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Almost contra-gpr-continuity

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Abstract: The object of the present paper is to study the basic properties of Almost contra−gpr−continuous functions.
Keywords: gpr-open sets, gpr-continuity, gpr-irresolute map, gpr-homeomorphisms and Almost contra-gpr-continuity.
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1. Introduction:

Norman Levine introduced generalized closed sets, K. Balachandaran and P. Sundaram studied generalized continuous functions and generalized homeomorphism. Y.Ganambal studied generalized preregular closed sets and generalized preregular continuous functions. Following Y.Gnanambal the authors of the present paper define a new variety of continuous function called contra gpr−continuous function, study its basic properties and interrelation with other type of contra continuous functions. Throughout the paper a space X means a topological space (X,τ). For any subset A of X its complement, interior, closure, gpr-interior, gpr-closure are denoted respectively by the symbols A^c, A^o, A^-, $gpr(A)^o$ and $gpr(A)^-$.

2. Preliminaries:

Definition 2.1: $A\subseteq X$ is called
(i) regularly open if A = int(cl(A)).
(ii) g-closed[resp: rg-closed] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open[resp: regular open].
(iii) αg-closed if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open.
(iv) gp-closed[resp: gpr-closed] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open[resp: regular-open].

Note 1: From definition 2.1 we have the following implication among the closed sets.

$$\begin{array}{ccccccccc}
\text{Closed} & \rightarrow & \text{g-closed} & \rightarrow & \alpha\text{g-closed} & \rightarrow & \text{gp-closed} & \rightarrow & \text{gpr-closed} \\
\uparrow & & & & & & & & \uparrow \\
\text{r-closed} & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \text{rg-closed.}
\end{array}$$

None is reversible

Definition 2.2: $A\subseteq X$ is called clopen[resp: nearly-clopen; g-clopen; gpr-clopen] if it is both open[resp: regular-open; g-open; gpr-open] and closed[resp: nearly-closed; g-closed; gpr-closed]

Definition 2.3: A function $f:X\rightarrow Y$ is said to be

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(i) Continuous[resp: nearly continuous, semi-continuous] if inverse image of open set is open[resp: regular-open, semi-open]
(ii) g-continuous[resp: gpr-continuous] if inverse image of closed set is g-closed[resp: gpr-closed]
(iii) irresolute[resp: nearly irresolute, gpr−irresolute] if inverse image of semi-open[resp: regular-open, gpr−open] set is semi-open[resp: regular-open, gpr−open]
(iv) g-irresolute[resp: gpr-irresolute] if inverse image of g-closed[resp: gpr-closed] set is g-closed[resp: gpr-closed]
(v) homeomorphism[resp: nearly homeomorphism] if f is bijective continuous[resp: nearly-continuous] and f^{-1} is continuous[resp: nearly-continuous]
(vi)rc-homeomorphism if f is bijective r-irresolute and f^{-1} is r-irresolute
(vii) g-homeomorphism[resp: gpr-homeomorphism] if f is bijective g-continuous[resp: gpr-continuous] and f^{-1} is g-continuous[resp: gpr-continuous]
(viii)gc-homeomorphism[resp: gprc-homeomorphism] if f is bijective g-irresolute[resp: gpr-irresolute] and f^{-1} is g-irresolute[resp: gpr-irresolute]

Theorem 2.1: If f is gpr-continuous[resp: gpr-irresolute; gpr-homeomorphism] and G is open[resp: gpr-open; gpr-closed] set in Y, then f^{-1}(G) is gpr-open[resp: gpr-open; gpr-closed] in X.

3 Almost contra-gpr-continuous functions:

Definition 3.1: A function $f:X \to Y$ is said to be almost contra-gpr-continuous if the inverse image of every regular open set is gpr-closed.

Note 2: Here after we call almost contra-gpr-continuous function as al.c.gpr.c function shortly.

Example 1: X = Y = {a, b, c}; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$.
(i) If $f : X \to Y$ defined by f(a) = b; f(b) = c and f(c) = a, then f is al.c.gpr.c.
(ii) If $f: X \to Y$ is identity function, then f is not al.c.gpr.c.

Theorem 3.1: f is almost contra-gpr-continuous iff inverse image of each closed set in Y is gpr-open in X.

