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A simple mathematical model for evaluating the effectiveness of solar water heating system with thermosyphon heat exchanger

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Abstract: A simple mathematical model for estimating energy transfer in a solar hot water tank fitted with thermosyphon heat exchanger and operating in a thermosyphon loop is developed. The model is used for evaluating the effectiveness of the heat exchanger operating under different mass flow rates. An experimental test facility for the heat exchanger is fabricated in which an electric geyser is used for hot water supply. Comparison of experimental results with the theoretical results yielded the effectiveness of the heat exchanger. It is found to vary between 0.7 to 1.0 for the normal range of flow rates encountered in a thermosphon solar water heating system.

Keywords: Solar water heating, Heat Exchanger, Effectivenes.

Introduction:

Thermosyphon solar water heating systems are widely used in many countries. The major factor that comes in the way of good performance of the system is the use of raw or untreated water having high total hardness, which results in scale formation. To overcome this problem, a heat exchanger is normally installed in hot water tank of the system. In the thermosyphon solar water heating system, during a day the collector mass flow rate does not remain constant due to solar radiation variation. To evaluate the effectiveness of any designed heat exchanger it is essential ot maintain steady flows inside and outside the heat exchanger. In this paper the effectiveness o f an existing heat exchanger of 1.5 sqm of surface area suitable for 100 lpd system is evaluated for varying operating conditions. An experimental testing facility with a geyser as a source of heat is used for studying the performance of the system consisting of a hot water tank with heat exchanger. A simple mathematical model for energy transfer in solar hot water tank with heat exchanger is developed. Using the model, temperatures at various sections within hot water tank and heat exchanger were evaluated for constant flow rates. Comparisons are made between theoretical and experimental results.

Literature: Heat exchangers fitted in the storage water tank are used in domestic solar water heating systems to transfer energy from collector fluid to the storage tank fluid. Water flow in the system is driven by the difference in hydrostatic pressure arising due to thermal gradients in the storage tank and the collector including the connecting pipes. To predict system performance, the simulation models require empirical heat transfer correlations for the heat exchanger. Because water flow rates are low and the flow passage is short the flow is likely to be developing, both thermally and hydro dynamically. Smith, et al [1] tested an unpressurized drain back system with a load side heat exchanger. They used a coiled tube, which carries the hot fluid placed inside a vertical cylindrical tank containing cold fluid (load side fluid) to be heated. They carried out analytical calculations for the heat exchanger effectiveness by dividing the tank with the heat exchanger into eight equal segments. The heat exchanger effectiveness was calculated at each data point by the equation

$$\in = \frac{\Delta Q}{\Delta Q_{max}} = \frac{\dot{m}_d C_p \left(T_{out} - T_{in}\right)}{\dot{m}_f C_p \left(T_{max} - T_{min}\right)}$$

and average effectiveness was obtained by summing all calculated effectiveness divided by the total number of data points and was found as 0.78. The overall thermal resistance of the heat exchanger is comprised of three terms: Resistance due to the inside convective coefficient due to the forced flow. Resistance through copper tubing Resistance due to outside convective heat transfer coefficient (natural convection). The inside convective coefficient h_i was determined using the Dittus-Boelter relationship $Nu_i = \frac{h_i D}{k} = 0.023 \mathrm{Re}_D^{0.8} \mathrm{Pr}^{0.4}$ for Re = 19,104(the flow was turbulent). The Nu was found to be 109. Neglecting conductance through the copper tubing the overall heat transfer conductance was calculated from $U_o = \left[A\left(\frac{1}{h_o A_{Do}} + \frac{1}{h_i A_{Di}} \right) \right]^{-1}$.

Different empirical correlations were used for evaluating h_o. They obtained best agreement with experimental results for the outside heat transfer coefficient for using correlation of free convection from horizontal tube. Heat transfer correlations for thermosyphon heat exchanger of tube in shell type are given by few researchers. The correlations are determined for uniform heat flux on the tube walls. Gruszezynski and Viskanta[2] and Hallinan and Viskanta [3] presented correlations for the Nusselt number for the thermosyphon water flows in triangular and rectangular arrays with P/D ratios of 1.25 and 1.33 inside a circular shell. On the tube side constant temperature water was mechanically pumped through the tubes giving a constant flux boundary condition.

