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Almost contra *vg*-continuity

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Abstract: The object of the paper is to study basic properties of Almost contra *vg*-continuous functions.

Keywords: *v*g-open sets, *v*g-continuity, *v*g-irresolute, *v*g-open map, *v*g-closed map,
*v*g-homeomorphisms and almost contra *v*g-continuity.
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1. Introduction:

In 1996, Dontchev introduced contra-continuous functions. C. W. Baker defined Subcontra-continuous functions in 1998 and almost contra β-continuous functions in 2006. J. Dontchev and T. Noiri introduced Contra-semicontinuous functions in 1999. S. Jafari and T. Noiri defined Contra-super-continuous functions in 1999; Contra-α-continuous functions in 2001 and contra-precontinuous functions in 2002. M. Caldas and S. Jafari studied Some Properties of Contra-β−Continuous Functions in 2001. T. Noiri and V. Popa studied unified theory of contra-continuity in 2002, Some properties of almost contra-precontinuity in 2005 and unified theory of almost contra-continuity in 2008. E. Ekici introduced almost contra-precontinuous functions in 2004 and studied another form of contra-continuity in 2006. A.A. Nasef studied some properties of contra-γ-continuous functions in 2005. M.K.R.S. Veera Kumar introduced Contra-Pre-Semi-Continuous Functions in 2005. During 2007, N. Rajesh studied total ω−Continuity, Strong ω-Continuity and almost contra ω-Continuity. Recently Ahmad Al-Omari and Mohd. Salmi Md. Noorani studied Some Properties of Contra-b-Continuous and almost contra-b-Continuous Functions in 2009 and Jamal M. Mustafa introduced almost contra Semi-I-Continuous functions in 2010. Inspired with these developments, we introduce almost contra *v*g-continuous function, obtain its basic properties, preservation theorems and relationship with other types of functions are verified.

2. Preliminaries:

Definition 2.1: A$\subseteq$ X is called

(i) regular open[pre-open; semi-open; α-open; β-open] if A = int(*cl*(A))[A$\subseteq$ int(*cl*(A); A$\subseteq$ *cl*(int(A)); A$\subseteq$ int(*cl*(int(A))); A$\subseteq$ *cl*(int(*cl*(A))].

(ii) *v*-open[rα-open] if $\exists$ a regular open set O such that O$\subset$ A$\subset$ *cl* (O)[O$\subset$ A$\subset$ α*cl*(O)]

(iii)semi-θ-open if it is the union of semi-regular sets and its complement is semi-θ-closed.

(iv) g-closed[rg-closed; gr-closed] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is open[r-open; open] in X.

(v) sg-closed[gs-closed] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is semi-open[open] in X.

(vi) pg-closed[gp-closed; gpr-closed] if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is pre-open[open; r-open] in X.

(vii)αg-closed[gα-closed;rgα-closed] if αcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\alpha-$open[open; r$\alpha-$open] in X.

(viii)*v*g-closed if *vcl*(A)$\subseteq$U whenever A$\subseteq$U and U is *v*$-$open in X.

(ix) *v*g-dense in X if *vgcl*(A) = X.

(x) The *v*g-frontier of A is defined by *v*gFr(A) = *vgcl*(A)-*vgcl*(X-A) = *vgcl*(A)-*vgint*(A).

(xi) θ-closed[θ-semi-closed] if A = Cl_θ(A) = {x$\in$X:*cl*(V)$\cap$A $\neq$ ϕ; for every V$\in\tau$}[A = sCl_θ(A) = {x$\in$X:*cl*(V)$\cap$A $\neq$ ϕ; for every V$\in$SO(X, x)}] and complement of θ-closed[θ-semi-closed] set is θ-open[θ-semi-open].Cl_θ(A)[sCl_θ(A)] is θ-closure[θ-semi-closure] of A.

It is shown that Cl_θ(V) = *cl*(V) for every V$\in\tau$ and Cl_θ(S) is closed in X for every S$\subset$ X.

