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A Comparative Analysis of different Methods for the tuning of PID Controller

Rajeev Kumar[1], Sunil K. Singla[2], Vikram[3]

(Department of Electrical & Instrumentation Engineering, Thapar University, Patiala, 147004, Punjab)

rajeev07ei35@gmail.com[1],sunilksingla2001@gmail.com[2],vikram.ee28@gmail.com[3]

Abstract. A proportional integral derivative (PID) controller is most widely used to control industrial processes. Tuning a PID controller is an important task for obtaining the desired closed loop specifications (rise time, settling time, peak time, overshoot and steady state error). This paper presents different PID tuning formulas for a third order process. They are based on the knowledge of the ultimate gain, ultimate period and minimization of integral squared error (ISE) and integral absolute error (IAE).The performance of various tuning methods has been compared by applying a step input to the given process. Simulation results show that tuning a PID controller with Ziegler Nichols (ZN) tuning method results in less rise time (t_r), peak time (t_P), and integral squared error (ISE). The Relay Auto tuning method is applicable when less ISE is required while Modulus Optimum (MO) tuning method is applicable when less settling time (t_S) and less overshoot is required and Computational Optimization (CO) method is helpful when the desired closed loop specifications are decided by the designer. The robustness factors gain margin (GM), phase margin (PM), gain crossover frequency, phase crossover frequency and stability are considered. The proposed approach is implemented in MATLAB.

Keywords: PID, ZN, Relay Auto tuning, MO, CO, IAE, ISE.

1 Introduction

A PID controller is most commonly used in industrial control systems. PID controller has three principal control effects. The proportional (P) action gives a change in the input (manipulated variable) directly proportional to the error signal. The integral (I) action gives a change in the input proportional to the integral of error, and its main purpose is to eliminate offset. Whereas the derivative (D) action is used to speed up the response or to stabilize the system and it gives a change in the input proportional to the derivative of the error signal. The overall controller output is the sum of the contributions from these three terms [1]. The general form of the PID controller is given below in equation (1) [2].

$$u(t)=K_P e(t)+\frac{1}{T_I}\int_0^t e(t)dt+T_D\frac{de(t)}{d(t)} \quad (1)$$

Fig. 1 shows that PID controller structure in parallel form which is more flexible than series form.

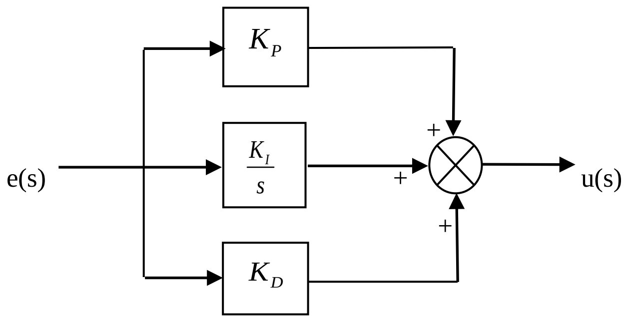

Fig. 1. PID controller structure [3]

Where u(t) and e(t) denote the control and the error signals, respectively, and proportional gain (K_P) integral time (T_I) and derivative time (T_D) are the parameters to be tuned. The goal of PID controller tuning is to determine parameters that meets the closed loop system performance specifications. In this paper various PID tuning formulas for a third order process has been analyzed, based on the knowledge of ultimate gain, ultimate period and minimization of ISE & IAE. In practical applications, the pure derivative action is never used because of “derivative kick” generated in the control signal for a step input, and to the undesirable noise amplification, which is shown in equation (3). It is usually cascaded by a first order low pass filter [4]. In the time domain, the controller transfer function can be expressed in equation (4). The PID controller improves the transient response as well as the steady-state error of the system.

A continuous-time PID controller is given by [5]

$$G_c(s) = \left(K_P+\frac{K_I}{s}+K_D s\right) \quad (2)$$

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$$G_c(s) = \left(K_P + \frac{K_I}{s} + \frac{K_D s}{1 + T_n s} \right) \tag{3}$$

$$G_c(s) = K_P \left(1 + \frac{1}{T_I s} + T_D s \right) \tag{4}$$

Where, K_P is the proportional gain, K_D is the derivative gain K_I is the integral gain, T_D is the derivative time and T_I is the integral time. The derivative term improves the transient response by adding a zero to the open loop plant transfer function. The integrator eliminates error by increasing the system type with additional pole at the origin. Generally, K_P will have the effect of reducing the rise time and it also reduce error but the steady-state error can never be eliminated. For eliminating the steady state error Integral gain K_I can be used, but it will make the transient response worse [5]. The block diagram of closed loop PID controller for a third order process is shown in fig. 2.