For seven tubes triangular array inside a circular shell

$Nu = 0.067\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.43}$ (Counter flow), $Nu = 0.081\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.43}$ (parallel flow) For operating condition of $80 < \mathrm{Re} < 500$ and $\mathrm{Pr} \approx 5.0$

For 21 tube rectangular array inside a circular shell

$Nu = 0.026\,\mathrm{Re}^{0.93}\,\mathrm{Pr}^{0.43}$ (Counter flow), $Nu = 0.051\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.43}$ (parallel flow) For operating condition of $80 < \mathrm{Re} < 500$ and $\mathrm{Pr} \approx 5.0$

Because the flow rate in this study was not controlled independently from the temperature difference, the authors did not differentiate between forced convection and natural convection effects. El-Genk and co workers [4] presented mixed convection heat transfer correlations for uniform heat flux boundary conditions. Kim and El-Genk[5] studied triangular arrays of seven tubes with P/D ratio of 1.38 and 1.51 enclosed in a hexagonal shroud. Their correlation was given as $Nu_M = 2.762 Ri^{0.163}\,\mathrm{Re}^{0.282}$ for $P/D = 1.38$, $Nu_M = 0.95 Ri^{0.25}\,\mathrm{Re}^{0.404}$ for $P/D = 1.51$. The parameter typically used to characterize mixed convection in tube bundles is the Richardson number, $Ri = Gr/\mathrm{Re}^2$. Richardson numbers up to 500 are well within the typical range of mixed convection flows. These studies covered a wide range of Reynolds number ($80 < \mathrm{Re} < 2300$) and Raleigh numbers ($5\mathrm{X}10^5 < \mathrm{Ra} < 7\mathrm{X}10^8$) corresponding to

Richardson numbers up to 500. Dahl and Davidson [6] presented mixed convection heat transfer and pressure drop correlations under uniform heat flux boundary conditions for three types tube in shell heat exchangers. The correlations are presented in the form of $Nu_M = \left[Nu_F^n \pm Nu_N^n \right]^{1/n}$, where n = 4, was first proposed by Churchill [7]. This expression separates the contribution of forced and natural convection to the total heat transfer. The negative sign corresponds to opposing flow conditions and the positive sign is for aiding the flow conditions. There is no clear method of comparing mixed convection heat transfer correlations for tube bundles with different P/D ratio and geometrics.

Formulation: Schematic arrangement of the hot water tank consisting of the heat exchange is shown in Figure: i. Hot water is assumed to be divided into 4 distinct layer of water causing temperature variation along the vertical height of, each layer. For each layer internal energy change is equal to the heat loss and water transport loss including the draw off for the utility from storage tank. During draw off an equal amount of makeup water is assumed to displace water from the tank. The following equations are formulated for each section of the tank.

$[\rho\, C_p\, V_i]_w\, [(T'_i - T_i) / \Delta\theta] = m_{d\, Cp}\, (T_{i+1} - T_i) + m_{f\, cp}\, e(t_{f,i-1} - t_{f,\,i}) - U\, A_i\, (T_i - T_a)$ _____________ (1)

Where **i = 1, 2, 3 & 4. i** represent layers of the system. $m_f = 0$ for top layer **i = 1** and

$T_4 = T_i$ for bottom layer

For fluid flow in heat exchanger the individual energy balance of each section is given by

$[\rho\, C_p\, V]_{f,i}\, [(t'_{f,i} - t_{f,i}) / \Delta\theta] = h\, a_i\, (T_i - t_{f,i})$ ___________ _______(2)

Where i = 2, 3, &4, represents the layer in heat exchanger.

Heat exchanger fluid temperature is given by $t_{f,i} = [(T_i + T_{i+1})/2]$

The fluid temperatures $t_{f\,i}$ of water in the heat exchanger are first evaluated from equation 2 and their values are substituted in equation 1 to get the hot water tank temperatures T_i. The obtained temperatures are used in subsequent time to get new temperatures.