Definition 2.2: A filter base Λ is said to be *v*-convergent (resp. rc-convergent) to a point x in X if for any U$\in$*v*O(X, x)(resp. U$\in$RC(X, x)), $\exists$ a *B*$\in\Lambda$ such that *B*$\subset$ U.

Definition 2.3: A function *f*: X $\to$ Y is called

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(i) almost-contra-[resp: almost-contra-semi-; almost-contra-pre-; almost-contra-nearly-; almost-contra-α-; almost-contra-$\beta-$; almost-contra- r$\alpha-$; almost-contra-$\omega-$; almost-contra-pre-semi-; almost contra-$\lambda-$]continuous if inverse image of every regular open set in Y is closed[resp: semi-closed; pre-closed; regular-closed; $\alpha-$closed; $\beta-$closed; r$\alpha-$closed; $\omega-$closed; pre-semi-closed; $\lambda-$closed] in X.

(ii) regular set-connected if inverse image of every regular open set is clopen.

(iii) perfectly continuous inverse image of every open set V is clopen.

(iv)almost s-continuous if for each $x\in X$ and each $V\in SO(Y)$ with $f(x) \in V$, $\exists$ an open set U in X containing x such that $f(U) \subset scl(V)$.

(v) (p, s)-continuous(resp. (θ, s)-continuous) if for each $x\in X$ and each $V\in SO(Y, f(x))$, $\exists$ $U\in PO(X, x)$ (resp. $U\in\tau$ containing x) such that $f(U) \subset Cl(V)$.

(vi) weakly continuous if for each $x\in X$ and each open set $V\in\sigma(Y), f(x))$, $\exists$ an open set U of X containing x such that $f(U) \subset cl(V)$.

(vii) (θ, s)-continuous iff for each θ-semi-open set V of Y, $f^{-1}(V)$ is open in X.

(viii)M-vg-open if the image of each vg-open set of X is vg-open in Y.

Definition 2.4: A graph $G(f)$ of a function f is said to be vg-regular if for each (x, y) in $(X\times Y) - G(f)$, $\exists$ $U\in vGC(X, x)$ and $V\in RO(Y, y)$ such that $(U\times V) \setminus G(f) = \phi$.

Lemma 2.1: The following properties are equivalent for a graph $G(f)$ of a function:

(1) $G(f)$ is vg$-$regular;

(2) for each $(x, y)\in(X\times Y)-G(f)$, $\exists$ $U\in vGC(X, x)$ and $V\in RO(Y, y)$ such that $f(U)\cap V = \phi$.

Lemma 2.2: If V is an regular-open set, then $sCl_\theta(V) = sCl(V) = Int(Cl(V))$

.

Lemma 2.3: For $V\subseteq Y$, the following properties hold:

(1) $\alpha cl(V) = cl(V)$ for every $V\in\beta O(Y)$,

(2) $v cl(V) = cl(V)$ for every $V\in SO(Y)$,

(3) $sclV = int(cl(V))$ for every $V\in RO(Y)$.

3. Almost contra *vg*-Continuous Functions:

Definition 3.1: A function *f* is said to be Almost contra *vg*-continuous if the inverse image of every regular open set is *vg*-closed.

Note 1: Here onwards we call Almost contra *vg*-continuous as al.c.*vg*.c., briefly.

Theorem 3.1: (i) *f* is al.c.*vg*.c. iff *f* is al.c.*vg*.c. at each $x \in X$.

(ii) *f* is al.c.*vg*.c. iff $f^{-1}(U) \in v\text{GO}(X)$ whenever $U \in \text{RC}(Y)$.

(iii) If *f* is c.*vg*.c., then *f* is al.c.*vg*.c. Converse is true if X is discrete space.

(iv) If *f* is al.c.*vg*.c. and $A \in \text{RO}(X)$, then $f_{/A}$ is al.c.*vg*.c.

Theorem 3.2: *f* is al.c.*vg*.c. iff $\forall$ $x \in X$ and $V \in \text{RGO}(Y, f(x))$[resp: $U_Y \in v\text{GO}(Y, f(x))$], $\exists$ $U \in vGO(X, x)$ s.t., $f(U) \subset V$[resp: $f(A) \subset U_Y$].

