitting a derivation d to satisfy new conditions we give some results about that.

§2.PRELIMINARIES

Throughout this paper, according to[15]near– ring is a triple (N,+,.) satisfying the condition :

(i) (N,+)is a group which may not be a belian .

(ii) (N,.) is a semigroup .

(iii) For all x,y,x∈N, x(y+z) = xy + xz . In fact, condition (iii).

makes N a left near-ring . If we replace (iii) by(iv) for all x,y,z ∈N, (x + y)z = xz + yz , then we obtain a right near-ring N, and N has no non-zero nilpotent elements with the center Z(N) A non-empty subset U of N will be called a semigroup ideal if UN⊆ U and NU⊆ U, Z(N) is the center of N . An additive map d : N → N is a derivation if d(xy)=xd(y)+d(x)y for all x,y ∈ N or equivalently (cf.[11]) that d(xy)=d(x)y+xd(y) for all x,y∈ N ,and d is called centralizing (resp. commuting) of N if xd(x)–d(x)x∈ Z(N) (resp. xd(x)=d(x)x is satisfied for each x∈N. Also d is called centralizer if d(x) ∈ Z(N) for each x∈N.

According to [1] a near-ring N is said to be semiprime if xNx = {o} for x∈N implies x = o, and is said to be n-torsion free , where n≠o is an integer , if whenever nx = o , with $x \in N$, then $x = o$.We write $[x,y]=xy-yx$ and note that important identities $[x,yz]=y[x,z]+[x,y]z$ and $[xy,z]=x[y,z]+[x,z]y$,also we write $xoy= xy + yx$.

To achieve our purpose , we mentions the following

Lemma2.1[16 :Problem14,Pag 9]: N has no non-zero nilpotent elements iff $a^2=0$ implies $a=0$ for all $a \in N$.

§3.THE MAIN RESULTS:

Theorem3.1: Let N be a semiprime near –ring. If N admits a non-zero derivation d satisfying $d([x,y])=[x,y]$ for all $x,y \in N$.Then d is commuting (resp.centralizing) on N.

Proof: We have $d([x,y])=[x,y]$ for all $x,y \in N$. (1)

Replacing y by xy in (1),we obtain

$d(x[x,y])=x[x,y]$ for all $x,y \in N$.Then

$xd([x,y])+d(x)[x,y]=x[x,y]$ for all $x,y \in N$.

According to (1) above equation become

$d(x)[x,y]=0$ for all $x,y \in N$. (2)

Replacing y by xy with using (2),we get

$d(x)x[x,y]=0$ for all $x,y \in N$. (3)

Left-multiplying (2) by x ,,we get

$xd(x)[x,y]=0$ for all $x,y \in N$. (4)

In (3) replacing y by zy with using (3),we get

$d(x)xz[x,y]=0$ for all $x,y,z \in N$. (5)

Similarly for (4) ,we obtain

$xd(x)z[x,y]=0$ for all $x,y,z \in N$. (6)

Now in(4)and (5) replacing y by d(x) with subtracting the results ,we obtain

$[x,d(x)]z[x,d(x)]=0$ for all $x, z \in N$. By using the semiprimeness of N ,we complete our proof.

Theorem 3.2: Let N be a semiprime near -ring.If N admits a non-zero derivation d satisfying $d([x,y])=-[x,y]$ for all $x,y \in N$.Then d is commuting(resp.centralizing) on N.

Proof: We have $d([x,y])=-[x,y]$ for all $x,y\in N$. (7)

Replacing y by xy in (7),we obtain

$d(x[x,y])=-x[x,y]$ for all $x,y\in N$.It follows that

$xd([x,y])+d(x)[x,y]=-x[x,y]$ for all $x,y \in N$.

According to (7),we get

$d(x)[x,y]=0$ for all $x,y \in N$. The proof is as in the proof of Theorem3.1.

Theorem3.3: Let N be a semiprime near -ring. If N admits a non-zero derivation d satisfying $d(xoy)=(xoy)$ for all $x,y \in N$.Then d is commuting(resp.centralizing) on N.

Proof: From our hypothesis ,we have

$d(xoy)=xy+yx$ for all $x,y\in N$. (8)

Replacing y by xy ,we get

$d(xo(xy))=x^2y+xyx$ for all $x,y \in N$. (9)

Since $xo(xy) =x(xoy)$,the by using the result in (9) ,we get

$d(x(xoy))=x^2y+xyx$ for all $x,y\in N$.

$d(x)(xoy) + xd(xoy)= x^2y+xyx$ for all $x,y\in N$. (10)

According to(8) ,above equation (10) reduces to

$d(x)(xoy) =0$ for all $x,y\in N$.

Replacing y by yz,we get

$-d(x)yzx= d(x)xyz=(-d(x)yx)z=d(x)y(-x)z$ for all $x,y,z \in N$. (11)

Since we have $-d(x)yzx= d(x)y(-x)z$, then above equation gives

$d(x)yz(-x)= d(x)y(-x)z$ for all $x,y,z\in N$.

Taking $-x$ instead of x in above ,we obtain

$d(-x)yzx= d(-x)yxz$ for all $x,y,z\in N$.

$d(-x)y[z,x]= 0$ for all $x,y,z\in N$. (12)

Replacing y by xy in (12) with z by d(x),we obtain

$d(-x)xy[d(x),x]= 0$ for all $x,y,z\in N$. (13)

Left-multiplying (12)by x with replacing z by d(x),we get

$xd(-x)y[d(x),x]= 0$ for all $x,y,z\in N$. (14)

Then by subtracting (13) and (14) with using N is semiprime ,we complete our proof.

By same method in above theorem we can prove the following.

Theorem 3.4: Let N be a 2-torsion free semiprime near -ring. If N admits a non-zero derivation d satisfying $d(xoy)=-(xoy)$ for all $x,y \in N$.Then d is commuting (resp.centralizing)on N.

Theorem 3.5: Let N be a 2-torsion free semiprime near -ring. If N admits a non-zero derivation d satisfying $d([x,y])= xoy$ for all $x,y\in N$.Then d is centralizer on N.

Proof: We have $d([x,y])= xoy$ for all $x,y\in N$.

Replacing y by x,we obtain

$2x^2=0$ for all $x\in N$. Since N is 2-torsion free ,we get

$x^2=0$ for all $x\in N$.

Replacing x by d(x) with using Lemma 2.1,we get

$d(x) =0$ for all $x\in N$. (15)

Then from (15),we obtain

$d(x) \in Z(N)$ for all $x \in N$. Thus we complete our proof.

By same method in above theorem we can prove the following.

Theorem3.6: Let N be a 2-torsion free semiprime near -ring. If N admits a non-zero derivation d satisfying $d(xoy)= [x,y]$ for all $x,y \in N$.Then $d(N^2)$ is centralizer on N.

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