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Hierarchical Importance Indices Based Approach for Reliability Redundancy Optimization of Flow Networks

Kumar Pardeep

Department of Instrumentation,
Kurukshetra University, Kurukshetra, – 136 119, India
E-mail: pardeepk@kuk.ac.in

Abstract. In flow networks, it is assumed that a reliability model representing telecommunications networks is independent of topological information, but depends on traffic path attributes like delay, reliability and capacity etc.. The performance of such networks from quality of service point of view is the measure of its flow capacity which can satisfy the customers demand. To design such flow networks, hierarchical importance indices based approach for reliability redundancy optimization using composite performance measure integrating reliability and capacity has been proposed. The method utilizes cardinality and other hierarchical importance indices based criterion in selecting flow paths and backup paths to optimize them. The algorithm is reasonably efficient due to reduced computation work even for large telecommunication networks.

Keywords: flow networks; capacity; telecommunication networks; heuristics.

1 Introduction

When a component is considered relatively more important in comparison to other for calculating system reliability, it is called an importance measure index. Many workers [1-5] have applied different hierarchical importance measures such as cutsets and pathsets criticality, Birnbaum importance, component importance, optimal assignment, structural importance and cardinality of pathsets, cutsets and subsystems etc. for solving reliability redundancy optimization problems of general systems. These importance measures are used to devise heuristics for optimal redundancy assignment like more important component is preferred over the less important component. The reliability redundancy optimization of flow networks is not only a function of network reliability but also depends on load carrying capacity of each node and link of the network. The modern flow networks like computer networks, telecommunication networks, transportation systems, electrical power transmission networks, internet etc. are mostly linked with the performance. The performance of such networks is the measure of maximum flow capacity per unit time. Therefore, some researchers [6-9] have proposed improved models (termed as capacity related reliability models) to represent performance degradation.

However, in modern telecommunication networks or transport systems all the flow paths of a network are never active for transportation of flow from source to destination because the selection of flow paths to transport flow are decided by routing mechanism and logical links assigned in physical layer. Therefore, the selection of specific routing paths out of various possibilities is done on certain attributes like reliability, performance, cost and quality etc.. Therefore, capacity related reliability (CRR) model must be modified incorporating attributes of routing paths and logical links assigned in physical layer. In the following sections a novel approach considering the above attributes and combining them with hierarchical importance indices such as cardinality of pathsets and cutsets, disjoint paths, the cardinality of subsystems and their flow capacity for the reliability redundancy optimization of flow networks using composite performance measure (CPM) integrating reliability and capacity has been proposed [10-11].

The proposed method is capable of addressing the ultrahigh reliability requirements of flow networks efficiently even for large telecommunication networks. Efficiency of the method is attributed to less computation work on account of reduced number of paths of the network considered for optimization.

2 Composite Performance Measure

A path is a sequence of arcs and nodes connecting a source to a sink. All the arcs and nodes of network have its own attributes like delay, reliability and capacity etc.. From the quality and performance point of view, measurement of the transmission ability of a network to meet the customers demand is very important [12]. When a given amount of flow is required to be transmitted through a flow network, it is desirable to have optimized network reliability to carry the desired flow. The capacity of each arc (the maximum flow passing the arc per unit time) has two levels, 0 and/or a positive integer value. The system reliability is the probability that the maximum flow through the network between the source and the sink is not less than the demand [10-15]. The earlier optimization methods were based on presumption that any amount of flow can be passed through any node or path. However, this assumption is neither valid nor justifiable for real life flow networks as links and nodes can pass only limited amount of flow. Hence, reliability under flow constraint is a more realistic performance measure for flow networks. A concept of weighted reliability was introduced by Pahuja (2004)[12], which requires that all the successful states qualifying the connectivity measure of the network be enumerated and the probability of each success state is evaluated and multiplied by the normalized weight to find out the composite performance of flow networks. The proposed algorithm utilizes the concept of weighted reliability to form composite performance measure (CPM) to optimize the capacity related reliability (CRR) of flow networks.

