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New compacton-like and solitary patterns-like solutions to the L-DBM equation

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[Abstract] Using a new transformation, the compacton-like and solitary patterns-like solutions of the L-DBM equation are obtained.
Keywords: Dodd-Bullough-Mikailov equation; companton-like solution; solitary patterns-like solution
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1.Introduction

The Dodd-Bullough-Mikhailov (DBM) equation[1], which appears in problems varying from fluid flow to quantum field theory, is given by

$$v_{xt} + pe^{v} + qe^{-2v} = 0 \tag{1}$$

Where p and q are real number and $p^2 + q^2 \neq 0$. When $p \neq 0, q = 0$, equation (1) become Liouville equation

$$v_{xt} + pe^{v} = 0$$

We notice that the transformation

$$v = \ln u \tag{2}$$

can change the DBM equation (1) into

$$uu_{xt} - u_x u_t + pu^3 + q = 0 \tag{3}$$

Where $v = v(x,t), u = u(x,t)$. Here, equation (3) is called the logarithmic DBM equation (L-DBM equation). Clearly, if we know the solutions of L-DBM equation (3), then we can obtain the solutions of DBM equation (1) by using the transformation (2).

In [2],Wazwaz studied the following special DBM equation

$$v_{xt} + e^{v} + e^{-2v} = 0 \tag{4}$$

and special Liouville equation

$$v_{xt} + e^{v} = 0 \tag{5}$$

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Using the transformation $v = \ln u$, the special DBM equation (4) changes to the following special L-DBM equation

$$uu_{xt} - u_x u_t + u^3 + 1 = 0 \tag{6}$$

In [2], by using the tanh method, Wazwaz obtained some solitons and periodic wave solutions for the special L-DBM equation (6). Engui Fan[3], by using the extended tanh method, build some exact explicit traveling solutions for the L-DBM equation..

In this paper, using a new transformation, the compacton-like and solitary patterns-like solutions of the L-DBM equation are obtained。

2. The generalized definitations

Recently, Kumar and Panigrahi [4] showed that the modified KdV equation

$$u_t + u^2 u_x + u_{xxx} = 0, \tag{7}$$

and some other nonlinear equations admitted the compacton-like solution in the form

$$u = \begin{cases} \dfrac{A\cos^2 k(x+\lambda t)}{1-2/3\cos^2 k(x+\lambda t)}, & |k(x+\lambda t)| \le \dfrac{\pi}{2}, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

Wazwaz [5] gave the solitary pattern-like solutions in the form

$$u = \frac{K\cosh^2 k(x-\lambda t)}{1-2/3\cosh^2 k(x-\lambda t)}, \tag{9}$$

for the following nonlinear wave equations with linear dispersion term

$$u_t + au^2 u_x - u_{xxx} = 0, a > 0,$$

$$u_t + (\frac{10}{3} + \frac{5}{4}u^4)u_x - 5u^2 u_{xxx} + u_{xxx} = 0,$$

$$u_t + \frac{1}{10}u^4 u_x - u_x^3 - u^2 u_{xx} + u_{xxx} = 0.$$

More recently, Yan [6] gave the generalized definitions of the compacton-like solutions and solitary pattern-like solutions , and then obtained the compacton-like solution and solitary pattern-like solutions of the mKdV equation, the nonlinear Schrodinger equation with a source term and the KdV-mKdV equation.

Definitation [6]. The solutions of the following forms

$$u = \begin{cases} \dfrac{A\cos k(x+\lambda t)+C}{1+B\cos k(x+\lambda t)}, & |k(x+\lambda t)| \le \dfrac{\pi}{2}, \\ 0, & \text{otherwise}, \end{cases} \tag{10}$$

and

$$u = \frac{A\cosh k(x+\lambda t)+C}{1+B\cosh k(x+\lambda t)}, \tag{11}$$

of some nonlinear wave equation are called the compacton-like solutions and solitary pattern-like solutions respectively.

If we use the relationships [6]

$$\cos^2(\xi)=\frac{1}{2}[1+\cos(2\xi)],\quad \cosh^2(\xi)=\frac{1}{2}[1+\cosh(2\xi)],$$

we find the formal functions (10) and (11) are generalizations of (8) and (9).