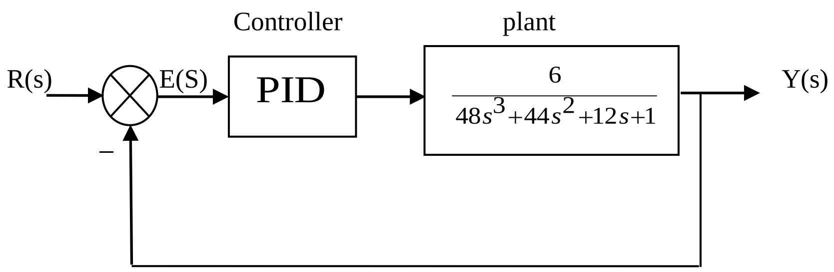

Fig. 2. PID controller for a third order process

2 Tuning methods

Tuning of a controller is a method of determining the parameters of a PID controller for a given system. A PID controller is described by three parameters; K_P , T_I and T_D. Four tuning methods discussed below have been used in this paper.

2.1 Ziegler Nichols Tuning Method

The most popular tuning methodology was proposed by Ziegler and Nichols in 1942 [6]. The closed-loop tuning method requires the determination of the ultimate gain and ultimate period. This can be achieved by adjusting the controller gain till the system undergoes sustained oscillations at the ultimate gain or critical gain (K_u), whilst maintaining the integral time constant at infinity and the derivative time constant at zero. The Ziegler-Nichols tuning method is based on the determination of process inherent characteristics such as the process gain, process time constant and process dead time. These characteristics are used to determine the controller tuning parameters.

Table 1. Tuning parameters for Ziegler Nichols closed loop ultimate gain method [7].

Controller	K_P	T_I	T_D
P	0.5 K_u	-------	--------
PI	0.45 K_u	0.83 P_u	--------
PID	0.6 K_u	0.5 P_u	0.125 P_u

2.2 Relay Auto tuning Method

Relay-based auto tuning is a simple way to tune PID controllers that avoids trial and error, and minimizes the possibility of operating the plant close to the stability limit [8]. Block diagram of simple feedback auto tuning system shown in Fig.3.

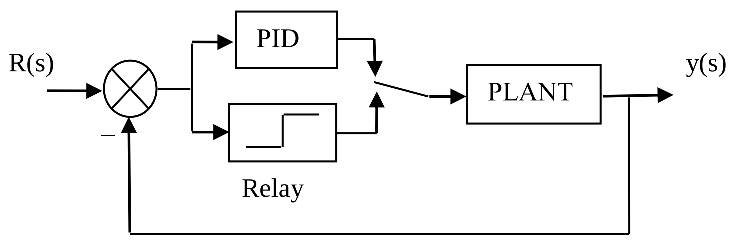

Fig. 3. Block diagram of auto tuning scheme

Auto tuning is based on the idea of using an on/off controller (called a relay controller). Initially, the plant oscillates without a definite pattern around the nominal output value until a definite and repeated output response can be identified. When the desired response pattern has been reached the oscillation period (P_u) and the amplitude (A) of the plant response can be measured and used for PID controller tuning. In fact, the ultimate gain can be computed as:

$$K_u = \frac{4h}{\Pi A} \tag{5}$$

Where, h=amplitude of the PID controller output
A=amplitude of the plant response

2.3 Computational Optimization Method

In this method an optimal set (or optimal sets) of values of PID controller to satisfy the transient response specifications is required to be obtained [9]. The PID controller with computational optimization approach has been shown in Fig.4. The objective is to find the combination of gain 'K' and 'a' such that the closed-loop system will have minimum rise time, settling time, peak time and overshoot. For designing the PID controller first specify the region to search for appropriate K and a. The values of K and a must specify

$$0.4 \leq K \leq 1 \tag{6}$$
$$0.08 \leq a \leq 0.3 \tag{7}$$

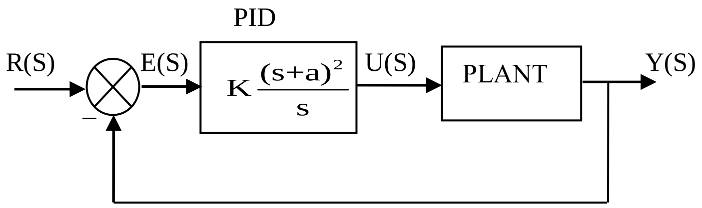

Fig. 4. Design of PID controller with computational optimization approach

2.4 Modulus Optimum

The modulus optimum (MO) method for optimization of regulators can be applied in a wide variety of cases in the control field. Modulus Optimum (MO) method is based on the transfer function of set point $G_{ref}(s)$ [10]. In ideal case the transfer function would be $G_{ref}(s)$ =1, i.e. step response of process variable is equal to set point. In frequency domain it corresponds with condition given in equation (8).