Experimental station: The experimental station Figure: ii consists of an electric geyser, hot water tank with the heat exchanger and a reservoir which supplies water to hot water tank is fabricated. The hot water tank is insulated all round with rock wool to a thickness of 0,05m to

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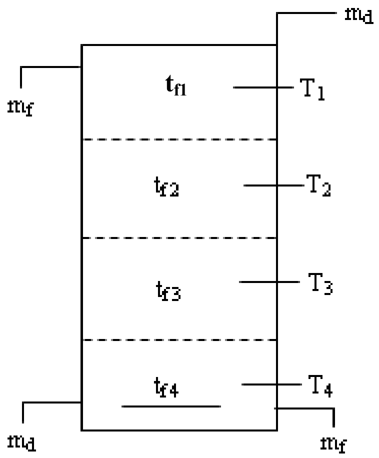
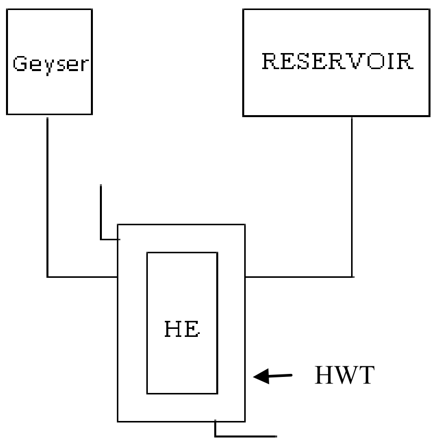
prevent het losses to atmosphere from the tank. The temperatures at different locations in the experimental station are measured with the help RTD sensors.





HE – Heat Exchanger,
HWT – Hot Water Tank

FIG i: Energy Balance – 4 layers of the tank **FIG: ii: Experimental Station**

Results and discussions: Experiment where conducted for mass flow rates of 37, 50, 62 & 83 1/hr of cold water flow rates. The theoretical evaluation of temperatures at various sections of the tank is carried out by using the computer programme developed. The initial values of temperatures required for theoretical evaluation are obtained from the experiment for each flow rate. Numerical results obtained from theoretical simulation programme for every 60 seconds are compared with experimental values. The difference in temperatures between the experimental and theoretical values for the four sections in the hot water tank are plotted to bring out the performance of the heat exchanger at different flow rates. Figures: 1-4 shows a temperature difference between theoretical and experimental values when the heat exchanger effectiveness value is assumed as 0.7. At lower flow rates of 37 and 501/hr, the temperature of the top two sections of the tank T_1 and T_2 are closely matching with experimental results as evidenced by small delta T of 1 degree or less. Larger variation in delta T is noticed for the next two sections of the tank i.e. T_3 and T_4, for mass flow rates of 62 and 83 1/hr the delta T remained constant through our study with a value of around 2 degrees of less. This clearly indicates that the theoretical results are matching well with experimental results for flow rates of 62 and 83 1/hr and the heat exchanger effectiveness taken as around 0.7 for flow rates of 62 and 83 1/hr. Figure's.5-8 are drawn for the heat exchanger effectiveness of 0.8 at higher flow rates of 62 & 83 1/hr, the delta T of top two sections of the tank is coming higher compared to the sections 3 and 4 as indicated by figures 7 and 8. Whereas for mass flow rates of 37 and 501/hr the top two sections, delta T is less compared to sections 3 and 4. Thus effectiveness value of 0.8 is giving higher temperature difference for sections 3 and 4 for lower mass flow rates of 37 and 50 1/hr and higher temperature difference for sections 1 and 2 for higher mass flow rates of 62 and 83 1/hr. Figure's. 9-12 are plotted for assumed effectiveness value of 0.9. It can be observe from figures 9 and 10 that the temperature difference between theoretical and experimental results remain constant and low in magnitude for all sections 1 to 4. In figure 11 and 12 the temperature difference in the top two sections is large indicating higher in accuracy. In other words for low flow rates of 37 and 50 1/hr. A heat exchanger of effectiveness 0.9 are yielding good results. Comparison is also made assuming highest possible heat exchanger effectiveness of 1.0. Figures: 13-16 shows the temperature difference for all the four flow rates. It can be observed that as the flow rate increases the delta t increases reaching a maximum of 5 degrees for the flow rates 62 and 831trs/hr. In other words effectiveness value of 1.0 results in larger variation between experimental and the theoretical values as the flow rates increased beyond 50 1/hr. On further comparison of results, figure's 4,8,12 and 16 it can be concluded that at higher flow rates of 831/hr the heat exchanger effectiveness of 0.7 is appropriate. Similarly comparing figures 3,7,11 and 15 it can be observed that is appropriate to use heat exchanger effectiveness of 0.7 for flow rate of 62 1/hr. Comparing figures 2,6,10 and 14 which are drawn for mass flow rate of 50 1/hr, the difference between theoretical and experimental results are large at lower effectiveness values of 0.7 and 0.8. Therefore it is appropriate to assume effectiveness of heat exchanger between 0.9 and 1.0 for flow rates of 501trs/hr and less. The comparison of the results at mass flow rate 37 1/hr, figure's 1, 5, 9 and 13. The temperature difference between theoretical and experimental values is higher with effectiveness value of 0.7 and 0.8. That is 37 1/hr mass flow rate it is appropriate to use effectiveness values of 0.9 to 1.0.