2.1 Notation

$a_l(X)$	Sensitivity factor of l^{th} minimal path set
$b_i(x_i)$	Subsystem selection factor for i^{th} subsystem with x_i components
C_j	Total amount of resource j available
$g^j{}_i(x_i)$	Amount of resources consumed for j^{th} constraint in subsystem-i with x_i components
$c_{ji}(x_i)$	Cost of subsystem i for j^{th} constraint with x_i components
cg	Number of different cardinality groups.
$cg_a(x_i)$	a^{th} cardinality group, a = 1, 2,..., d.
$h(.)$	Function yielding system reliability; dependent on number of subsystems (n) and configuration of subsystems
k	Number of constraints, $j = 1, 2,\ldots, k$
$L(x)$	$(L_{x1}, L_{x2},\ldots, L_{xn})$ lower limit of each subsystem i.
m	Number of main minimal path sets, $l = 1, 2,., m$
n	Number of subsystems, $i = 1, 2,\ldots, n$
P_l	l^{th} minimal path set of the system
Ps	$(l^1,l^2,\ldots,l^{min})$: priority vector s.t. l^1 and l^{min} are the number of minimal path sets arranged in decreasing order of path selection parameter $a_l(X)$.
$Q_i(x_i)$	Unreliability of subsystem i with x_i components.
r_i	Reliability of a component at subsystem i.
$R_i(x_i)$	Reliability of subsystem i with x_i components.
R_r	Residual resources [total resource available (C_j) - resources consumed ($\sum g_i^j x_i$)]
R_s(X)	System reliability
$S(x)$	Set of variables that have been used as key-elements in a given decomposed expressions
$U(x)$	$(U_{x1}, U_{x2},\ldots, U_{xn})$ upper limit of each of subsystem i.
x^*	Optimal solution

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x_i	Number of components in subsystem i; $i = 1,2,\ldots.n$
X	A vector $(x_1,\ldots\ldots\ldots x_n)$
Y	Finite set of traffic paths
Z	Finite set of cuts of the network
ΔR_i	Increment in i^{th} stage reliability when a unit is added in parallel to the i^{th} stage

2.2 Assumptions

Following are the assumptions for the rest of the sections:

1. The system and all its subsystems are coherent.
2. Subsystem structures (other than coherence) are not restricted.
3. The networks are modelled with the help of graphs, the paths (ordered pair of arcs and the members of the ordered pair are reliability and capacity respectively) where in are assigned as the weight of each link.
4. Both the system and components are bi-state, either operative or not.
5. All component states are mutually and statistically independent.
6. All constraints are separable and additive among components.
7. Each constraint is an increasing function of x_i for each subsystem.
8. Redundant components cannot cross subsystem boundaries.
9. Components are functionally interchangeable.

2.3 New Model

In simple networks the reliability is defined as the probability of connection between source and sink with the assumption that each node and arc of the network is capable of transporting any amount of flow. However, the performance of flow networks is considered as the maximum flow capacity and one should consider immediate states of communication that manifest as performance degradation [16] by considering a simple case illustrated in Fig.1. The network has two different flow paths having flow capacity 200Mbps and 100 Mbps respectively. A total flow of 300 Mbps is possible in normal states. However, the performance of the network degrades as either of the path flow 1 or flow 2 is not assured. During the degraded state the total flow of the network reduces to either 200 Mbps or 100 Mbps from 300 Mbps in case of failure of flow 1 or flow 2 respectively. The reliability model ignores such degradation [7-10]. To address such realistic issues improved capacity related reliability model was introduced [7-10]. It assumes that each node and link has a failure probability, and capacity. That assumption is used to define the reliability of the telecommunications network as the probabilistic distributions or the expectations of 'maximum flow' between source and destination nodes [16].