$$G_{ref}(j\omega) = 1 \Rightarrow \left|G_{ref}(j\omega)\right| = A_{ref}(\omega) = 1 \tag{8}$$

This condition cannot be satisfied in reality; however it can be proven that control process ends the fastest when amplitude characteristics $A_{ref}(j\omega)$ will be flat at first and then monotonically decrease. The setting of PID parameters K_P , T_I and T_D by MO method is sorted in the table for practical use and it depends on the type of controlled plant, Table 2.

Table 2. PID Controller's Parameters by MO Method

Model of Controlled plant	K_P	T_I	T_D
$\frac{k}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}$ $T_1 \geq T_2 \geq T_3$	$\frac{T_1}{2kT_3}$	$T_1 + T_2$	$\frac{T_1 T_2}{T_1 + T_2}$

3 Simulation Results

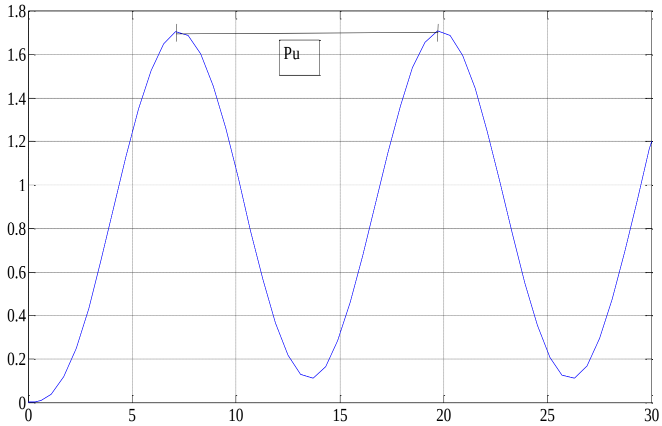

Fig. 5. Steady oscillation illustrating the ultimate period for ZN

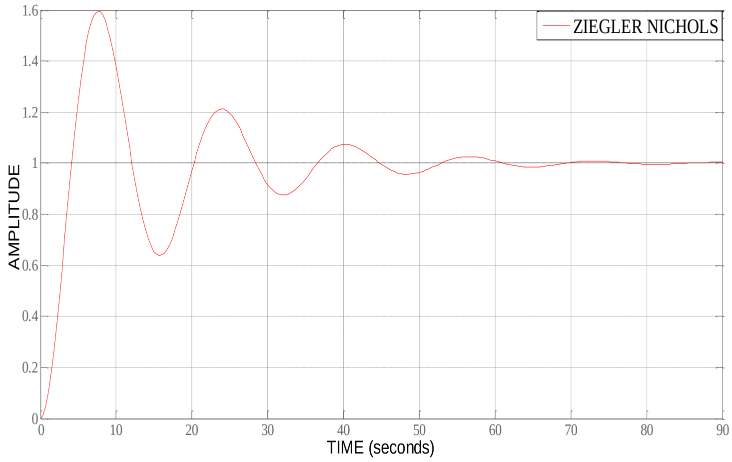

Fig. 6. Unit step response for the plant using ZN method.