Conclusions: A simple mathematical model for the energy transfer in a solar hot water tank fitted with a heat exchanger is developed. The model is able to estimate precisely the energy transfer with in the tank under varying mass flow rates of collector fluid and service water. The model is also used for evaluating the effectiveness of a given heat exchanger. Using the experimental setup fabricated the heat exchanger effectiveness is evaluated at difference mass flow rates. The results indicate that at higher flow rates of 62 and 831/hr, the heat exchanger effectiveness is found to be around 0.7 and at low flow rates the value lie around 0.9 reaching unity for very low flow rates of 32 1/hr.

Nomenclature:

A : Surface area of the tank, m^2
a : Surface area of the heat exchanger, m^2
C_p : Specific heat exchanger.
h : Heat transfer coefficient, W/m^2 K
m_f : Mass flow rate of hot fluid, Kg/s
m_d : Mass flow rate of cold fluid, Kg/s
T : Temperature of the fluid in the tank, °C
T_a : Ambient temperature, °C
T_I : Inlet temperature of cold fluid, °C
T^I : Initial temperature, °C
t :Temperature of fluid in the heat exchanger, °C.
U : Overall heat transfer coefficient, W/m^2K.
V : Volume, m^3
ρ : Density, kg/m^3.
$\Delta\theta$: Time interval, s.

Suffixes:

i = 1, 2, 3&4 represents sections with respect to tank.
f,i = 2,3&4 represents sections with respect to heat exchanger.
w = Tank with fluid