Theorem 3.2: If f is almost contra-gpr-continuous and A∈RC(X), then the restriction $f_{/A}$ is almost contra-gpr-continuous.

Theorem 3.3: (i) We have the following diagram for the function $f: X \to Y$

$$\begin{tikzcd}
\text{Al.c.c} \arrow[r] & \text{al.c.g.c} \arrow[r] & \text{al.c.}\alpha\text{g.c} \arrow[r] & \text{al.c.gp.c} \arrow[r] & \text{al.c.gpr.c} \\
\text{Al.c.r.c} \arrow[u] \arrow[rrrr] & & & & \text{al.c.rg.c} \arrow[u]
\end{tikzcd}$$

None is reversible

(ii) If GPRC(X) = RC(X). We have the following diagram for the function $f: X \to Y$

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$$\begin{array}{ccccccccccc} \text{Al.c.c} & \leftrightarrow & \leftrightarrow & \text{al.c.g.c} & \leftrightarrow\leftrightarrow & \text{al.c.}\alpha\text{g.c} & \leftrightarrow\leftrightarrow & \text{al.c.gp.c} & \leftrightarrow\leftrightarrow & \text{al.c.gpr.c} \\ \uparrow\downarrow & & & & & & & & & \uparrow\downarrow \\ \text{Al.c.r.c} & \leftrightarrow & \leftrightarrow & \leftrightarrow & \leftrightarrow\leftrightarrow & \leftrightarrow & \leftrightarrow & \leftrightarrow & \leftrightarrow & \text{al.c.rg.c} \end{array}$$

Example 2: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {a, b},X} and σ = {ϕ, {a}, {b}, {a, b}, Y }. Let $f: X \to Y$ be defined as f(a) = b; f(b) = c; f(c) = a, then f is almost contra-gpr-continuous; almost contra-rg-continuous but not almost contra-continuous; almost contra-g-continuous; almost contra-gp-continuous and almost contra-αg-continuous.

Example 3: Let X = Y = {a, b, c, d}; τ = {ϕ, {a}, {b}, {d},{a, d}, {b, d}, {a, b}, {a, b, c},{a, b, d}, X} = σ. Let $f: X \to Y$ be defined as f (a) = b; f (b) = c; f (c) = d; f (d) = a. Then f is almost contra-gpr-continuous but not almost contra rg-continuous.

Theorem 3.4: Let f_i:$X_i \to Y_i$ be almost contra-gpr-continuous for i = 1, 2. Let f: $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f:$X_1\times X_2 \to Y_1\times Y_2$ is almost contra-gpr-continuous.
Proof: Let $U_1\times U_2 \in RO(Y_1\times Y_2)$. Then $f^{-1}(U_1\times U_2) = f^{-1}_1(U_1)\times f^{-1}_2(U_2) \in GPRC(X_1\times X_2)$, since $f^{-1}_i(U_i) \in GPRC(X_i)$ for i = 1, 2, respectively. Now if $U \in RO(Y_1\times Y_2)$, then $f^{-1}(U) = f^{-1}(\cup Ui) = \cup f^{-1}(Ui) \in GPRC(X_1\times X_2)$ where $Ui = Ui_1\times Ui_2$.

Theorem 3.5: Let h:$X \to X_1\times X_2$ be almost contra-gpr-continuous. h(x) = (h_1(x), h_2(x)). Then h_i:X $\to$ Xi is almost contra-gpr-continuous for i = 1, 2.
Proof: Let $U_1 \in RO(X_1)$. Then $U_1\times X_2 \in RO(X_1\times X_2)$ and $h^{-1}_1(U_1) = h^{-1}(U_1\times X_2) \in GPRC(X)$, therefore h_1 is almost contra-gpr-continuous. Similar argument gives h_2 is almost contra-gpr-continuous and thus h_i is almost contra-gpr-continuous for i = 1, 2.

Theorem 3.6: Let Y and {X_i:i$\in$I} be Topological Spaces. If $f: Y \to \Pi X_i$ is almost contra gpr-continuous, then $\pi_i \bullet f$:Y$\to X_i$ is almost contra gpr-continuous but the converse is not true.
Proof: Suppose f is almost contra gpr-continuous. Since π_i:$\Pi X_i \to X_i$ is nearly continuous for each i$\in$I, it follows that $\pi_i \bullet$f is almost contra gpr-continuous.