Proof: Let $U_Y \in \text{RO}(Y)$ and let $x \in f^{-1}(U_Y)$. Then $f(x) \in U_Y$ and $\exists$ $A_x \in v\text{GO}(X, x)$ and $f(A_x) \subset U_Y$. Then $x \in A_x \subset f^{-1}(U_Y)$ and $f^{-1}(U_Y) = \cup A_x$. Hence $f^{-1}(U_Y) \in v\text{GO}(X)$.

Example 1: $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Then (i) the identity function *f* on X is al.c.*vg*.c., al.c.gs.c alc.βg.c., but not al.c.g.c., al.c.sg.c., al.c.pg.c., al.c.gp.c., al.c.rg.c; al.c.gr.c., al.c.gpr.c., alc.rpg.c, al.c.αg.c., al.c.gα.c., al.c.rgα.c.

(ii) *f* defined by $f(a) = c$; $f(b) = a$; $f(c) = b$ is al.c.*vg*.c., but not al.c.gs.c alc.βg.c.,

Example 2: $X = Y = \{a, b, c, d\}$: $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\} = \sigma$. Then (i) *f* defined by $f(a) = b$; $f(b) = c$; $f(c) = d$; $f(d) = a$ is al.c.sg.c., al.c.gs.c., and al.c.gpr.c., but not al.c.*vg*.c; al.c.g.c., al.c.sg.c., al.c.pg.c., al.c.gp.c., al.c.rg.c; al.c.gr.c., alc.rpg.c, al.c.αg.c., al.c.gα.c., and al.c.rgα.c.

(ii) the identity function *f* is al.c.sg.c., al.c.gs.c., and al.c.gpr.c., but not al.c.*vg*.c;

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(iii) f defined by $f(a) = b$; $f(b) = a$; $f(c) = d$; $f(d) = c$ is al.c.sg.c., al.c.gs.c., and al.c.gpr.c., but not al.c.*v*g.c; al.c.g.c., al.c.sg.c., al.c.pg.c., al.c.gp.c., al.c.rg.c; al.c.gr.c., alc.rpg.c, al.c.αg.c., al.c.gα.c., and al.c.rgα.c.

under usual topology on $\Re$ both al.c.g.c and al.c.rg.c. as well al.c.sg.c. and al.c.*v*g.c. are same.

Theorem 3.3: Let f_i: $X_i \rightarrow Y_i$ be al.c.*v*g.c. for i = 1, 2. Let f: $X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is al.c.*v*g.c.

Theorem 3.4: Let h:$X \rightarrow X_1 \times X_2$ be al.c.*v*g.c., where $h(x) = (h_1(x), h_2(x))$. Then h_i:$X \rightarrow X_i$ is al.c.*v*g.c. for i = 1, 2.

In general we have the following extension of theorems 3.3 and 3.4:

Theorem 3.5: (i) f:$\Pi X_\lambda \rightarrow \Pi Y_\lambda$ is al.c.*v*g.c, iff f_λ: $X_\lambda \rightarrow Y_\lambda$ is al.c.*v*g.c for each $\lambda \in \Lambda$.
(ii)If f:$X \rightarrow \Pi Y_\lambda$ is al.c.*v*g.c, then $P_\lambda \bullet f$: $X \rightarrow Y_\lambda$ is al.c.*v*g.c for every $\lambda \in \Lambda$; P_λ:ΠY_λ onto Y_λ.

Note 2: With respect to usual topology on $\Re$, open sets and regular open sets are one and the same. So converse of theorem 3.5 is not true in general, as shown by.

Example 3: Let $X = X_1 = X_2 = [0, 1]$. Let f_1: $X \rightarrow X_1$ and f_2: $X \rightarrow X_2$ are defined as follows: $f_1(x) = 1$ if $0 \le x \le 1/2$ and $f_1(x) = 0$ if $1/2 < x \le 1$. $f_2(x) = 1$ if $0 \le x < 1/2$ and $f_2(x) = 0$ if $1/2 < x < 1$. Then f_i: $X \rightarrow X_i$ is clearly al.c.*v*g.c. for i = 1, 2., but $h(x) = (f_1(x_1), f_2(x_2))$:$X \rightarrow X_1 \times X_2$ is not al.c.*v*g.c., for $S_{1/2}(1, 0) \in RO(X_1 \times X_2)$, but $h^{-1}(S_{1/2}(1, 0)) = \{1/2\} \notin v GO(X)$.