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effectiveness = 0.7

Figure 1 m = 37 1/hr

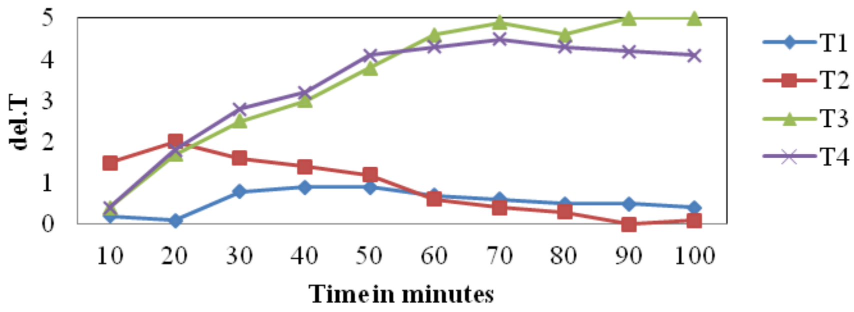


Figure 2 m = 50 1/hr

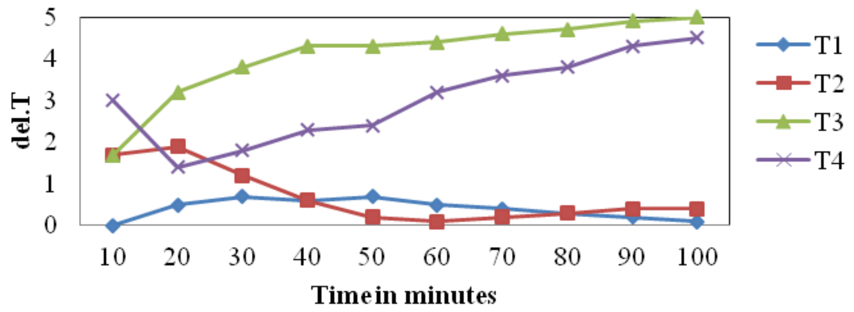


Figure 3 m = 62 1/hr

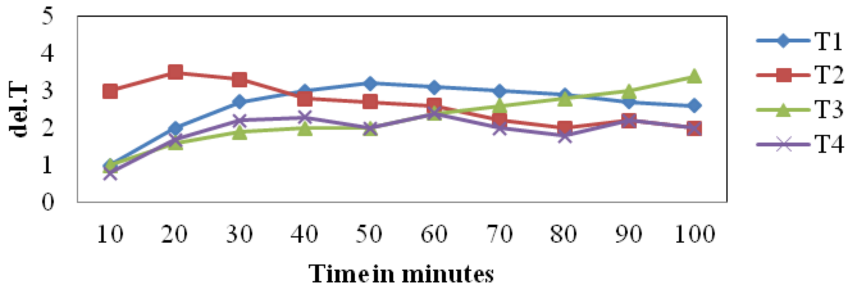
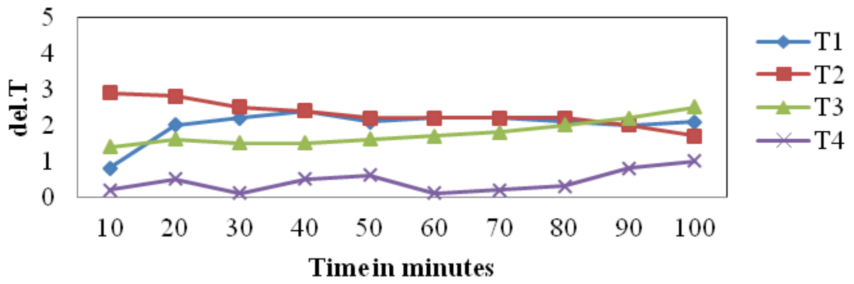