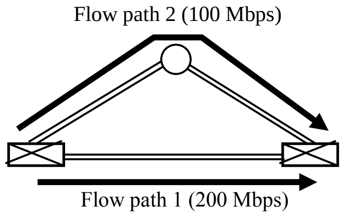

Fig.1 A simple flow network

However, in present days modern flow networks the situation discussed above is not justifiable as all the arcs are not simultaneously connected to carry flow from source to sink. The selection of paths to transport flow is decided by routing mechanism and logical links assigned in physical layer. Thus in practical systems the entire pathsets are never utilized for transfer of information [16]. The flow is transmitted through the main path(s) only and in case of failure of main path(s), backup path(s) takes over the task of main path(s).

2.4 Composite Performance Measure (CPM)

The weighted reliability measure [10,11,13] i.e. composite performance measure (CPM), integrating both capacity and reliability may be stated as:

$$\mathrm{CPM} = \sum_{i \in S(x)} \omega t_i \, R_i \tag{1}$$

Where ωt_i is the normalized weight and is defined as: $\omega t_i = Cap_i / Cap_{max}$

i.e. the ratio of capacity in the i^{th} state to the maximum capacity (Cap_{max}) of the system and R_i probability of the system being in state S_i and is computed as:

$$R_i = P_r\{S_i\} = \prod_{j/S_{ij}=1} p_j \times \prod_{k/S_{ik}=0} q_k \tag{2}$$

2.5 Capacity Functions of Networks

The capacity function of different arcs connected in parallel is (Ramirez et al. 2005):

$$C(X)_{Par} = \sum_{i \in x} Cap_i \tag{3}$$

and the capacity function of different arcs connected in series is:

$$C(X)_{Ser} = \min\{Cap_i\} \tag{4}$$

The rules for connecting series and parallel arcs to integrate capacity and reliability to give composite performance measure are expressed as:

$$CR(X)_{Ser} = \{\min_{i \in x} Cap_i\} \prod_{i=1}^{n} r_i \tag{5}$$

$$CR(X)_{Par} = \sum_{i=1}^{n} Cap_i \,.\, \bigcup_{i=1}^{n} r_i \tag{6}$$

CPM for series and parallel networks can be defined as:

$$\text{CPM}_{Par} = CR(X)_{Par} / Cap_{max} \tag{7}$$

and $$\text{CPM}_{Ser} = \text{CR(X)}_{Ser} / \text{Cap}_{max} \tag{8}$$

3 Problem Formulation and Heuristic Method

3.1 Problem Formulation

The general constrained redundancy optimization problem in complex systems can be reduced to the following integer programming problem [17-21]:

Maximise $$R_s(X) = h(R_1(x_1),..., R_n(x_n)) \quad (9)$$

subject to $$\sum_{i=1}^{n} g_i^j(x_i) \le C_j, j = 1, 2, \ldots, k \quad (10)$$

and $$1 \le x_i \le U_{xi}, \quad i = 1, 2, \ldots, n.$$

3.2 Proposed Heuristic Method

As discussed in Sec. 2.3 above, the main and backup paths of flow networked are decided by the routing mechanism. Hence, the proposed algorithm first ascertain the main path(s) and back up path(s) using hierarchical importance indices like cardinality of pathsets and cutsets, disjoint paths, capacity etc. and then optimize the main flow path and backup paths using a heuristic method. The cardinality is defined as number of elements in a mathematical set. On the basis of this definition the cardinality of a subsystem is defined as frequency of its occurrence in all pathsets and cutsets of the network whereas, the cardinality of a pathset is the number of subsystems contained in the pathset. The algorithm first combine the cardinality of different pathsets and cutsets, disjoints paths and capacity of the node and arcs to form different groups of subsystems to be optimized on priority basis using three phases. Unlike existing heuristics, a switching criterion has been applied to switch from CRR optimization of highest priority group to lower priority group. Using this approach network designer can utilize generally limited resources more efficiently [10,11,21-25] and also can meet the performance goal of the network. The three phases of the proposed method for optimization are:

In the first phase, the path sets having minimum cardinality are given highest priority and the path sets having maximum cardinality are given least priority then all the subsystems having maximum cardinality are found. All highest priority minimal pathsets containing the highest cardinality subsystems form first group. If highest priority minimal path sets are different than that of containing highest cardinality subsystem(s), these path sets along with the path sets containing highest cardinality subsystems are grouped together in the first group. Using this criterion all subsystems of the network are arranged in different groups with decreasing priority importance.