Remark 1: In general, (i) al.c.*v*g.c. function of al.c.*v*g.c. function is not al.c.*v*g.c.
(ii) The algebraic sum; product and composition of two al.c.*v*g.c. functions is not al.c.*v*g.c. However the scalar multiple of al.c.*v*g.c. function is al.c.*v*g.c.

(iii)The pointwise limit of a sequence of al.c.*vg*.c. functions is not al.c.*vg*.c. as shown by the following examples.

Example 4: Let $X = X_1 = X_2 = [0, 1]$. Let $f_1:X\to X_1$ and $f_2:X\to X_2$ are defined as follows: $f_1(x) = x$ if $0 < x < 1/2$ and $f_1(x) = 0$ if $1/2 < x < 1$; $f_2(x) = 0$ if $0 < x < 1/2$ and $f_2(x) = 1$ if $1/2 < x < 1$. Then their product is not al.c.*vg*.c.

Example 5: Let $X = Y = [0, 1]$. Let $f_n:X\to Y$ is defined as follows: $f_n(x) = x_n$ for $n \geq 1$ then f: $X\to Y$ is the limit of the sequence where $f(x) = 0$ if $0 \leq x < 1$ and $f(x) = 1$ if $x = 1$. Therefore f is not al.c.*vg*.c. For $(1/2, 1]\in v$GO(Y), $f^{-1}((1/2, 1]) = (1)\notin v$GO(X).

However we can prove the following theorem.

Theorem 3.6: Uniform Limit of a sequence of al.c.*vg*.c. functions is al.c.*vg*.c.

Problem: (i) Are sup$\{f, g\}$ and inf$\{f, g\}$ are al.c.*vg*.c if f, g are al.c.*vg*.c
(ii) Is $C_{al.c.vg.c}(X, R)$, the set of all al.c.*vg*.c functions,
(1) a Group. (2) a Ring. (3) a Vector space. (4) a Lattice.

Example 6: Let $X = Y = [0, 1]$. Let f: $X\to Y$ be defined as follows: $f(x) = 1$ if $0 \leq x < 1/2$ and $f(x) = 0$ if $1/2 < x \leq 1$. Then obviously f is al.c.*vg*.c. but not r-continuous.

Example 7: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. The identity map f is al.c.s.c., and al.c.*vg*.c. but not al.c.c., and r-irresolute.

Example 8: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. f defined as $f(a) = f(b) = b$; $f(c) = c$ is al.c.s.c., and al.c.c., but not al.c.*vg*.c., and r-irresolute.

Remark 2: We have the following implication diagram for a function *f* and reverse implication is true if *v*g-open sets are r-open.

al.c.g.c   al.c.gs.c
↓   ↓   ↑
al.c.rgα.c → al.c.rg.c → al.c.*v*g.c ← al.c.sg.c ← al.c.βg.c
↑   ↑   ↑   ↑   ↑
al.c.rα.c → → → al.c.*v*.c
al.c.r.c. → al.c.π.c → al.c.c → al.c.α.c → al.c.s.c → al.c.β.c
↓   ↓   ↓
al.c.πg.c al.c.p.c→ al.c.ω.c.→ al.c.gα.c
↓   ↓
al.c.gp.c ← al.c.pg.c al.c.rω.c

Theorem 3.7: (i) If *f* is al.c.*vg*.c. [a l.c.rg.c.] and *g* is r-irresolute then *g*•*f* is al.c.*vg*.c.

(ii)If *f* is c.*vg*.c.[al.c.*vg*.c.] *g* is al.g.c.[al.rg.c.] and Y is $T_{1/2}$[r$T_{1/2}$], then *g*•*f* is al.c.*vg*.c.