Theorem 3.7: f :$\Pi X_i \to \Pi Y_i$ is al.c.gpr.c, iff f_i :$X_i \to Y_i$ is al.c.gpr.c for each i$\in$I.

Theorem 3.8: If Y is r$T_{1/2}$ and {X_i:i$\in$I} be Topological Spaces. Let f:Y$\to \Pi X_i$ be a function, then f is almost contra gpr-continuous iff $\pi_i \bullet f$: Y $\to X_i$ is almost contra gpr-continuous.

Corollary 3.2: Let f_i:$X_i \to Y_i$ be a function and let f:$\Pi X_i \to \Pi Y_i$ be defined by $f((x_i)_{i\in I}) = (f_i(x_i))_{i\in I}$. If f is almost contra gpr-continuous then each f_i is almost contra gpr-continuous.

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Corollary 3.3: For each i, let X_i be $rT_{1/2}$ and let $f_i:X_i \to Y_i$ be a function and let $f:\Pi X_i \to \Pi Y_i$ be defined by $f((x_i)_{i\in I}) = (f_i(x_i))_{i\in I}$, then f is almost contra gpr-continuous iff each f_i is almost contra gpr-continuous.

Remark 1: In general,
(i) The algebraic sum and product of two almost contra-gpr-continuous functions is not almost contra-gpr-continuous. However the scalar multiple of almost contra-gpr-continuous function is almost contra-gpr-continuous.
(ii)The pointwise limit of a sequence of almost contra-gpr-continuous functions is not almost contra-gpr-continuous as shown by the following example.

Example 4: Let X = Y = [0, 1]. Let $f_n:X\to Y$ be defined as $f_n(x) = x_n$ for $n \geq 1$then $f:X\to Y$ is the limit of the sequence where $f(x) = 0$ if $0 \leq x < 1$ and $f(x) = 1$ if $x = 1$. Therefore f is not almost contra-gpr-continuous. For $(1/2, 1]$ is open in Y, $f^{-1}((1/2, 1]) = (1)$ is not gpr-closed in X.

However we can prove the following theorem.

Theorem 3.9: Let f_n: $(X, d_X) \to (Y, d_Y)$, be almost contra-gpr-continuous, for n = 1, 2... and let f: $(X, d_X)\to (Y, d_Y)$ be the uniform limit of $\{f_n\}$, then $f:(X, d_X) \to (Y, d_Y)$ is Almost contra-gpr-continuous.

Remark 2: It is clear from Remark 1 that (i) $C_{al.c.gpr.c}(X,R)$, the set of all almost contra-gpr-continuous functions is not a Vector space.

Note 3: With respect to usual topology on the set of real numbers, open sets and regular open sets are one and the same. So we rewrite the converse of the theorem 3.5 is not true in general, as shown by the following example.

Example 5: Let $X = X_1 = X_2 = [0, 1]$.
Let $f_1 : X \to X_1$ be defined as follows: $f_1(x) = 1$ if $0 \leq x \leq 1/2$ and $f_1(x) = 0$ if $1/2 < x \leq 1$.
Let $f_2 : X \to X_2$ be defined as follows: $f_2(x) = 1$ if $0 \leq x < 1/2$ and $f_2(x) = 0$ if $1/2 < x \leq 1$.
Then $f_i : X \to X_i$ is clearly almost contra-gpr-continuous for i = 1, 2., but $h(x) = (f_1(x_1), f_2(x_2)):X\to X_1 \times X_2$ is not almost contra-gpr-continuous, for $S_{1/2}(1, 0)$ is regular open in $X_1 \times X_2$, but $h^{-1}(S_{1/2}(1, 0)) = \{1/2\}$ which is not gpr-closed in X.

Remark 3: In general, the product of almost contra-gpr-continuous function of almost contra-gpr-continuous function is not almost contra-gpr-continuous.

Theorem 3.10: If f is gpr-irresolute and g is al.c.gpr.c.[almost contra-continuous], then $g\bullet f$ is almost contra-gpr-continuous.