In the second phase highest selection-factor $b_i(x_i)$ is computed for the chosen priority group using

$$b_i(x_i) = \frac{\Delta R_i}{\sum_{j=1}^{k}(g_i^j(x_i)/kC_j)}, \tag{11}$$

for each $i \in cg_a(x_i)$

where $\Delta R_i = R_i(x_i) - R_i(x_i - 1)$ (12)

$cg_a(x_i)$ is the a^{th} cardinality group such that a = 1, 2,..., d and

$cg_a(x_i)$ is the a^{th} cardinality group such that a = 1, 2,..., d.

In the third phase, a redundant parallel subsystem is added to the unsaturated subsystem belonging to the chosen $cg_a(x_i)$ with highest selection factor and also satisfying the capacity requirements as well as maximum reliability need of each path. The three phases are repeated till optimal solution is reached.

After obtaining the optimal solution for the network; calculate the composite performance measure for each subsystem of the network. Then evaluate the system reliability using the CPM of the each subsystem. Novelty of the method is that unlike other existing heuristic for complex systems it requires only one selection factor instead two. To determine the total capacity, if system is working normally then capacity for each primary path is ensured otherwise the flow capacity of the primary path is the minimum, and is the summation of the capacities of reserved backup paths that are working. Finally, total capacity is computed by summing the ensured capacities.

3.3 Steps of the Proposed Method

Step1: Find all path sets and cut sets for the network then using cardinality approach:

I. The path sets having minimum cardinality are given highest priority and the path sets having maximum cardinality are given least priority then all the subsystems having maximum cardinality are found.

II. All highest priority minimal path sets containing the highest cardinality subsystems form first group. If highest priority minimal path sets are different than that of containing highest cardinality subsystem(s), these path sets along with the path sets containing highest cardinality subsystems are grouped together in the first group.

III. Using this criterion all subsystems of the network are arranged in different *d* groups with decreasing priority importance.

Step2: Let a = 1; from a = 1, 2,…, *d*.

Step3: Let x_i = 1 for all *i; i* = 1, 2,…, *n*.

Step4: Compute $b_i(x_i)$ using (11) for each subsystem belonging to selected cardinality group cg_a, find $i^* \in cg_a(x_i)$ such that $b_{i^*}(x_i)$ $= \max[b_i(x_i)]$.

Step5: Check, if by adding one redundant subsystem to unsaturated subsystem i^* :

I. no constraints are violated and reliability of the subsystem satisfies the stopping criterion ΔR_i > .001, then check if the capacity of the subsystem is <= required flow of network, add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$, and go to step 4.

II. if at least one constraint is exactly satisfied and other are not violated, also and reliability of the subsystem satisfies the stopping criterion ΔR_i > .001, then check if the capacity of the subsystem is <= required flow of network, then add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$. The $x^* = X$ is the optimal solution. Go to step 6.

III. if at least one constraint is violated, then remove subsystem *i** from further consideration and consider the next subsystem having maximum $b_{i^*}(x_i)$ value and go to step 5.

IV. if all $i^* \in cg_a(x_i)$ have now been exhausted, check if a < *d*; then $a = a + 1$ and go to step 4;

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V. if $a \geq d$ then $x^* = X$ is the optimal solution, go to step 6.

Step6: Evaluate the composite performance measure (CPM) for each subsystem of the network.

Step7: Evaluate the system reliability using the CPM of the each subsystem.

4 Computation and Results

To illustrate the performance of the proposed algorithm a network having six arcs $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and five minimal path sets $\{y_1, y_2, y_3, y_4, y_5\}$ as shown in the Figure 1 is considered and solved for capacity related redundancy reliability optimization using CPM [7,8]. System reliability is determined using Bayes method. The network shown in Figure 2 is a bench mark problem, considered by Hayashi & Abe (2008) [16].