(iii) If *f* is al.c.*vg*.c.;[resp: *vg*.c.;] *g* is al.g.c.[al.rg.c.] and every g-open set[rg-open] in Y is r-open, then *g*•*f* is al.c.*vg*.c.

(iv) If *f* is *v*g-irresolute and *g* is al.c.*v*g.c.[al.c.g.c], then *g*•*f* is al.c.*v*g.c.

(v) If *f* is al.c.*v*g.c. and *g* is al.c.,[resp: nearly continuous] then *g*•*f* is al.c.*v*g.c.

(vi)If *f* is c.*v*g.c.[al.c.rg.c.] *g* is al.c.g.c[al.c.rg.c] and Y is T_{12}[rT_{12}], then *g*•*f* is al.c.*v*g.c.

Theorem 3.8: (i) If *f* is *vg*-irresolute, *vg*-open and *v*GO(X) = τ and *g* be a function, then *g*•*f* is al.c.*vg*.c iff *g* is al.c.*vg*.c.

(ii) If *f* is *v*g-irresolute, *v*g-open[al-*v*g-open; M-*v*g-open] and bijective, *g* is a function. Then *g* isal.c.*v*g.c. iff *g*•*f* is al.c.*v*g.c.

Corollary 3.1: (i) If *f* is c.c.[c.r.c.], *g* is al.c.,[r-irresolute], then *g*•*f* is al.c.*v*g.c.

(ii) If *f* is c.c.[c.r.c.], *g* is al.g.c.,[al.rg.c.,] and Y is T_{12}{rT_{12}}, then *g*•*f* is al.c.*v*g.c.

(iii)If *f* be r-open, al.c.*vg*.c. and *g* be al.c.*vg*.c., then *g*•*f* is al.c.*vg*.c.

Theorem 3.9: Let X, Y, Z be spaces and every *vg*-open set is r-open in Y, then the composition of two al.c.*vg*.c. maps is al.c.*vg*.c.

Note 3: Pasting Lemma is not true with respect to al.c.*vg*.c. functions. However we have the following weaker versions.

Theorem 3.10: Pasting Lemma: Let X; Y be such that X = A∪B. Let $f_{/A}$ and $g_{/B}$ are al.c.*vg*.c.[resp: r-irresolute] such that *f*(x) = *g*(x) for every x∈A∩B. If A, B∈RO(X) and *v*GO(X)[resp: RO(X)] is closed under finite unions, then the combination α:X→Y is al.c.*vg*.c.

Theorem 3.11: The following statements are equivalent for a function *f*:

(1) *f* is al.c.*vg*.c.;

(2) f^{-1}(F) ∈*v*GO(X) for every F∈ RC(Y);

(3) for each x∈X and each F∈RC(Y,*f*(x)), ∃ U∈*v*GO(X, x) such that *f*(U) ⊂ F;

(4) for each x∈X and each F∈RO(Y) non-containing *f*(x), ∃ K∈*v*GC(X) non-containing x such that f^{-1}(V) ⊂ K;

(5) f^{-1}(int(*cl*(G))⊂ *v*GC(X) for every regular open subset G of Y;

(6) f^{-1}(*cl*(int(F)))⊂ *v*GO(X) for every regular closed subset F of Y.

Example 9: Let X = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {a, c},X} and σ = {ϕ, {b}, {c}, {b, c},X}. Then the identity function *f* on X is al.c.*vg*.c., but it is not regular set-connected.

Example 10: Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c},X} and σ = {ϕ, {a}, {a, b},X}.Then the identity function *f* on X is al.c.vg.c. which is not c.*v*g.c.

Remark 3: Every restriction of an al.c.*vg*.c. function is not necessarily al.c.*vg*.c.

Theorem 3.12: Let *f* be a function and $\Sigma = \{U_\alpha: \alpha\in I\}$ be a *vg*-cover of X. If for each $\alpha\in I$, $f_{|U\alpha}$ is al.c.*vg*.c., then *f* is an al.c.*vg*.c.

