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gpr-closed and gpr-open mappings

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Abstract: In this paper we discuss new type of closed and open mappings called *gpr*-closed and *gpr*-open mappings; its properties and interrelation with other such functions are studied.

Keywords: closed mapping; semi-closed mapping; pre-closed mapping; β-closed mapping; γ-closed mapping, ν-closed mapping, open mapping; semi-open mapping; pre-open mapping; β-open mapping; γ-open mapping and ν-open mapping

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1 INTRODUCTION

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Norman Levine introduced the notion of generalized closed sets. After him different mathematicians worked and studied on different versions of generalized closed sets and related topological properties. In this paper we are going to further study weak form of closed and open mappings namely *gpr*-closed and *gpr*-open mappings using *gpr*-closed and *gpr*-open sets. Basic properties are verified. Throughout the paper X, Y means a topological spaces (X, τ) and (Y, σ) unless otherwise mentioned without any separation axioms.

2 PRELIMINARIES

Definition 2.01: $A\subset X$ is called
(i) closed[semi-closed; regular-closed] if its complement is open[semi-open; regular-open].
(ii) g-closed[rg-closed] if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(iii) pg-closed[gp-closed; gpr-closed] if pcl(A) $\subseteq U$ whenever $A\subseteq U$ and U is pre-open[open; regular-open] in X.
(iv) αg-closed if αcl(A) $\subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.02: A function *f*:X→ Y is said to be
(i) continuous[resp**:** nearly-continuous; pre-continuous; g-continuous; rg-continuous] if inverse image of each open set is open[resp**:** regular-open; preopen; g-open; rg-open].
(ii) nearly-irresolute; [resp**:** g-rresolute; rg-irresolute] if inverse image of each regular-open set[resp**:** g-open; rg-open] is regular-open; [resp**:** g-open; rg-open].
(iii) closed if image of each closed set is closed[resp**:** regular-closed; g-closed; rg-closed].
(iv) open if image of each open set is open[resp**:** regular-open; g-open; rg-open].

Definition 2.03: X is said to be $T_{1/2}[rT_{1/2}]$ if every g-[rg-]closed set is [regular-]closed.

Note 1: From Definition 2.1 we have the interrelations among closed and open sets.

Closed → → g-closed → → αg-closed → → gp-closed → → gpr-closed
↑ ↑
r-closed → → → → → → → → → → → → → rg-closed

Open → → g-open → → αg-open → → gp-open → → gpr-open
↑ ↑
r-open → → → → → → → → → → → → → → rg-open

3 GPR-CLOSED MAPPINGS

Definition 3.01: A function *f*: X→ Y is said to be *gpr*-closed if image of every closed set in X is *gpr*-closed in Y

Note 2: By note 1 and Definitions 2.02(iii) and 3.01 we have the following diagram.

Closed map → g-closed map → αg-closed map → gp-closed map → *gpr*-closed map
↑ ↑
r-irresolute map → → → → → → → → → → → rg-closed map

However, we have the following converse part**:**

Note 3: If GPRC(Y) = RC(Y) we have the following implication diagram.

Closed map ↔ g-closed map ↔ αg-closed map ↔ gp-closed map ↔ *gpr*-closed map
↑↓ ↑↓
r-irresolute map ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔rg-closed map

Example 1: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X}. and σ = {ϕ, {a}, Y} and let f: X→ Y be defined as f(a) = b; f(b) = c; f(c) = a, then f is ***gpr*-closed**; rg-closed but not g-closed; gp-closed; r-closed; closed; αg-closed.

Theorem 3.01: If (Y, σ) is a discrete space, then f is closed of all types:

Example 2: Let X = Y = {a, b, c} and τ = {ϕ, {b}, {a, b}, {b, c}, X}; σ = ℘(Y) and let f:X→ Y be defined as f(a) = b; f(b) = a; f(c) = c, then f is ***gpr*-closed**; rg-closed; g-closed; gp-closed; αg-closed; r-closed; closed.