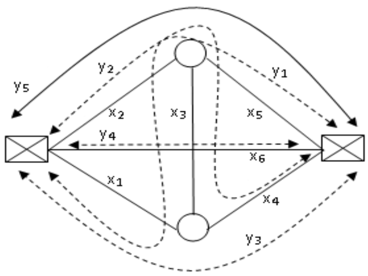

Fig. 2 Illustration Network

Using Baye's method, the Reliability of the system can be expressed as:

$$R_s(X) = R_3\,[1- Q_6\,\{1-(1-Q_1Q_2)(1-Q_4Q_5)\}] + Q_3[1-(1-R_2R_5)(1-R_1R_4)]*Q6 \quad (13)$$

The problem is solved for data given in Table 1. For this first determine all the simple minimal pathsets and cutsets of the Network:

$\boldsymbol{Y} = \{\boldsymbol{y_1, y_2, y_3, y_4, y_5}\}$

where $\boldsymbol{y_1} = \{1, 3, 5\}$, $\boldsymbol{y_2} = \{2, 3, 4\}$, $\boldsymbol{y_3} = \{1, 4\}$, $\boldsymbol{y_4} = \{6\}$, $\boldsymbol{y_5} = \{2, 5\}$

$\boldsymbol{Z} = \{\boldsymbol{z_1, z_2, z_3, z_4}\}$

where $\boldsymbol{z_1} = \{1, 2, 6\}$, $\boldsymbol{z_2} =\{4, 5, 6\}$, $\boldsymbol{z_3} =\{2, 3, 4, 6\}$, $\boldsymbol{z_4} =\{1, 3, 5, 6\}$

and then as discussed in Sec. 3.3 above, on the basis of cardinality of pathsets and cutsets of the network main flow paths and backup flow paths are ascertained as { $\boldsymbol{y_2}$, $\boldsymbol{y_3}$, $\boldsymbol{y_4}$ } and { $\boldsymbol{y_1}$, $\boldsymbol{y_5}$ } respectively. Then all the subsystems of the network are arranged in two groups $cg_1(x_i)$ = {6} having cardinality 5 and $cg_2(x_i)$ = {1, 2, 3, 4, 5} of cardinality 4. The general problem of constrained reliability redundancy allocation has been solved using the steps discussed in Sec. 3.3. The problem is solved by considering that every flow path has a capacity of 100. The total flow through network at any time should not be less than 200 in any case. The proposed algorithm gives the optimal solution (2, 2, 2, 2, 1, 3) with system reliability R_s = 0.9578, the optimized subsystem reliability probability R_i and unreliability probabilities Q_i are shown in Table 2.

Table 1 Data for Fig. 1

i	1	2	3	4	5	6
r_i	0.70	0.75	0.8	0.85	0.70	0.90
c_{1i}	2	3	2	3	1	3
C_1	30					

Table 2 Optimized subsystem reliability/unreliability for Fig. 1

i	x_1	x_2	x_3	x_4	x_5	x_6
X^*	2	2	2	2	1	3
R_i	0.9100	0.9375	0.9600	0.9775	0.7000	0.9990
Q_i	0.090	0.0625	0.0400	0.0225	0.3000	0.0009

The capacity of each subsystem of the flow path is taken as 100 and the capacity of flow paths of the network is determined using (3) of proposed approach as:

$$\left.\begin{aligned}
Cap\{y_1\} &= \min Cap\{1,3,5\} \\
&= \min Cap\{2*100, 2*100, 100\} = 100 \\
Cap\{y_2\} &= \min Cap\{2,3,4\} \\
&= \min Cap\{2*100, 2*100, 2*100\} = 200 \\
Cap\{y_3\} &= \min Cap\{1,4\} \\
&= \min Cap\{2*100, 2*100\} = 200 \\
Cap\{y_4\} &= \min Cap\{6\} = \min Cap\{3*100\} = 300 \\
Cap\{y_5\} &= \min Cap\{2,5\} = \min Cap\{2*100, 100\} = 100
\end{aligned}\right\} \qquad (14)$$

Next the CPM expressions (15-20) are derived using (7 and 8). The value for CPM for an assumed flow of 200 is supposed to pass through the flow path and it comes out to be 1.0000.