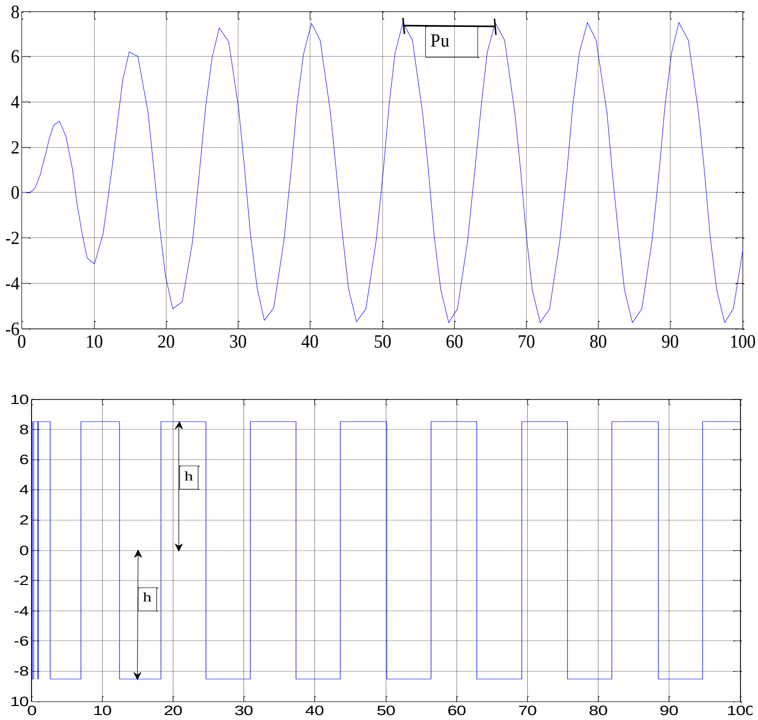

Fig. 7. A plant oscillating under relay feedback with the PID regulator temporarily disabled