Proof: Let $F\in RC(Y)$. $f_{|U\alpha}$ is al.c.*vg*.c. for each $\alpha\in I$, $f_{|U\alpha}^{-1}(F)\in vGO_{|U\alpha}$. Since $U_\alpha\in vGO(X)$, $f_{|U\alpha}^{-1}(F)\in vGO(X)$ for each $\alpha\in I$. Then $f^{-1}(F) = \cup_{\alpha\in I} f_{|U\alpha}^{-1}(F)\in vGO(X)$. Thus *f* is al.c.*vg*.c.

Theorem 3.13: Let *f* be a function and $x\in X$. If $\exists\ U\in vGO(U, x)$ [resp: $U\in RO(X, x)$] and $f_{|U}$ is al.c.*vg*.c. at x, then *f* is al.c.*vg*.c. at x.

Proof: Let $F\in RC(Y,f(x))$. Since $f_{|U}$ is al.c.*vg*.c. at x, $\exists\ V\in vGO(U, x)$ such that $f(V) = (f_{|U})(V) \subset F$. Since $U\in RO(X, x)$, it follows that $V\in vGO(X, x)$. Hence *f* is al.c.*vg*.c. at x.

Theorem 3.14: Let g:$X\to X\times Y$ be the graph function of *f*, defined by $g(x) = (x, f(x))$ for every $x\in X$. If *g* is al.c.*vg*.c., then *f* is al.c.*vg*.c.

Proof: Let $V\in RC(Y)$, then $X\times V = X\times cl(int(V)) = cl(int(X))\times cl(int(V)) = cl(int(X\times V))\in RC(X\times Y)$. Since *g* is al.c.*vg*.c., then $f^{-1}(V) = g^{-1}(X\times V)\in vGC(X)$. Thus, *f* is al.c.*vg*.c.

Theorem 3.15: For *f* and *g*. The following properties hold:

(1) If *f* is al.c.*vg*.c.[c.*vg*.c.] and *g* is regular set-connected, then $g\bullet f$ is al.c.*vg*.c.

(2) If *f* is al.c.*vg*.c. and *g* is perfectly continuous, then $g\bullet f$ is *vg*.c. and c.*vg*.c.

Theorem 3.16: If *f* is a surjective M-*vg*-open[resp:M-*vg*-closed] and *g* is a function such that $g\bullet f$ is al.c.*vg*.c., then *g* is al.c.*vg*.c.

Theorem 3.17: If *f* is al.c.*vg*.c., then for each point $x\in X$ and each filter base Λ in X *vg*-converging to x, the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.

Definition 3.2: A function *f* is called (*vg*, s)-continuous if for each $x\in X$ and each $V\in SO(Y, f(x))$, $\exists\ U\in vGO(X, x)$ such that $f(U) \subset cl(V)$.

Theorem 3.18: For *f*, the following properties are equivalent:

(1) *f* is (*v*g, s)-continuous;

(2) *f* is al.c.*v*g.c.;

(3) f^{-1}(V) is *v*g-open in X for each θ-semi-open set V of Y;

(4) f^{-1}(F) is *v*g-closed in X for each θ-semi-closed set F of Y.

Theorem 3.17: The following are equivalent:

(1) *f* is al.c.*v*g.c.;

(2) f^{-1}(cl(V)) is *v*g-open in X for every V∈βO(Y);

(3) f^{-1}(cl(V)) is *v*g-open in X for every V∈ SO(Y);

(4) f^{-1}(int(cl(V))) is *v*g-closed in X for every V∈RO(Y).

Corollary 3.2: For *f*, the following are equivalent:

(1) *f* is al.c.*v*g.c.;

(2) f^{-1}(αcl(V)) is *v*g-open in X for every V∈βO(Y);

(3) f^{-1}(*v*cl(V)) is *v*g-open in X for every V∈SO(Y);

(4) f^{-1}(scl(V)) is *v*g-closed in X for every V∈RO(Y).

Proof: This is an immediate consequence of Theorem 3.17 and Lemma 2.3.

Remark 4: al.*v*g.c. and al.c.*v*g.c. are independent of each other.