Figure 4 m = 83 1/hr



del. T = Temperature difference between theoretical and experimental values

effectiveness = 0.8

Figure 5 m = 37 ltrs/hr

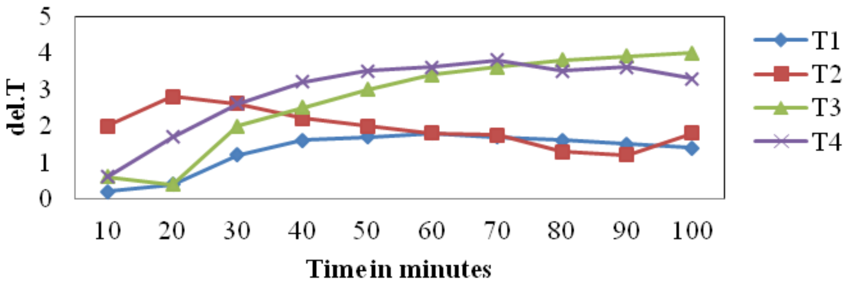


Figure 6 m = 50 1/hr

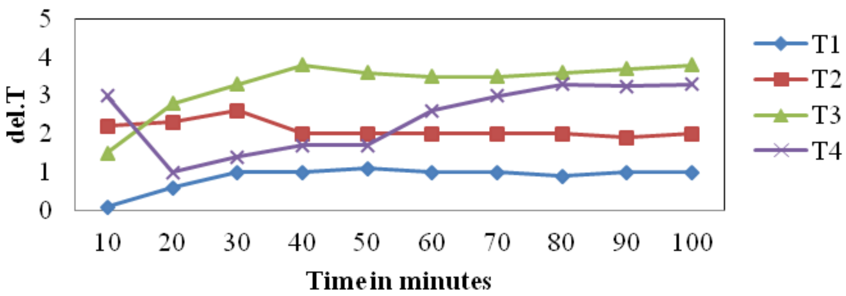


Figure 7 m = 62 1/hr

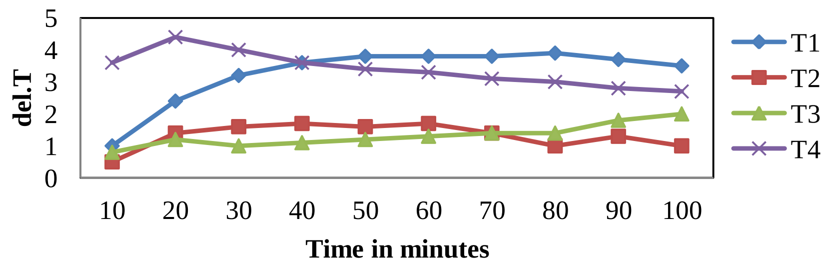
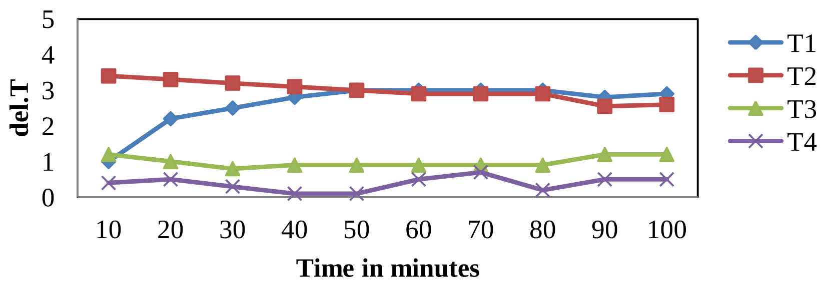


Figure 8 m = 83 l/hr



del. T = Temperature difference between theoretical and experimental values

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effectiveness = 0.9

Figure 9 m = 37 1/hr

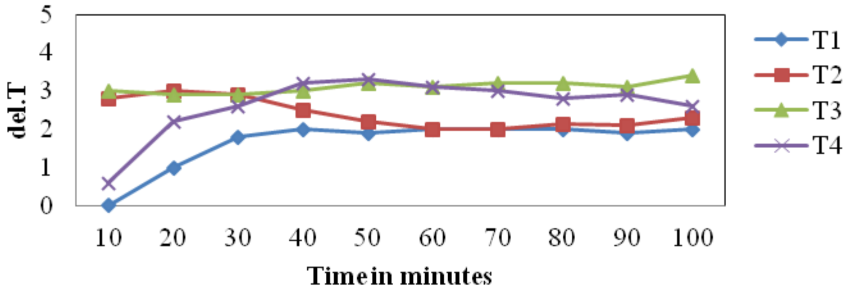


Figure 10 m = 50 1/hr

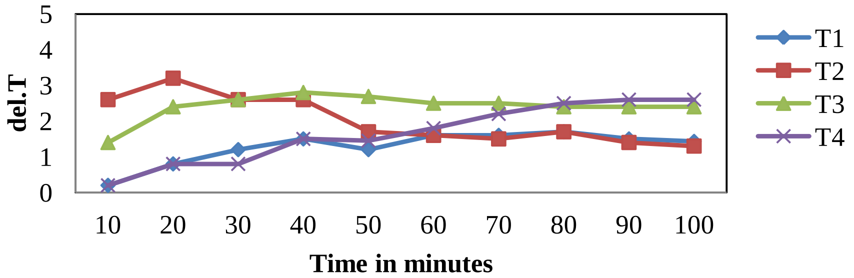


Figure 11 m = 62 1/hr

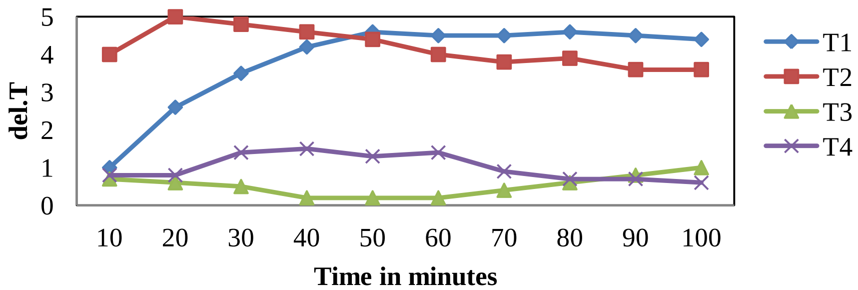
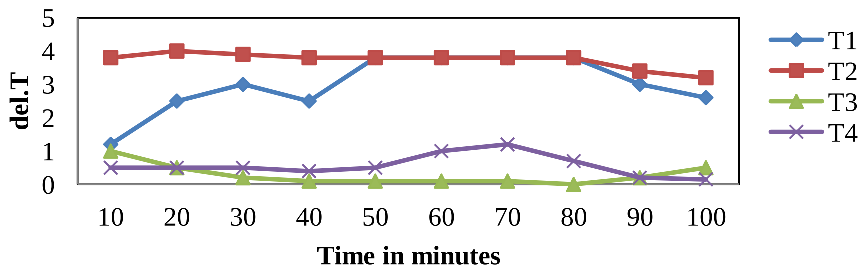