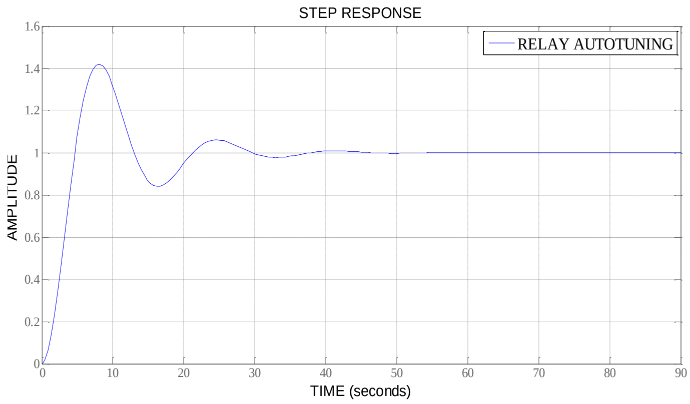

Fig. 8. Unit step response for the plant using Relay auto tuning method

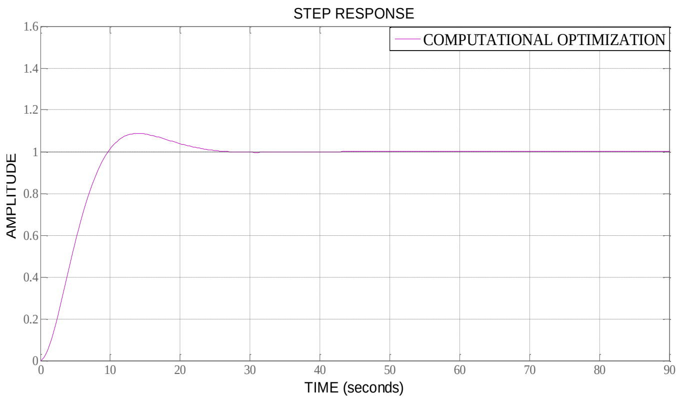

Fig. 9. Unit step response for the plant using Computational Optimization

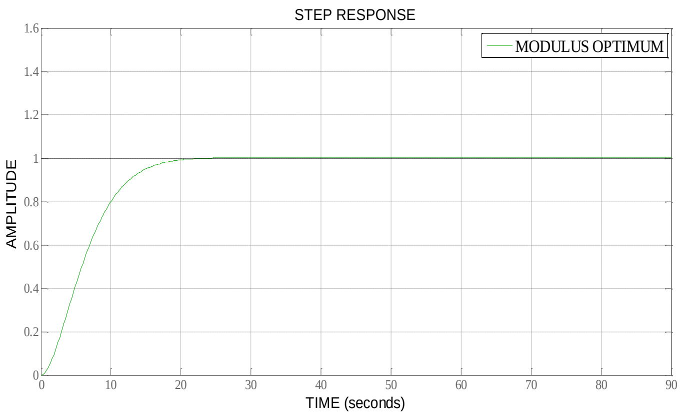

Fig. 10. Unit step response for the plant using Modulus Optimum method