Theorem 3.18: For *f*, the following properties are equivalent:

(1) *f* is al.c.*v*g.c.;

(2) $f(vg(cl\ A)) \subset sCl_\theta(f(A))$ for every subset A of X;

(3) $vgcl\{(f^{-1}(B))\} \subset f^{-1}(sCl_\theta(B))$ for every subset B of Y.

4. The preservation theorems:

Theorem 4.1: (i) If *f* is al.c.*v*g.c.[resp: al.c.rg.c] surjection and X is *v*g-compact[*v*g-lindeloff],then Y is nearly closed compact[nearly closed lindeloff].

(ii) If *f* is al.c.*v*g.c., surjection and X is *v*g-compact[*v*g-lindeloff] then Y is mildly closed compact[mildly closed lindeloff].

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Theorem 4.2: If *f* is al.c.*v*g.c.[al.c.rg.c.], surjection and

(i) X is locally *v*g-compact[locally *v*g-lindeloff], then Y is locally nearly closed compact[resp:locally mildly compact; locally nearly closed Lindeloff; locally mildly lindeloff].

(ii) If *f* is al.c.*v*g.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

(iii)X is *v*g-compact[resp: countably *v*g-compact] then Y is S-closed[resp: countably S-closed].

(iv) X is *v*g-Lindelof, then Y is S-Lindelof and nearly Lindelof.

Theorem 4.3: If *f* is an al.c.*v*g.c. and al.c., surjection and X is mildly compact (resp. mildly countably compact, mildly Lindelof), then Y is nearly compact (resp. nearly countably compact, nearly Lindelof) and S-closed (resp. countably S-closed, S-Lindelof).

Theorem 4.4: (i) If *f* is al.c.*vg*.c.[contra *vg*-irreolute] surjection and X is *vg*-connected, then Y is connected[*vg*-connected]

(ii) If X is *vg*-ultra-connected and *f* is al.c.*vg*.c. and surjective, then Y is hyperconnected.

(iii) The inverse image of a disconnected[*vg*-disconnected] space under al.c.*vg*.c.,[contra *vg*-irreolute] surjection is *vg*-disconnected.

Theorem 4.5: If *f* is al.c.*vg*.c., injection and

(i) Y is UT_i[resp: UC_i; UD_i], then X is vg_i[resp:$vg\ C_i$; $vg\ D_i$] i = 0,1,2.

(ii) Y is UR_i, then X is vgR_i i = 0, 1.

(iii)Y is weakly Hausdorff[resp: rT_2], then X is vg_1[vg_i; i = 0,1,2.]

(iv) If *f* is closed, Y is UT_i, then X is vg_i i = 3, 4.

Theorem 4.6: (i) If *f* is al.c.*vg*.c.[resp: al.c.g.c.; al.c.sg.c.; al.c.rg.c] and Y is UT_2,

(a) then the graph G(*f*) of *f* is *vg*-closed in X×Y.

(b) then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is *vg*-closed in X×Y.

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(ii) If *f* is al.c.rg.c.[al.c.g.c.]; *g* is c.*vg*.c., and Y is UT_2, then E = {x∈X: *f*(x) = *g*(x)} is *vg*-closed in X.

5. Relations to weak forms of continuity:

Definition 5.1: A function *f* is said to be faintly *vg*-continuous if for each x∈ X and each θ-open set V of Y containing *f*(x), ∃ U∈*v*GO(X, x) such that *f*(U) ⊂ V.

Example 11: Let X = {a, b, c}, τ = {ϕ, {a, b}, X} and σ = {ϕ, {a}, {b, c}, X}. Then, the identity function *f* is al.c.*vg*.c but it is not weakly continuous.

Example 12: Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c}, X} and σ = {ϕ, {a}, {a, b}, X}. Then, the identity function *f* is (θ, s)-continuous and al.c.*vg*.c.

Example 13: Let ℜ be the reals with the usual topology and *f*: ℜ → ℜ the identity function. Then *f* is continuous, weakly continuous, al.c.p.c., and al.c.*vg*.c.

Example 14: Let X = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {a, c}, X} and σ = {ϕ, {b}, {c}, {b, c}, X}. Then, the identity function on X is c.c., c.s.c., and al.c.*vg*.c.