Figure 12 m = 83 1/hr



del. T = Temperature difference between theoretical and experimental values

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effectiveness = 1.0

Figure 13 m = 37 1/hr

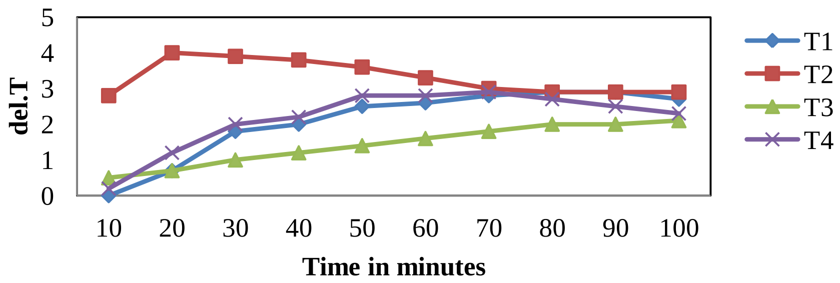


Figure 14 m = 50 1/hr

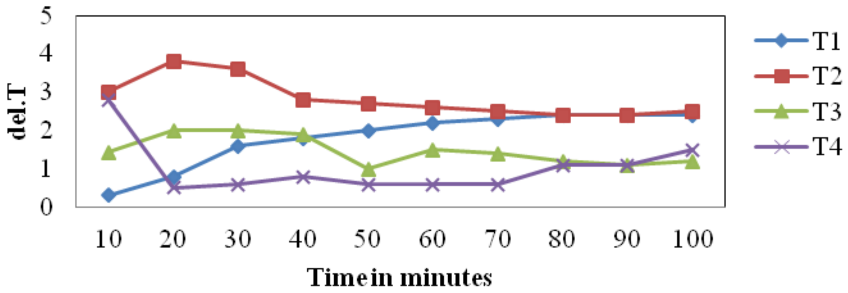


Figure 15 m = 62 1/hr

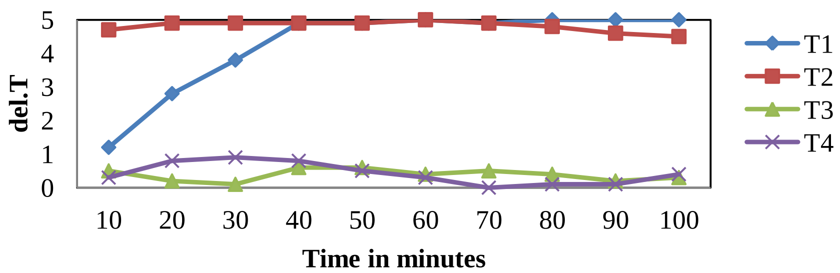
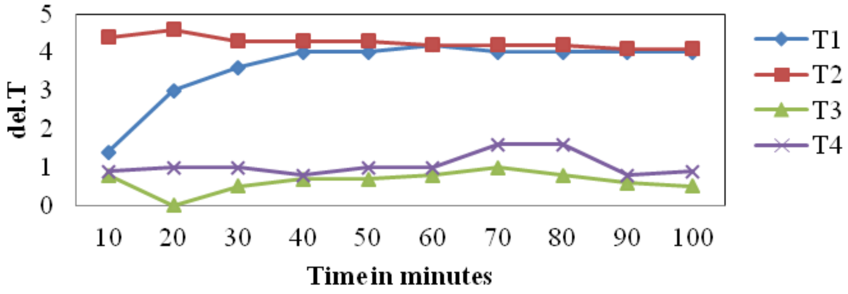


Figure 16 m = 83 1/hr



01del. T = Temperature difference between theoretical and experimental values