Example 3: Let X = Y = {a, b, c} and τ = ℘(X); σ = {ϕ, {b}, {a, b}, {b, c}, Y} and let f:X→ Y be defined as f(a) = c; f(b) = a; f(c) = b, then f is ***gpr*-closed**; rg-closed; but not g-closed; r-closed; closed.

Theorem 3.02: (i) If f is closed and g is *gpr*-closed[rg-closed; r-irresolute] then g•f is *gpr*-closed
(ii) If f and g are r-irresolute then g•f is *gpr*-closed
(iii)If f is r-irresolute and g is *gpr*-closed[r-irresolute] then g•f is *gpr*-closed

Theorem 3.03: If f: X→ Y is *gpr*-closed, then *gpr(*cl{f(A)}) ⊂ f(cl{A})
Proof: Let A⊂ X and f:X→Y is *gpr*-closed gives f(cl{A}) is *gpr*-closed in Y and f(A) ⊂f(cl{A}) which in turn gives *gpr(*cl{f(A)}) ⊂ *gpr*cl(f(cl{A})) - - - - - - - - - - - (1)
Since f(cl{A}) is *gpr*-closed in Y, *gpr*cl(f(cl{A})) = f(cl{A})- - - - - - - - - - - - (2)
combaining (1) and (2) we have *gpr(*cl{f(A)}) ⊂ (f(cl{A})) for every subset A of X.

Remark 1: converse is not true in general, as shown by

Example 4: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X}, and σ = {ϕ, {a}, {c}, {a, c}, Y}. f:(X, τ)→(Y, σ) be the identity map then *gpr(*cl{f(A)}) ⊂ f(cl{A}) for every subset A of X but f is not *gpr*-closed. Since f({c}) = {c} is not *gpr*-closed.

Theorem 3.04: If f is *gpr*-closed[rg-closed] and A∈RC(X), then f(A) is τ_{gpr}-closed in Y.
Proof: Let A⊂X and f:X→Y is *gpr*-closed ⇒ *gpr(*cl{f(A)})⊂f(cl{A}) which in turn implies *gpr(*cl{f(A)})⊂ f(A), since f(A) = f(cl{A}). But f(A)⊂*gpr(*cl{f(A)}). Combining we get f(A) = *gpr(*cl{f(A)}). Hence f(A) is τ_{gpr}-closed in Y.

Corollary 3.01: (i) If f: X→ Y is rg-closed, then *gpr(*cl{f(A)}) ⊂ f(cl{A})
(ii) If f: X→ Y is r-closed, then *gpr(*cl{f(A)}) ⊂ f(cl{A})
(iii)If f: X→Y is r-closed, then f(A) is τ_{gpr}-closed in Y if A is closed[r-closed] set in X.

Theorem 3.05: If *gprcl*(A) = *rgcl*(A) for every A⊂Y, the following are equivalent:
(i) f:X→ Y is *gpr*-closed map
(ii)*gpr(*cl{f(A)}) ⊂ f(cl{A})
Proof: (i) ⇒ (ii) follows from Theorem 3.3

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(ii) $\Rightarrow$ (i) Let A be closed in X, then $f(A) = f(cl\{A\}) \supset gpr(cl\{f(A)\})$ by hypothesis. We have $f(A) \subset gpr(cl\{f(A)\})$. Combining we get $f(A) = gpr(cl\{f(A)\}) = rg(cl\{f(A)\})$[by given condition] $\Rightarrow f(A)$ is rg-closed and hence *gpr*-closed. Thus *f* is *gpr*-closed.