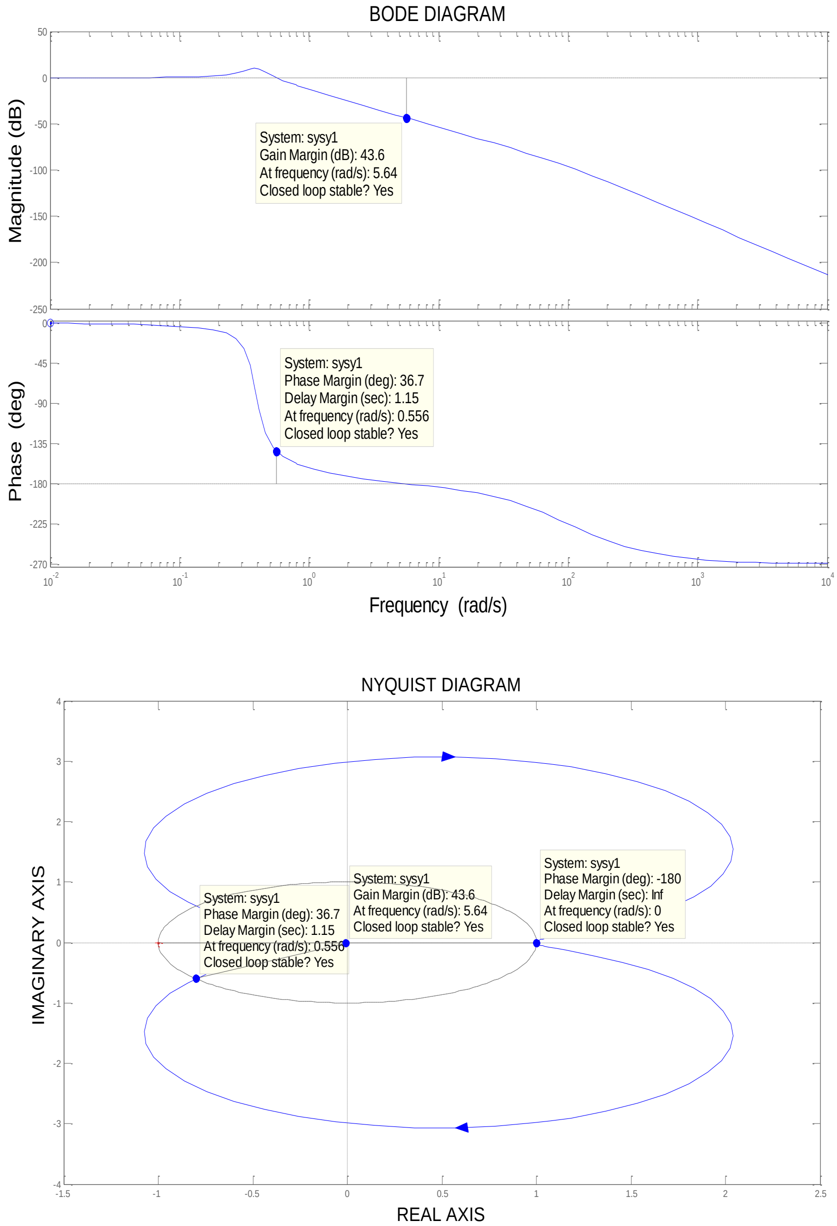

Fig. 11. Bode and nyquist plot of Zeigler Nichols method

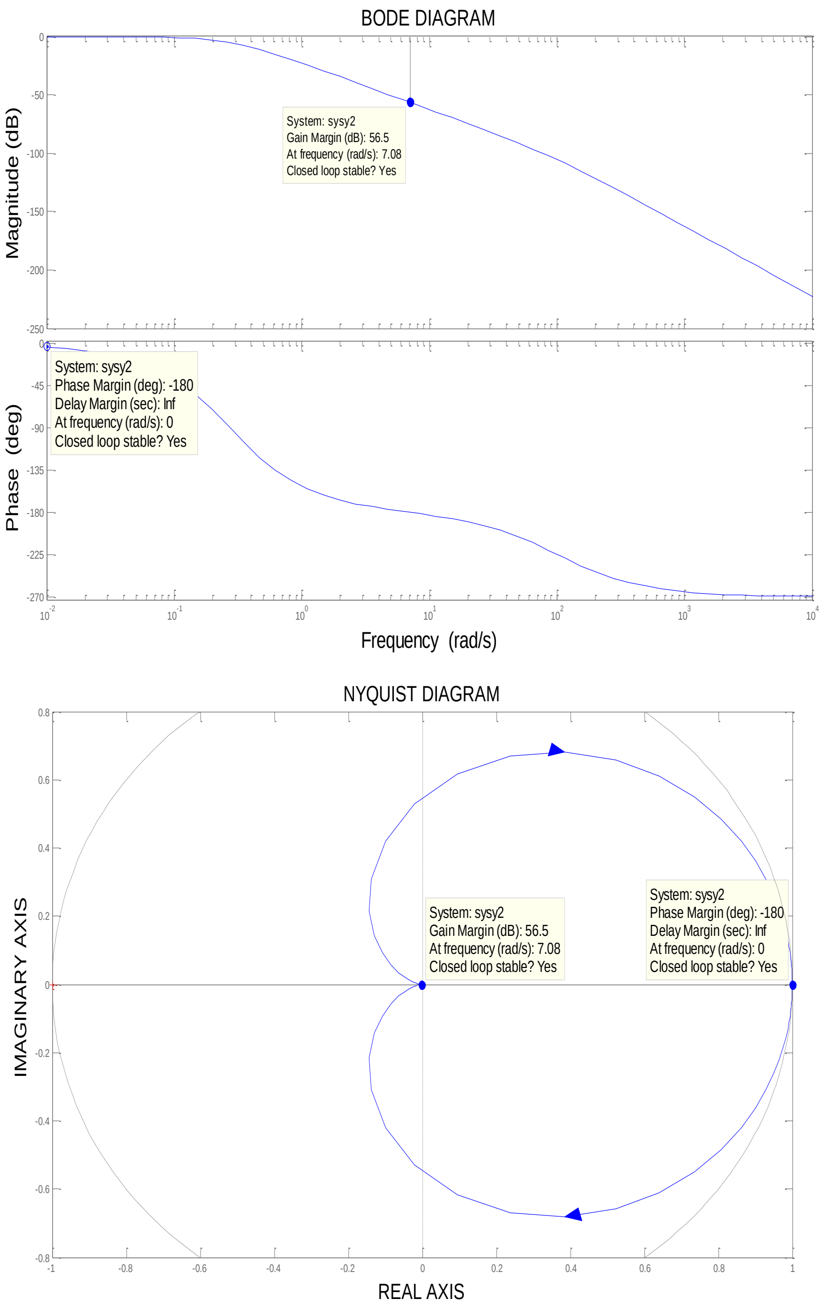

Fig. 12. Bode and nyquist plot of Modulus Optimum method

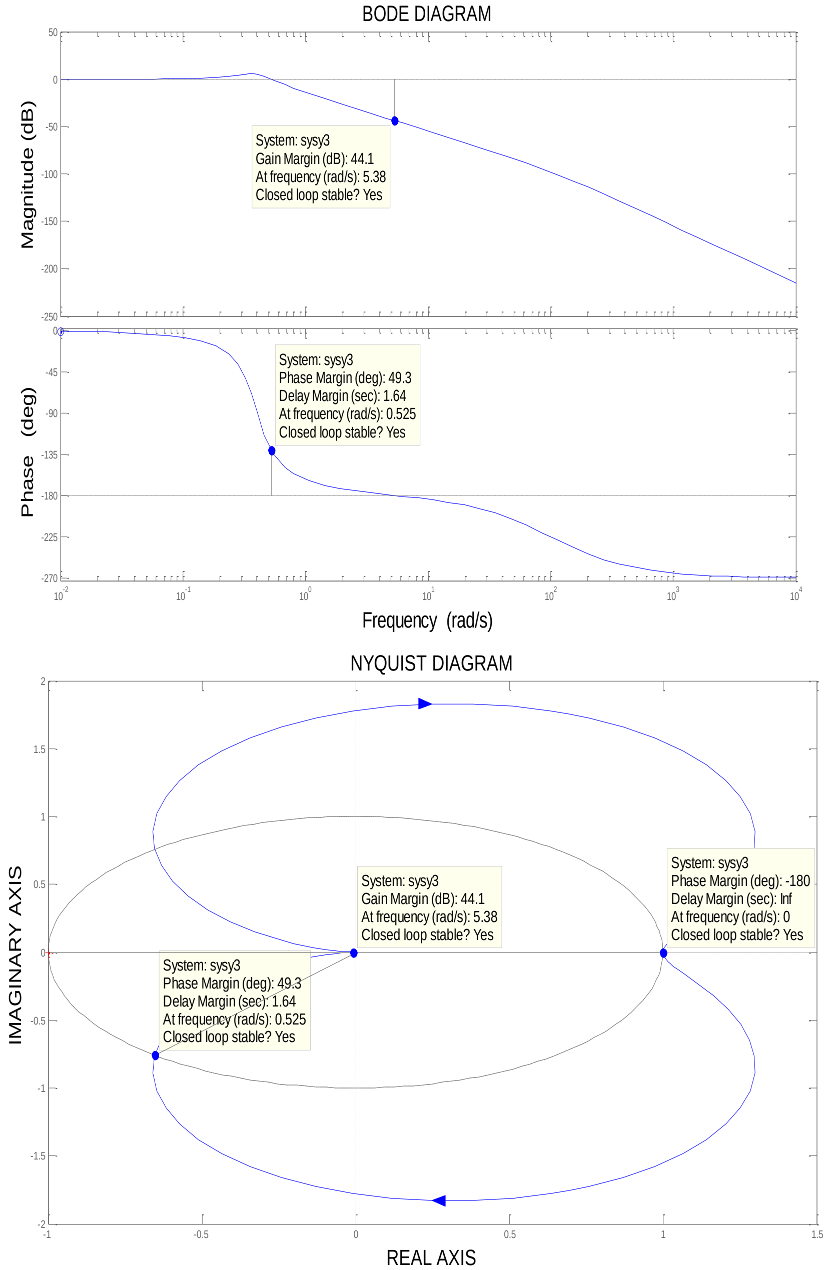

Fig. 13. Bode and nyquist plot of Relay Auto tuning method

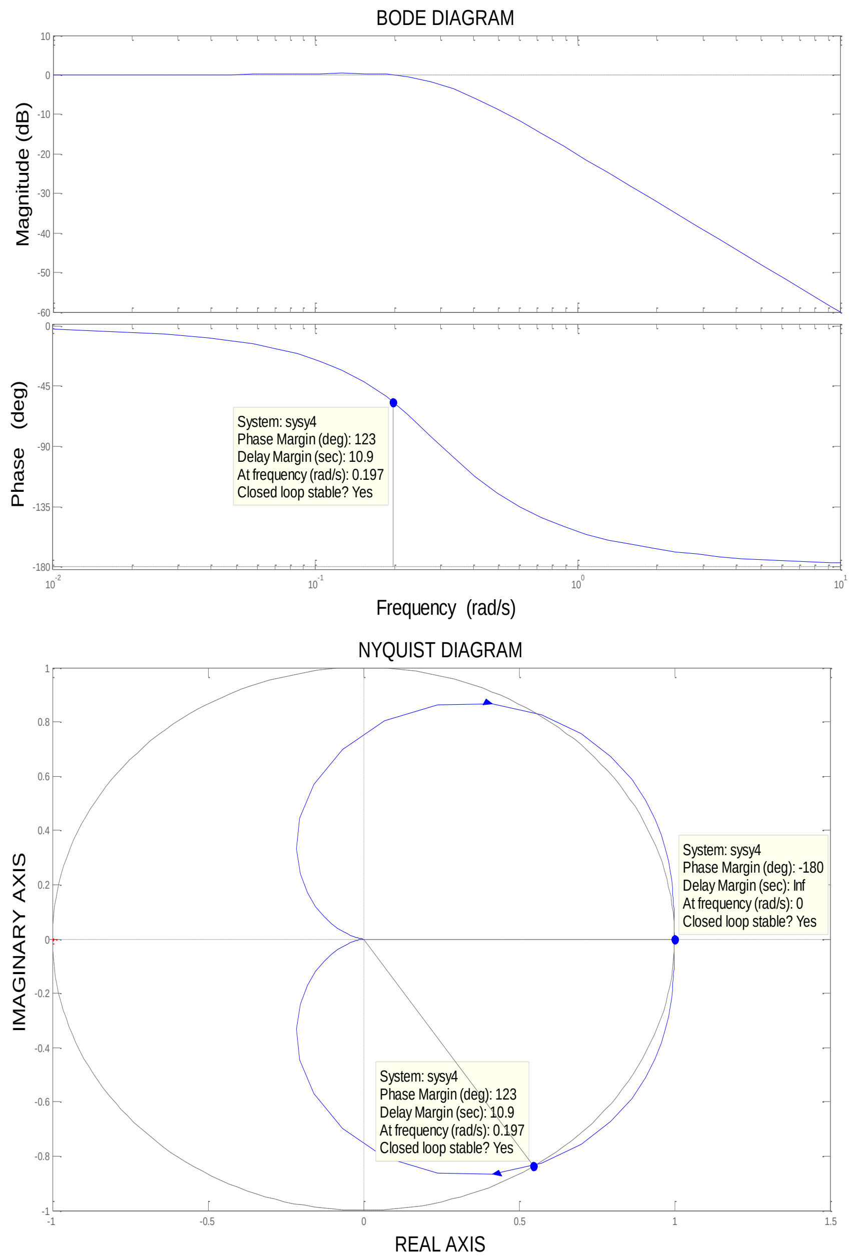

Fig. 14. Bode and nyquist plot of computational optimization method

3.1 Comparisons of all the above four tuning methods

Table 3.The results of PID tuning parameters for simple plant

Method	K_P	K_D	K_I
ZN	1.002	1.6733	0.25
Relay Auto tuning	0.9	1.4288	0.09
Computational optimization	0.368	0.8	0.04232
MO	0.25	0.6	0.025