Theorem 3.11: If $f:X\to Y$ is gpr-irresolute, gpr-open and GPRO(X) = τ and $g: Y \to Z$ be any function, then $g\bullet f: X \to Z$ is al.c.gpr.c iff $g: Y \to Z$ is al.c.gpr.c.
Proof: If part: Theorem 3.10

Only if part: Let A be r-closed subset of Z. Then $(g\bullet f)^{-1}(A)$ is a gpr-open subset of X and hence open in X[by assumption]. Since f is gpr-open $f(g\bullet f)^{-1}(A) = g^{-1}(A)$ is gpr-open in Y. Thus $g: Y \to Z$ is al.c.gpr.c.

Corollary 3.4: If f is gpr-irresolute, gpr-open and bijective, g is a function. Then g is al.c.gpr.c. iff $g\bullet f$ is al.c.gpr.c.

Theorem 3.12: If $g: X \to X\times Y$, defined by $g(x) = (x, f(x))$ $\forall$ $x \in X$ be the graph function of $f: X \to Y$. Then g is al.c.gpr.c iff f is al.c.gpr.c.

Proof: Let $V\in RO(Y)$, then $X\times V$ is open in $X\times Y$. Since g is al.c.gpr.c., $f^{-1}(V) = f^{-1}(X\times V) \in GPRC(X)$. Thus f is al.c.gpr.c.

Conversely, let $x\in X$ and f be a r-closed subset of $(X\times Y, g(x))$. Then $F\cap(\{x\}\times Y)$ is closed in $(\{x\}\times Y, g(x))$. Also $\{x\}\times Y$ is homeomorphic to Y. Hence $\{y\in Y:(x, y)\in F\}$ is r-closed subset of Y. Since f is al.c.gpr.c. $\cup\{f^{-1}(y):(x, y)\in F\}$ is gpr-open in X. Further $x\in\cup\{f^{-1}(y):(x, y)\in F\}\subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is gpr-open. Thus $g: X \to Y$ is al.c.gpr.c.

Problem:

(i) Are $\sup\{f, g\}$ and $\inf\{f, g\}$ are almost contra-gpr-continuous if f, g are almost contra-gpr-continuous

(ii)Is $C_{al.c.gpr.c}(X,R)$ a Lattice?

(iii)Suppose $f_i: X \to X_i (i = 1, 2)$ are almost contra-gpr-continuous. If $f: X \to X_1 \times X_2$ defined by $f(x) = (f_1(x), f_2(x))$, then f is almost contra-gpr-continuous .

Solution: No. Since union and intersection of two gpr-closed sets are not gpr-closed.

Theorem 3.13: Let $f: X \to Y$ and $g: Y \to Z$ be such that

(i) If f is contra-gpr-continuous and g is continuous[resp: nearly-continuous] then $g\bullet f$ is almost contra-gpr-continuous

(ii) If f is almost contra-gp-continuous[resp: almost contra-g-continuous; almost contra-rg-continuous; almost contra-αg-continuous] and g is r-irresolute, then $g\bullet f$ is gpr-continuous.

Remark 4: In general, composition of two almost contra-gpr-continuous functions is not almost contra-gpr-continuous. However we have the following example:

Example 6: Let X = Y = Z = {a, b, c} and $\tau = \wp(X)$; $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$, and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Let f and g be identity maps which are almost contra-gpr-continuous but $g\bullet f$ is almost contra-gpr-continuous.

Theorem 3.14: Let X, Y, Z be topological spaces and every gpr-closed set be open[resp: r-open] in Y, then the composition of two almost contra-gpr-continuous maps is almost contra-gpr-continuous.

Proof: Let V be r-open in Z $\Rightarrow$ $g^{-1}(V)$ is gpr-closed in Y $\Rightarrow$ $g^{-1}(V)$ is open in Y[by assumption] $\Rightarrow f^{-1}(g^{-1}(V)) = (g\bullet f)^{-1}(A)$ is gpr-closed in X $\Rightarrow$ $g\bullet f$ is almost contra-gpr-continuous.