Theorem 3.06: *f*: X$\rightarrow$ Y is *gpr*-closed iff for each subset S of Y and each open set U containing $f^{-1}(S)$, there is a *gpr*-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.
Proof: If *f* is *gpr*-closed, $S \subset Y$ and $U \in (\tau, f^{-1}(S))$, then $f(X\text{-}U) \in GPRC(Y)$ and $V = Y\text{-}f(X\text{-}U) \in GPRO(Y)$. $f^{-1}(S) \subset U \Rightarrow S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X\text{-}U)) \subset X\text{-}(X\text{-}U) = U$.

Conversely let F be closed in X, then $f^{-1}(f(F^c)) \subset F^c$. By hypothesis, there exists $V \in GPRO(Y)$ such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f[(f^{-1}(V))^c] \subset V^c \Rightarrow f(F) \subset V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is *gpr*-closed in Y and therefore *f* is *gpr*-closed.

Remark 2: composition of two *gpr*-closed maps is not *gpr*-closed.

Theorem 3.07: Let X, Y, Z be topological spaces and every *gpr*-closed set is r-closed in Y, then the composition of two *gpr*-closed maps is *gpr*-closed.
Proof: Let A be closed in X $\Rightarrow f(A)$ is *gpr*-closed in Y $\Rightarrow f(A)$ is regular-closed in Y[by assumption] $\Rightarrow g(f(A)) = g \bullet f(A)$ is *gpr*-closed in Z $\Rightarrow g \bullet f$ is *gpr*-closed.

Theorem 3.08: If *f* is rg-closed; *g* is *gpr*-closed[rg-closed] and Y is $rT_{1/2}$, then $g \bullet f$ is *gpr*-closed.

Corollary 3.02: If *f* is rg-closed; *g* is r-closed and Y is r-$T_{1/2}$, then $g \bullet f$ is *gpr*-closed.

Theorem 3.09: If *f; g* be such that $g \bullet f$ is *gpr*-closed[rg-closed]. Then the following are true
(i) If *f* is continuous[r-irresolute] and surjective, then *g* is *gpr*-closed
(ii) If *f* is rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is *gpr*-closed

Corollary 3.03: For *f;g* if $g \bullet f$ is r-irresolute. Then the following are true
(i) If *f* is continuous[r-irresoloute] and surjective, then *g* is *gpr*-closed
(ii) If *f* is rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is *gpr*-closed

Theorem 3.10: If X is *gpr*-regular, *f*:X$\rightarrow$Y is r-open, rg-continuous, *gpr*-closed surjection and $cl\{A\} = A$ for every *gpr*-closed set in Y, then Y is *gpr*-regular.
Proof: Let $p \in U \in GPRO(Y)$, $\exists$ $x \in X \ni f(x) = p$. Since X is *gpr*-regular and *f* is rg-continuous $\exists$ $V \in RGO(X) \ni x \in V \subset cl(V) \subset f^{-1}(U) \Rightarrow p \in f(V) \subset f(cl(V)) \subset U$- - - - - - - (1)
Since *f* is *gpr*-closed, $f(cl(V)) \subset U$ is *gpr*-closed and $cl(f(cl(V))) = f(cl(V))$ and $cl\{f(cl(V))\} = cl\{f(V)\}$ - (2)
From (1) and (2) $p \in f(V) \subset cl(f(V)) \subset U$ and $f(V)$ is rg-open. Hence Y is *gpr*-regular.

Corollary 3.04: If X is *gpr*-regular, *f*:X$\rightarrow$Y is r-open, rg-continuous, *gpr*-closed surjection and $cl(A) = A$ for every rg-closed set in Y, then Y is *gpr*-regular.

Theorem 3.11: (i) If f is *gpr*-closed[rg-closed] and $A \in RC(X)$, then f_A is *gpr*-closed.
(ii) If f is *gpr*-closed[rg-closed], X is $rT_{1/2}$ and $A \in RGC(X)$, then f_A is *gpr*-closed.

Corollary 3.05: (i) If f is r-closed and A is r-closed set of X, then f_A is *gpr*-closed.
(ii)If f is r-closed, X is $rT_{1/2}$ and A is rg-closed set of X, then f_A is *gpr*-closed.