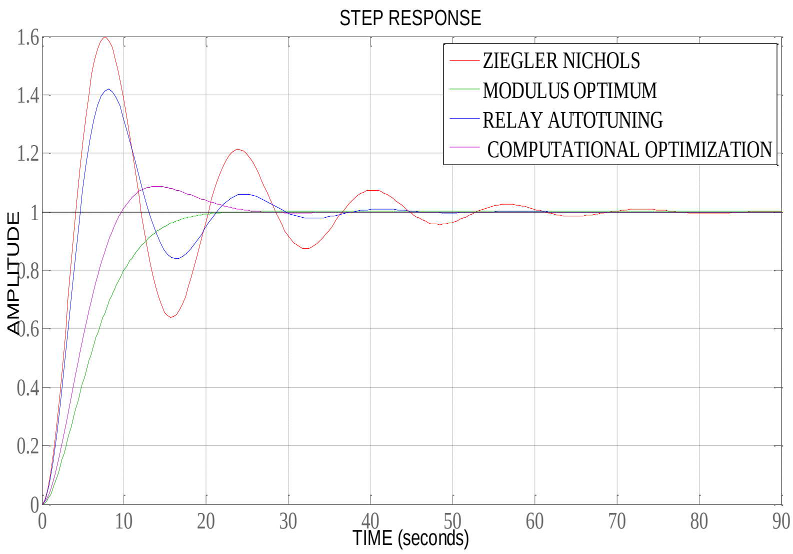

Fig. 15. Comparison of step response among various tuning methods

Fig. 15 shows the closed- loop system responses of all the PID controllers for a step set point. According to the results, it is observed that the Ziegler Nichols method is used when less rise time and peak time is required while Modulus Optimum method is applicable when require less settling time and overshoot.

3.2 Performance index

The integral error is generally accepted as a good measure for system performance. The followings are some commonly used criteria based on the integral error for a step set point or disturbance response [11].

$$\text{IAE}=\int_0^\infty |e(t)|dt$$

$$\text{ISE}=\int_0^\infty e^2(t)dt$$

Where,

IAE=Integral of absolute error

ISE=Integral of square error

3.3 Robustness Analysis

Frequency response analysis is performed because, it provides a measurement of robustness of the controller tuning. It provides a measure of the amount of model uncertainty that can be tolerated before the controller will become unstable. The frequency response of a system consists of the magnitude response and phase response [12]. To investigate the effect of stability due to addition of disturbance, bode plot and nyquist plot is plotted. The gain margin is the reciprocal of the magnitude |G (jω)| at the frequency at which the phase angle is -180°. If gain margin is greater than unity it means that the system is stable, where as if the gain margin is less than unity it means that the system is unstable. The phase margin is that amount of additional phase lag at gain crossover frequency required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which |G (jω)|, the magnitude of open loop transfer function is unity. It is the frequency at which the phase angle of open loop transfer function is -180°.The robustness factors considered here are gain margin(GM), phase margin(PM) , gain crossover frequency, phase crossover frequency and stability.

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Table 4. The results of PID Performance Criteria

Performance Index	ZN	Relay auto tuning	Computational optimization	MO
Rise time(t_r) in sec	2.7668	3.1507	6.4775	10.7301
Settling time(t_S) in sec	58.3869	34.3307	22.1642	17.6936
Peak time(t_P) in sec	7.8374	8.1142	13.9401	28.2871
Overshoot (%)	59.3524	41.8925	8.6943	0.0839
Steady state error (e_{SS}) in sec	0	0	0	0
ISE	4.1191	2.9196	3.3679	4.3345
IAE	0.6667	1.1759	3.9382	6.6667

Table 5. The results of robustness analysis of different system configurations

Parameters	ZN	Relay Auto tuning	Computational Optimization	MO
Gain Margin in (rad/sec)	151.13	160.45	Inf	667.99
Gain Margin in (dB)	43.6	44.1	Inf	56.5
GM Frequency (rad/sec)	5.64	5.38	Inf	7.08
Phase Margin (deg)	[-180 36.75]	[-180 49.28]	[-180 123.24]	[-180]
PM Frequency (rad/sec)	0.56	0.53	[0 0.20]	0
Stability	Stable	Stable	Stable	Stable

Conclusions

In this work, various methods for tuning a PID controller have been proposed. Several well-known PID tuning formulas were analyzed. The performances of various tuning methods have been compared by applying a step input to the given process. Simulation results show that tuning a PID controller with Ziegler Nichols (ZN) tuning method results in less rise time (t_r), peak time (t_P) and error ISE. The Relay Auto tuning method is applicable when less ISE is required. The Modulus Optimum (MO) tuning method is applicable when less settling time (ts) and less overshoot is required while the Computational Optimization (CO) method is helpful when the desired closed loop specifications are decided by the designer**.**

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