Theorem 3.15: (i) If f is almost contra-gpr-continuous; g is g-continuous and Y is $T_{1/2}$ space, then $g\bullet f$ is almost contra-gpr-continuous.
(ii) If f is almost contra r-continuous[resp: almost contra-gpr-continuous] g is rg-continuous and Y is $rT_{1/2}$ space, then $g\bullet f$ is almost contra-gpr-continuous.
Proof: (i) Let V be r-open in Z $\Rightarrow g^{-1}(V)$ is g-open in Y $\Rightarrow g^{-1}(V)$ is open in Y[since Y is $T_{1/2}$] $\Rightarrow f^{-1}(g^{-1}(V)) = (g\bullet f)^{-1}(V)$ is gpr-closed in X $\Rightarrow g\bullet f$ is almost contra-gpr-continuous.
(ii) Let V be r-open in Z $\Rightarrow g^{-1}(V)$ is rg-open in Y $\Rightarrow g^{-1}(V)$ is r-open in Y[since Y is $r-T_{1/2}$] $\Rightarrow g^{-1}(V)$ is open in Y[since every r-open set is open] $\Rightarrow f^{-1}(g^{-1}(V))$ is r-closed in Y $\Rightarrow (g\bullet f)^{-1}(V)$ is gpr-closed in X $\Rightarrow g\bullet f$ is almost contra-gpr-continuous.
(iii) Let V be r-open in Z $\Rightarrow g^{-1}(V)$ is rg-open in Y $\Rightarrow g^{-1}(V)$ is r-open in Y[since Y is $r - T_{1/2}$] $\Rightarrow g^{-1}(V)$ is open in Y[since every r-open set is open] $\Rightarrow f^{-1}(g^{-1}(V))$ is r-closed in Y $\Rightarrow (g\bullet f)^{-1}(V)$ is gpr-closed in X $\Rightarrow g\bullet f$ is almost contra-gpr-continuous.

Note 4: Pasting Lemma is not true with respect to almost contra-gpr-continuous functions. However we have the following weaker versions.

Theorem 3.16: Pasting Lemma Let X and Y be spaces such that $X = A\cup B$. Let $f_{/A}$ and $g_{/B}$ are almost contra-gpr-continuous[resp: almost contra r-continuous] such that $f(x) = g(x)$ $\forall x\in A\cap B$. If A; $B\in RC(X)$ and GPRC(X)[resp: RC(X)] is closed under finite unions, then the combination α:X$\rightarrow$Y is almost contra-gpr-continuous.
Proof: Let f be r-open set in Y, then $\alpha^{-1}(F) = f^{-1}(F)\cup g^{-1}(F)$ where $f^{-1}(F)$ is gpr-closed in A and $g^{-1}(F)$ is gpr-closed in B $\Rightarrow f^{-1}(F)$ and $g^{-1}(F)$ are gpr-closed in X $\Rightarrow f^{-1}(F)\cup g^{-1}(F)$ is gpr-closed in X[since GPRC(X) is closed under finite unions] $\Rightarrow \alpha^{-1}(F)$ is gpr-closed in X. Hence α is almost contra-gpr-continuous.

4. Covering and Separation properties:

Theorem 4.1: If f is al.c.gpr.c.[resp: al.c.r.c] surjection and X is gpr-compact, then Y is nearly closed compact.
Proof: Let $\{G_i : i\in I\}$ be any r-closed cover for Y. Then for G_i is r-closed in Y and $f: X \rightarrow Y$ is al.c.gpr.c., $f^{-1}(G_i)$ is gpr-open in X. Thus $\{f^{-1}(G_i)\}$ forms a gpr-open cover for X and hence have a finite subcover, since X is gpr-compact. Since f is surjection, $Y = f(X) = \cup^{n}_{i=1} G_i$. Therefore Y is nearly closed compact.

Theorem 4.2: If f is al.c.gpr.c., surjection and X is gpr-compact[gpr-lindeloff] then Y is mildly closed compact[mildly closed lindeloff].

Corollary 4.1: (i) If f is al.c.gpr.c. surjection and X is locally gpr-compact[gpr-lindeloff], then Y is locally nearly closed compact[nearly closed Lindeloff].
(ii) If f is al.c.gpr.c. surjection and X is locally gpr-lindeloff, then Y is locally nearly closed Lindeloff.
(iii) If f is al.c.gpr.c.[al.c.r.c.], surjection and X is locally gpr-compact then Y is locally mildly compact.

(iv) If f is al.c.gpr.c.[al.c.r.c.], surjection and X is gpr-lindeloff[locally gpr-lindeloff] then Y is locally mildly lindeloff.