3.The compacton-like solutions to the L-DBM

Here we consider compacton-like solutions of the L-DBM equation (3):
With the aid of Mathematica , the substitution of the solution

$$u = \frac{A\ \cos\ k(x+\lambda t)+C}{1+B\ \cos\ k(x+\lambda t)} \qquad |k(x+\lambda t)| \le \frac{\pi}{2} \tag{12}$$

into (3) yields a ploynomial equation in $\cos^i k(x+\lambda t)\sin^j k(x+\lambda t)$. where $j=0,1$ and $i=0,1,2,3,\cdots$.Setting to zero their coefficients yields a set of algebraic equations in unknowns A,B,C,k,λ as

$$\begin{cases} C^3p+q-A^2k^2\lambda+B^2C^2k^2\lambda=0 \\ 3AC^2p+BC^3p+4Bq-2A^2Bk^2\lambda-ACk^2\lambda+2AB^2Ck^2\lambda+BC^2k^2\lambda=0 \\ 3A^2Cp+3ABC^2p+6B^2p=0 \\ A^3p+3A^2BCp+4B^3q+A^2Bk^2\lambda-AB^2Ck^2\lambda=0 \\ A^2Bp+B^4q=0 \end{cases}, \tag{13}$$

By solving the system of algebra Eq.(13), we can determined these unknown parameters A,B,C,k,λ:

$$\lambda=-\frac{3p^{2/3}q^{1/3}}{k^2},\ C=\frac{2q^{1/3}}{p^{1/3}},\ A=\mp\frac{q^{1/3}}{p^{1/3}},\ B=\pm 1 \tag{14}$$

Thus we have the following compacton-like solution of the L-DBM equation (3):

$$u=\begin{cases}\dfrac{\mp\dfrac{q^{1/3}}{p^{1/3}}\cos\ k(x-\dfrac{3p^{2/3}q^{1/3}}{k^2}t)+\dfrac{2q^{1/3}}{p^{1/3}}}{1\pm\cos\ k(x-\dfrac{3p^{2/3}q^{1/3}}{k^2}t)}, & \left|k(x-\dfrac{3p^{2/3}q^{1/3}}{k^2}t)\right|\le\dfrac{\pi}{2}\ , \\ 0 & \text{otherwise}\end{cases}\tag{15}$$

When p=q=k=1, from (14) we can obtain a set of special solution $\lambda=-3, A=-1, B=1, C=2$ and the compacton-like solution (15) of the L-DBM equation reduces to

$$u=\begin{cases}\dfrac{-\cos(x-3t)+2}{1+\cos(x-3t)} & |x-3t|\le\dfrac{\pi}{2}, \\ 0 & \text{otherwise}\end{cases}\tag{16}$$

Which is shown in Fig 1(a). and Fig 1(b) shows its the graph in the plane with t=0.

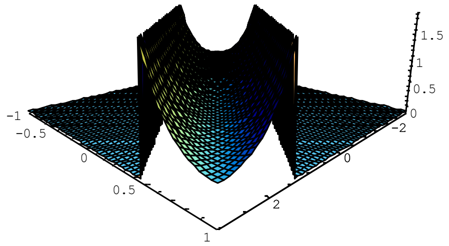

Fig. 1(a)

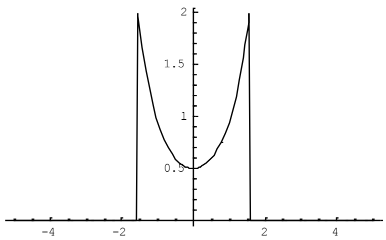

Fig.1(b)

4.The Solitary patterns-like solutions to L-DMB equation

Similarly, the substitution of the solution

$$u=\frac{A\cosh k(x+\lambda t)+C}{1+B\cosh k(x+\lambda t)}, \tag{17}$$

into (3) yields new solitary patterns-like solutions of L-DBM equation

$$u=\frac{\pm\dfrac{q^{1/3}}{p^{1/3}}\cosh k(x+\dfrac{3p^{2/3}q^{1/3}}{k^2}t)+\dfrac{2q^{1/3}}{p^{1/3}}}{1\mp\cosh k(x+\dfrac{3p^{2/3}q^{1/3}}{k^2}t)} \tag{18}$$

When p=q=k=1, from (18) we can obtain a special solitary patterns-like solution of the L-DBM equation which reduces to

$$u=\frac{\cosh\ (x+3t)+2}{1+\ \cosh\ (x+3t)} \tag{19}$$

Which is shown in Fig 2(a). and Fig 2(b) shows its the graph in the plane with t=0.

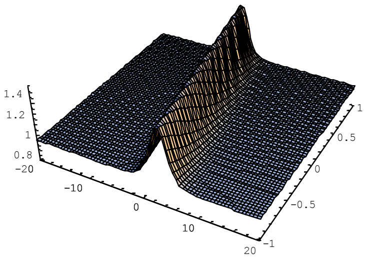

Fig.2(a)

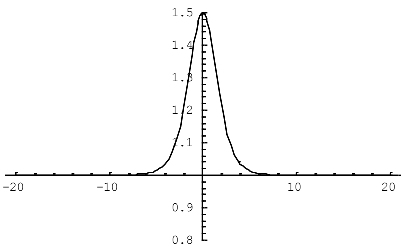

Fig.2(b)

5. Conclusion

To seek exact solutions of nonlinear wave equations is one of important work in the study of nonlinear equation. There is no general and uniformly method to solve nonlinear equations for its complication. In this paper we used the transformations to construct the compacton-like and solitary patterns-like solutions of the L-DBM equation. These solutions may be useful to explain some physical phenomena.

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