Theorem 3.12: If $f_i: X_i \to Y_i$ be *gpr*-closed [rg-closed] for i = 1, 2. If $f: X_1 \times X_2 \to Y_1 \times Y_2$ is defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1 \times X_2 \to Y_1 \times Y_2$ is *gpr*-closed.
Proof: Let $U_1 \times U_2 \subset X_1 \times X_2$ where $U_i \in RC(X_i)$ for i = 1, 2. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2) \in GPRC(Y_1 \times Y_2)$. Hence f is *gpr*-closed.

Theorem 3.13: Let $h: X \to X_1 \times X_2$ be *gpr*-closed[rg-closed]. Let $f_i: X \to X_i$ be defined as: $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \to X_i$ is *gpr*-closed for i = 1, 2.
Proof: Let $U_1 \times X_2 \in RC(X_1 \times X_2)$, then $f_1(U_1) = h(U_1 \times X_2) \in GPRC(X)$, therefore f_1 is *gpr*-closed. Similarly we have f_2 is *gpr*-closed and thus f_i is *gpr*-closed for i = 1, 2.

Corollary 3.06: (i) If $f_i: X_i \to Y_i$ be r-closed for i = 1, 2. Let $f: X_1 \times X_2 \to Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1 \times X_2 \to Y_1 \times Y_2$ is *gpr*-closed.
(ii) Let $h: X \to X_1 \times X_2$ be r-closed. Let $f_i: X \to X_i$ be defined as: $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \to X_i$ is *gpr*-closed for i = 1, 2.

4 GPR-OPEN MAPPINGS

Definition 4.01: A function $f: X \to Y$ is said to be *gpr*-open if image of every open set in X is *gpr*-open in Y

Note 4: By note 1 and Definitions 2.02(iv) and 4.01 we have the following diagram.

Open map → g-open map → αg-open map → gp-open map → *gpr*-open map
↑ ↑
r-open map → → → → → → → → → rg-open map

However, we have the following converse part:

Note 5: Converse is true if GPRO(Y) = RO(Y)

Open map ↔ g- open map ↔ αg- open map ↔ gp-open map ↔ *gpr*-open map
↑↓ ↑↓
r-irresolute map ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ rg-open map

Example 5: Let X = Y = {a, b, c}; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$ and let $f: X \to Y$ be defined as $f(a) = b$; $f(b) = c$; $f(c) = a$, then f is ***gpr*-open**; rg-open but not g- open; gp-open; r-open; open; αg-open.

Theorem 4.01: If (Y, σ) is a discrete space, then *f* is open of all types**:**

Example 6: Let X = Y = {a, b, c} and τ = {ϕ, {b}, {a, b}, {b, c}, X}; σ = $\wp$(Y) and let *f*:X$\rightarrow$Y be defined as *f*(a) = b; *f*(b) = a; *f*(c) = c, then *f* is ***gpr*-open**; rg-open; g-open; gp-open; αg-open; r-open; open.

Example 7: Let X = Y = {a, b, c} and τ = $\wp$(X); σ = {ϕ, {b}, {a, b}, {b, c}, Y} and let *f*:X$\rightarrow$Y be defined as *f*(a) = c; *f*(b) = a; *f*(c) = b, then *f* is ***gpr*-open**; rg-open; but not g-open; r-open; open.