$$\text{CPM}_{y1} = \frac{\min Cap_i}{Cap_{\max}} [R_1 R_3 R_5] \quad (15)$$

$$= (100/200) * 0.91 * .96 * .7 = 0.3058$$

$$\text{CPM}_{y2} = \frac{\min Cap_i}{Cap_{\max}} [R_2 R_3 R_4] \quad (16)$$

$$= (200/200) * 0.9375 * .96 * .9775$$
$$= 0.8797$$

$$\text{CPM}_{y3} = \frac{\min Cap_i}{Cap_{\max}} [R_1 R_4] \quad (17)$$

$$= (200/200) * 0.91 * .9775 = 0.8895$$

$$\text{CPM}_{y4} = \frac{\min Cap_i}{Cap_{\max}} [R_6] \quad (18)$$

$$= (300/200) * 0.9990$$
$$= (1.5) * 0.9990 \text{ and } \quad \text{as } 0 \le (\min Cap_i / Cap_{\max}) \le 1$$

so $= 1 * 0.9990 = 0.9990$

$$\text{CPM}_{y5} = \frac{\min Cap_i}{Cap_{\max}} [R_2 R_5] \quad (19)$$

$$= (100/200) * 0.90 * .70 = 0.3281$$

Composite performance measure integrating the reliability with capacity is calculated as:

$$\text{CPM}_{\text{Network}} = 1 - (1 - \text{CPM}_{y1}) * (1 - \text{CPM}_{y2}) * (1 - \text{CPM}_{y3}) *$$
$$(1 - \text{CPM}_{y4}) * (1 - \text{CPM}_{y5})$$
$$= 1 - (1 - 0.3058)(1 - 0.8787)$$
$$(1 - 0.8895)(1 - 0.9990)(1 - 0.3281)$$
$$\cong 1.0000 \quad (20)$$

The above result shows that proposed method is capable of optimizing the flow network to transport the desired capacity through the network with highest reliability. However, the selection of main paths and backup paths will affect the quality of composite performance measure. Hence the proper choice of these paths may be done using cardinality criteria [23] or any other hierarchical measures of importance.

5 Conclusions

This paper presented a new model for designing reliable flow networks capable of transmitting required flow. The proposed algorithm utilizes the concept of main and backup flow paths. The choice of backup and flow paths is application specific and paths with minimum cardinality may be selected as main path and disjoint paths can be the backup paths. The numerical example demonstrates that the proposed algorithm is fast for designing large, reliable telecommunications networks because the task of optimization is reduced, as only few paths are selected as main paths.

References

1. **W. Wang, J. Loman,** and **P. Vassiliou,** "Reliability Importance of components in a complex system", *RAM* , pp. 6-10, 2004.
2. **F. K. Hwang,** "A Hierarchy of Importance Indices", *IEEE Trans. Reliability*, vol. 54 (1), pp. 169-172, 2005.
3. **C. Ha, and W. Kuo**, "Multi-Path Heuristic for Redundancy Allocation: The Tree Heuristic", IEEE Trans. Reliability, vol. 55(1), pp.37-43, 2006.
4. **J. E. Ramirez-Marquez** and **D. W. Coit, "**Composite importance measures for multi-state systems with multi-state components", *IEEE Trans. Reliability*, vol. 54(3), pp 517-529, 2005
5. **J. E. Ramirez-Marquez, C. M. Rocco, B. A. Gebre, D. W. Coit,** and **M. Tortorella,** "New insights on multi-state component criticality and importance", *Reliability Engineering and System Safety*, vol. 91, pp. 894–904, 2006.
6. **H. Nagamochi and T. Ibaraki**, "On Onaga's upper bounds on the mean values of probabilistic maximum flows," *IEEE Trans. Reliability*, vol. R-41, pp. 225–229, 1992.
7. **P. K. Varshney, A. R. Joshi, and P. L. Chang**, "Reliability modelling and performance evaluation of variable link-capacity networks," *IEEE Trans. Reliability*, vol. R-43, pp. 378–382, 1994.
8. **Y. Chan, E. Yim, and A. Marsh**, "Exact & approximate improvement to the throughput of a stochastic network," *IEEE Trans. Reliability*, vol. R-46, pp. 473–486, 1997.
9. **S. Soh and S. Rai**, "An efficient cutset approach for evaluating communication- network reliability with heterogeneous link-capacities," *IEEE Trans. Reliability*, vol. R-54, pp. 133–144, 2005.
10. **P. Kumar, D. K. Chaturvedi** and **G. L. Pahuja,** "A Heuristic Method for Reliability Redundancy Optimization of Flow Networks", *Reliability Theory and Applications*, vol. 1, #02(25), pp.69-77, 2012.
11. **P. Kumar,** "Cardinality Based Approach for Reliability Redundancy Optimization of Flow Networks", *Reliability Theory and Applications*, accepted for publication in Dec. 2012.
12. **Y. K. Lin**, "Reliability of a computer network in case capacity weight varying with arcs, nodes and types of commodity", *Reliability Engineering and System Safety*, vol. 2(5), pp. 1-7, 2006.
13. **G. L. Pahuja,** "*Reliability Evaluation and Optimization Recent and New Approaches",* Ph.D. thesis, Kurukshetra University, Kurukshetra, 2004.