Theorem 4.3: If f is al.c.gpr.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
Proof: Let $\{V_i : V_i \in CO(Y): i \in I\}$ be a cover of Y, then $\{f^{-1}(V_i): i \in I\}$ is gpr-open cover of X[by Thm 4.1] and so there is finite subset I_0 of I, such that $\{f^{-1}(V_i): i \in I_0\}$ covers X. Therefore $\{V_i: i \in I_0\}$ covers Y since f is surjection. Hence Y is mildly compact.

Theorem 4.4: If f is al.c.gpr.c. surjection and X is gpr-connected, then Y is connected.
Proof: If $Y = A \cup B$ where A and B are disjoint clopen sets in Y. Since f is al.c.gpr.c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint gpr-open sets in X, which is a contradiction. Hence Y is connected.

Corollary 4.2: (i) The inverse image of a disconnected space under a al.c.gpr.c., surjection is gpr-disconnected.
(ii) The inverse image of a gpr-disconnected space under a al.c.gpr.c., surjection is gpr-disconnected.

Theorem 4.5: If f is al.c.gpr.c., injection and Y is rT_i, then X is $gprT_i$, $i = 0, 1, 2$.
Proof: Let $x_1 \neq x_2 \in X$. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is rT_2 $\exists$ V_1, $V_2 \in RO(Y)$ s.t. $f(x_1) \in V_1$; $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$ implies $x_1 \in f^{-1}(V_1) \in GPRO(X)$; $x_2 \in f^{-1}(V_2) \in GPRO(X)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is $gprT_2$.

Remaining parts are omitted as they are consequence of above part.

Theorem 4.6: If f is al.c.gpr.c., injection and closed and Y is rT_i, then X is $gprT_i$; $i = 3, 4$.
Proof:(i) Let $x \in X$ and f be disjoint r-closed subset of X not containing x, then $f(x)$ and $f(F)$ are disjoint closed subset of Y, since f is closed and injection. Since Y is rT_3, $f(x)$ and $f(F)$ are separated by disjoint r-open sets U and V respectively. Hence $x \in f^{-1}(U)$; $f \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in GPRO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is gpr-regular.
(ii) For F_1, F_2 are disjoint r-closed sets in X, $f(F_1)$ and $f(F_2)$ are disjoint r-closed subsets of Y, since f is closed and injection and Y is rT_4, $f(F_1)$ and $f(F_2)$ are separated by disjoint r-open sets V_1 and V_2 respectively. Hence $F_i \subseteq f^{-1}(V_i)$, $i = 1,2$; $f^{-1}(V_i) \in GPRO(X)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is gpr-normal.

Theorem 4.7: If f is al.c.gpr.c., injection and
(i) Y is rC_i[resp : rD_i] then X is $gprC_i$[resp: $gprD_i$] $i = 0, 1, 2$.
(ii)Y is rR_i, then X is $gprR_i$ $i = 0, 1$.

Theorem 4.8: If f is al.c.gpr.c.[resp: al.c.r.c] and Y is rT_2, then the graph $G(f)$ of f is gpr-closed in $X \times Y$.

Proof: Let $(x, y) \notin G(f) \Rightarrow y \neq f(x) \Rightarrow \exists$ disjoint open sets V and W such that $f(x) \in V$ and $y \in W$. Since f is al.c.gpr.c., $\exists$ $U \in GPRO(X)$ such that $x \in U$ and $f(U) \subseteq W$. Thus $(x, y) \in U \times V \subseteq X \times Y - G(f)$. Hence $G(f)$ is gpr-closed in $X \times Y$.

Theorem 4.9: (i) If f is al.c.gpr.c.[al.c.r.c] and Y is rT_2, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is gpr-closed in $X \times X$.
(ii) If f is al.c.r.c.[al.c.c]; g is al.c.gpr.c; and Y is rT_2, then $E = \{x \in X : f(x) = g(x)\}$ is gpr-closed in X.
Proof: (i)Let $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$. If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$, then $\exists$ disjoint open sets V_1 and V_2 such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$, then by almost contra-gpr-continuity of f, $f^{-1}(V_j) \in GPRO(X, x_j)$ for each j. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \subseteq X \times X - A$ $\Rightarrow X \times X - A$ is gpr-open. Hence A is gpr-closed.

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