Theorem 4.02: (i) If *f* is open and *g* is *gpr*-open[rg-open; r-open] then *g•f* is *gpr*-open
(ii) If *f* and *g* are r-irresolute then *g•f* is *gpr*-open
(iii)If *f* is r-irresolute and *g* is *gpr*-open then *g•f* is *gpr*-open

Theorem 4.03: If *f***:** X$\rightarrow$ Y is *gpr*-open, then $f(A^o) \subset gpr(\{f(A)\}^o)$
Proof: Let A$\subset$ X and *f*:X$\rightarrow$ Y is *gpr*-open gives $f(A^o)$ is *gpr*-open in Y and $f(A^o)$ $\subset f(A)$ which in turn gives $gpr(f(\{A\}^o))^o \subset gpr(\{f(A)\}^o$ - (1)
Since $f(\{A\}^o)$ is *gpr*-open in Y, $gpr(f(\{A\}^o))^o = f(\{A\}^o)$- - - - - - - - - - - - - - - - - - - (2)
combaining (1) and (2) we have $f(A^o) \subset gpr(\{f(A)\}^o)$ for every subset A of X.

Remark 3: converse is not true in general, as shown by
Example 8: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X}, and σ = {ϕ, {a}, {c}, {a, c}, Y}. *f***:**(X, τ) $\rightarrow$ (Y, σ) be the identity map then $f(A^o) \subset gpr(\{f(A)\}^o)$ for every subset A of X but *f* is not *gpr*-open. Since *f*({a, b}) = {a, b} is not *gpr*-open.

Theorem 4.04: If *f* is *gpr*-open[rg-open] and A$\in$RO(X), then *f*(A) is τ_{gpr}-open in Y.
Proof: Let A$\subset$ X and *f*:X$\rightarrow$Y is *gpr*-open $\Rightarrow$ $gpr(\{f(A)\}^o) \subset f(\{A\}^o)$ which in turn implies $gpr(\{f(A)\}^o) \subset f(A)$, since $f(A) = f(\{A\}^o)$. But $f(A) \subset gpr(\{f(A)\}^o)$. Combining we get $f(A) = gpr(\{f(A)\}^o)$. Hence *f*(A) is τ_{gpr}-open in Y.

Corollary 4.01: (i) If *f***:** X$\rightarrow$ Y is rg-open, then $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$
(ii) If *f***:** X$\rightarrow$ Y is r-open, then $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$
(iii)If *f***:** X$\rightarrow$ Y is r-open, then *f*(A) is τ_{gpr}-open in Y if A is open[r-open] set in X.

Theorem 4.05: If $gpr(\{A\}^o) = rg(\{A\}^o)$ for every A$\subset$Y, the following are equivalent**:**
(i) *f***:**X$\rightarrow$ Y is *gpr*-open map
(ii) $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$

Theorem 4.06: *f***:** X$\rightarrow$ Y is *gpr*-open iff for each subset S of Y and each open set U containing $f^{-1}(S)$, there is a *gpr*-open set V of Y such that S$\subset$V and $f^{-1}(V) \subset$ U.
Proof: Assume *f* is *gpr*-open, S$\subset$Y and U$\in$(τ, f^{-1}(S)), then *f*(U)$\in$GPRO(Y) and V =

Y-f(X-U)∈GPRO(Y). f^{-1}(S)⊂U ⇒ S⊂V and f^{-1}(V) = X–f^{-1}(f(X-U))⊂X-(X-U) = U.

Conversely let F∈τ, then f^{-1}(f(F^c))⊂F^c and there exists V∈GPRO(Y) such that f(F^c)⊂V and f^{-1}(V)⊂F^c and so F⊂(f^{-1}(V))^c. Hence V^c⊂f(F) ⊂f[(f^{-1}(V))^c]⊂V^c ⇒ f(F)⊂V^c ⇒ f(F) = V^c. Thus f(F)∈GPRO(Y) and therefore f is *gpr*-open.

Remark 4: composition of two *gpr*-open maps is not *gpr*-open.

Theorem 4.07: Let X, Y, Z be topological spaces and every *gpr*-open set is r-open in Y, then the composition of two *gpr*-open maps is *gpr*-open.
Proof: Let A be r-open in X ⇒ f(A) is *gpr*-open in Y ⇒ f(A) is r-open in Y[by assumption] ⇒ g(f(A)) is *gpr*-open in Z ⇒ g•f(A) is *gpr*-open in Z ⇒ g•f is *gpr*-open.