Corollary 5.1: If *f* is M-*vg*-open and c.*vg*.c., then *f* is al.c.*vg*.c.

Lemma 5.1: For *f*, the following properties are equivalent:
(1) *f* is faintly-*vg*-continuous;
(2) f^{-1}(V)∈*v*GO(X) for every θ-open set V of Y;
(3) f^{-1}(K)∈*v*GC(X) for every θ-closed set K of Y.

Theorem 5.1: If for each $x_1 \neq x_2$∈ X, ∃ *f* of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and *f* is al.c.*vg*.c., at x_1 and x_2, then X is vg_2.

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Proof: For $x_1 \neq x_2$, $\exists$ $V_i \in (\sigma, f(x_i))$ s.t., $\cap cl(V_i) = \phi$ for i = 1,2. For f is al.c.*vg*.c., at x_i, $\exists$ $U_i \in v$GO(X, x_i) s.t., $f(U_i) \subset cl(V_i)$ for i = 1, 2., and $\cap U_i = \phi$. Hence X is vg_2.

Corollary 5.2: If f is al.c.*vg*.c. injection and Y is Urysohn, then X is vg_2.

Theorem 5.2: {x∈ X: f is not al.c.*vg*.c.} is identical with the union of the *vg*-frontier of the inverse images of regular closed sets of Y containing f(x).

Proof: If f is not al.c.*vg*.c. at x∈X. By Theorem 3.11, ∃ F∈RC(Y, f(x)) s.t., $f(U) \cap (Y - F) \neq \phi$ for every U∈*v*GO(X, x). Then $x \in vgcl(f^{-1}(Y - F)) = vgcl(X - f^{-1}(F))$. On the other hand, we get $x \in f^{-1}(F) \subset vgcl\{(f^{-1}(F))\}$ and hence $x \in vg\,Fr(f^{-1}(F))$.

Conversely, If f is al.c.*vg*.c. at x and F∈ RO(Y, f(x)), ∃ U∈*v*GO(X, x) s.t $x \in U \subset f^{-1}(F)$. Hence $x \in vgint(f^{-1}(F))$, which contradicts $x \in vgFr(f^{-1}(F))$. Thus f is not al.c.*vg*.c.

Theorem 5.3: Let Y be E.D. Then, f is al.c.*vg*.c. iff it is al.*vg*.c..

Definition 5.2: A function f is said to have a strongly contra-*vg*-closed graph if for each (x, y)∈(X×Y) - $g(f)$ ∃ U∈*v*GO(X, x) and V∈RC(Y,y) such that $(U \times V) \cap \{g(f)\} = \phi$.

Lemma 5.2: f has a strongly contra-*vg*-closed graph iff for each (x, y) ∈ (X× Y) - $g(f)$ ∃ U∈*v*GO(X, x) and V∈ RC(Y,y) such that $f(U) \cap V = \phi$.

Theorem 5.4: If f is al.c.*vg*.c. and Y is Hausdorff, then $g(f)$ is strongly contra-*vg*-closed.

Theorem 5.5: If f is injective al.c.*vg*.c. with strongly contra-*vg*-closed graph, then X is vg_2.

Proof: Let x ≠ y∈ X. Since f is injective, we have $f(x) \neq f(y)$ and (x, f(y)) ∈ (X× Y) - $g(f)$. Since $g(f)$ is strongly contra-*vg*-closed, by Lemma 5.2 ∃ U∈*v*GO(X, x) and V∈RC(Y, f(y)) such that $f(U) \cap V = \phi$. Since f is al.c.*vg*.c., by Theorem 3.11, ∃ G∈*v*GO(X, y) such that $f(G) \subset V$. Therefore $f(U) \cap f(G) = \phi$; hence $U \cap G = \phi$. Thus X is vg_2.

Corollary 5.3: If *f* is al.c.*vg*.c. and Y is Urysohn, then *g*(*f*) is strongly contra-*vg*-closed and contra-*vg*-closed.

CONCLUSION: In this paper we defined Almost contra *vg*-continuous functions, studied its properties and their interrelations with other types of such functions.

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