14. **Lin Y. K.**, "Reliability evaluation for an information network with node failure under cost constraint," *IEEE Trans. Systems, Man and Cybernetics-Part A: Systems and Humans*, vol. 37, no. 2, pp. 180–188, 2007.
15. **Y. K. Lin,** "On a multi-commodity stochastic-flow network with unreliable nodes subject to budget constraint," *European Journal of Operational Research*, vol. 176, no. 1, pp. 347–360, 2007.
16. **M. Hayashi** and **T. Abe**, "Evaluating Reliability of Telecommunications Networks Using Traffic Path Information", *IEEE Trans. Reliability*, vol. 57(2), pp. 283-294, 2008.
17. **J. A. Abraham**, "An improved algorithm for network reliability," *IEEE Trans. Reliability*, vol. R-28, pp. 58–61, 1979.
18. **A. Satyanarayana and R. K. Wood**, "A linear time algorithm for computing K-terminal reliability in series-parallel networks," *SIAM J. Computing*, vol. 14, pp. 818–832, 1985.
19. **W. G. Schneeweiss**, *Boolean Functions With Engineering Applications and Computer Programs*. : Springer-Verlag, 1989.
20. **O. R. Theologou and J. G. Carlier**, "Factoring & reductions for networks with imperfect vertices," *IEEE Trans. Reliability*, vol. R-40, pp. 210–217, 1991.
21. **S. Y. Kuo, F. M. Yeh, and H. Y. Lin**, "Efficient and exact reliability evaluation for networks with imperfect vertices," *IEEE Trans. Reliability*, vol. R-56, pp. 288–300, 2007.
22. **P. Kumar, D. K. Chaturvedi** and **G. L. Pahuja, "**Heuristic algorithm for constrained redundancy reliability optimization and performance evaluation," *Proc. IMechE, Part O: J. Risk and Reliability*, vol. **223** (4), 381-386, 2009.
23. **P. Kumar, D. K. Chaturvedi** and **G. L. Pahuja,** "Heuristic Methods for Solving Redundancy Allocation in Complex Systems", *Int. J. of Reliability and Safety*, vol. 4, No. 2/3, pp. 285-298, 2010.
24. **P. Kumar, D. K. Chaturvedi** and **G. L. Pahuja,** "An efficient heuristic algorithm for determining optimal redundancy allocation of complex networks", *Reliability Theory and Applications*, vol. 3(18), pp.15-28, 2010.
25. **P. Kumar, D. K. Chaturvedi** and **G. L. Pahuja,**"Constrained Reliability Redundancy Optimization of Complex Systems using Genetic Algorithm", *MIT Int. J of Electrical and Instrumentation Engineering*, vol. 1, No. 1, pp. 41-48, 2011**.**