Theorem 4.08: If f is rg-open; g is *gpr*-open[rg-open] and Y is $rT_{1/2}$, then g•f is *gpr*-open.
Proof:(i) Let A be r-open in X ⇒ f(A) is rg-open in Y ⇒ f(A) is r-open in Y[since Y is r-$T_{1/2}$] ⇒ g(f(A)) is *gpr*-open in Z ⇒ g•f(A) is *gpr*-open in Z ⇒ g•f is *gpr*-open.

(ii) Since every g-open set is rg-open, this part follows from the above case.

Corollary 4.02: If f is g-open[rg-open]; g is r-open and Y is $T_{1/2}${$rT_{1/2}$}, then g•f is *gpr*-open.

Theorem 4.09: If f and g be two mappings such that g•f is *gpr*-open[rg-open]. Then the following are true
(i) If f is continuous[r-irresolute] and surjective, then g is *gpr*-open
(ii) If f is rg-continuous, surjective and X is r-$T_{1/2}$, then g is *gpr*-open

Corollary 4.03: If f; g be such that g•f is r-open. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is *gpr*-open
(ii) If f is rg-continuous, surjective and X is r-$T_{1/2}$, then g is *gpr*-open

Theorem 4.10: If X is *gpr*-regular, f:X→ Y is r-open, rg-continuous, *gpr*-open surjection and A^o = A for every *gpr*-open set in Y, then Y is *gpr*-regular.
Proof: Let p∈U∈*GPRO*(Y), ∃ x∈X ∋ f(x) = p. Since X is *gpr*-regular and f is rg-continuous ∃ V∈RGO(X) ∋ x∈V^o⊂V⊂f^{-1}(U) which implies p∈f(V^o)⊂f(V)⊂U- - - (1)
Since f is *gpr*-open, f((V)^o)⊂U is *gpr*-open and {f(V^o)}^{o} = f(V^o) and {f(V^o)}^{o} = {f(V)}^{o}- - - - - - - - -- - - - - - - - - - -- - - - - - (2)
From (1) and (2) p∈{f(V)}^{o} ⊂f(V) ⊂ U and f(V) is rg-open. Hence Y is *gpr*-regular.

Corollary 4.04: If X is *gpr*-regular, f:X→Y is r-open, rg-continuous, *gpr*-open surjection and A^o = A for every rg-open set in Y, then Y is *gpr*-regular.

Theorem 4.11: (i) If f is *gpr*-open[rg-open] and A∈RO(X), then f_A is *gpr*-open.
(ii) If f is *gpr*-open[rg-open], X is $rT_{1/2}$ and A∈RGO(X), then f_A is *gpr*-open.

Corollary 4.05: (i) If *f* is r-open and A is r-open set of X, then f_A is *gpr*-open.
(ii)If *f* is r-open, X is $rT_{1/2}$ and A is rg-open set of X, then f_A is *gpr*-open.

Theorem 4.12: If f_i:$X_i \rightarrow Y_i$ be *gpr*-open[rg-open] for i = 1, 2. If *f*:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f*:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is *gpr*-open.

Theorem 4.13: Let *h*: $X \rightarrow X_1 \times X_2$ be *gpr*-open[rg-open]. Let f_i: $X \rightarrow X_i$ be defined as: $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \rightarrow X_i$ is *gpr*-open for i = 1, 2.

Corollary 4.06: (i) If f_i: $X_i \rightarrow Y_i$ be r-open for i = 1, 2. Let *f*: $X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f*: $X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is *gpr*-open.
(ii) Let *h*: $X \rightarrow X_1 \times X_2$ be r-open. Let f_i: $X \rightarrow X_i$ be defined as: $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \rightarrow X_i$ is *gpr*-open for i = 1, 2.

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