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Projective Geometry Method in the Theory of Electric Circuits with Variable Parameters of Elements

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Abstract. Projective geometry is used for interpreting of circuit regime changes and mutual influence of loads. The changes of regime parameters are introduced through the cross ratio of four points. The formulas of the recalculation of the currents, which possess the group properties at change of conductivity of the loads, are obtained. The projective coordinates allows receiving the equation of the network in a normalized form as well as to define the scales for the currents and conductivity of the loads. This approach makes it possible to compare the regime of different circuits. The given approach is applicable to the analysis of «flowed » form processes of various physical natures.

Keywords: active multi- port, recalculation of currents, projective coordinates, changeable loads.

1 Introduction

In the theory of electric circuits, an attention is given to the circuits with variable parameters of elements or loads. A mutual influence of loads on the value of their currents is feature of these circuits.

One of the analysis problems is to obtain the formulas of the recalculation of load currents. The traditional approach, based on use of change of load resistances in the form of increments, is known. However, at a number or group of changes of these resistances, the increments should be counted concerning an initial circuit and the solution of the equation is repeated. Therefore, the non-fulfillment of group properties (when the final result should be obtained through the intermediate results) complicates recalculation and limits possibilities of this approach.

The definition of parameters of an operating regime in a normalized form, with respect to some characteristic values (as scales), is importance for electric circuits with changeable parameters of loads. Such a definition of regimes (hereinafter referred to as relative regimes) allows comparing or setting the regimes of different systems. For example, for a circuit in the form of an active two-pole, open circuit voltage, short circuit current, internal resistance can be the scales for the voltage, current, and resistance of load. Then, the normalized equation of the circuit (load straight line) is obtained evidently. Therefore, the regimes of compared active two-poles will be identical if the normalized values of parameters of loads are also identical.

However, the consideration of more complex circuits reveals no triviality of the problem, because a number of characteristic values are increased and well-founded approaches to the formation of relative expressions are missing. For example, suppose that any resistance is changed in the active two-pole. Then, the open circuit voltage is changed also. This change in the scale of the load voltage complicates the estimation of the regime of the circuit, creates uncertainty for comparison of the regimes of the different circuits. Further, consider an active two- port network with changeable loads. Therefore, an interference of changes of the load resistances on the value of voltage of these loads takes place. This bring up the problem of choice of the scales for the voltages or currents of loads and the problem of the normalized formulation of the circuit equation.

In a number of papers of the author, the approach is developed for interpretation of changes or "kinematics" of the circuit regimes on the basis of projective geometry [1], [2], [3]. The changes of regime parameters are introduced otherwise through the cross ratio of four points. Therefore, the obvious changes in a form of increments are formal without reflecting the substantial aspect of the mutual influences: resistances → currents.

Easy-to-use formulas of the recalculation of the currents, which possess the group properties at change of conductivity of the loads, are obtained. The projective coordinates allow receiving the equation of the active two-port network in a normalized form as well as to define the scales for the currents and conductivity of the loads. Such approach makes it possible to compare the regime of the different circuits. In this paper some of the main results are presented, which show other aspects of use of projective geometry.

2 Show of Projective Geometry in Electric Circuits

Let us give the necessary knowledge about the projective geometry [4] for interpretation of changes or "kinematics" of regimes of electric circuits (an active two-pole A) in Fig.1. Let two cases of change of elements R_L, R_i be considered.

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Case 1. $R_i = const$. At change of the load $R_L > 0$ from a regime of the short circuit SC $(R_L = 0)$ to the open circuit OC $(R_L = \infty)$, a load straight line

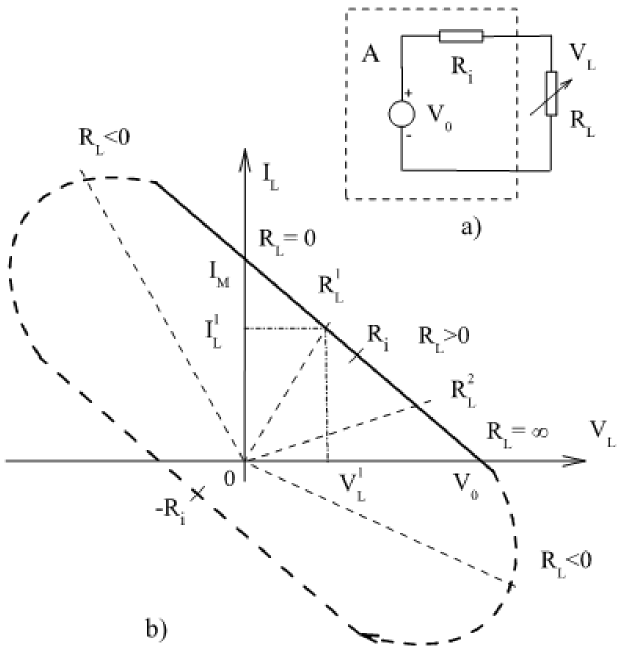

Fig.1. An active two-pole – a), $I - V$ characteristic – b).

or the $I - V$ characteristic $I_L(V_L)$ is given by

$$I_L = \frac{V_0}{R_i} - \frac{V_L}{R_i} = I_M - \frac{V_L}{R_i},$$

where I_M is the SC current. The bunch of straight lines with the parameter R_L and the centre in the point 0 corresponds to this load straight line.

Further, it is possible to calibrate the load straight line in values of load resistance. This internal area of change corresponds to a regime of energy consumption by the load. If to continue the graduation for negative values $R_L < 0$, the regime passes into external area, which physically means a return of energy to the voltage source. Therefore, in an infinite point $R_L = -R_i$ the graduations of the $I-V$ characteristic will coincide for areas $V_L > V_0, V_L < 0$. Thus, the straight line of the $I-V$ characteristic is closed; it is typical for projective geometry.

Further, the equation $V_L(R_L)$ has the characteristic linear-fractional species

$$V_L = V_0 \frac{R_L}{R_i + R_L}.$$

It gives the grounds for considering the transformation of a straight line R_L into a straight line V_L as projective, which is in accordance with the positions of projective geometry. Generally, the projective transformation is set by the centre of a projection or by three pairs of respective points.

It is convenient to use the points of characteristic regimes, as pairs of the respective points, which can be easily determined at the qualitative level, i.e. short circuit, open circuit, maximum load power. The projective transformations preserve a cross ratio m_L of four points, where the fourth point is the point of a running regime, R_L^1, V_L^1, I_L^1. The cross ratio m_L has the form

$$m_L^1 = (0\ R_L^1\ R_i\ \infty) = \frac{R_L^1 - 0}{R_L^1 - \infty} : \frac{R_i - 0}{R_i - \infty} = \frac{R_L^1}{R_i},$$

$$m_L^1 = (\ 0\ V_L^1\ 0{,}5V_0\ V_0\ ) = \frac{V_L^1}{V_0 - V_L^1},$$

$$m_L^1 = (I_M\ I_L^1\ 0{,}5I_M\ 0\ ) = \frac{I_M - I_L^1}{I_L^1}.$$

The point $R_L = R_i$ is a scale or unit point and the points $R_L = 0, R_L = \infty$ correspond to extreme or base values. Thus, the coordinate of a running regime point is set by this value m_L, which is defined in the invariant manner through the various regime parameters, R_L, V_L, I_L.

The regime change $R_L^1 \to R_L^2$ can be expressed similarly

$$m_L^{21} = (0\ R_L^2\ R_L^1\ \infty) = \frac{R_L^2}{R_L^1} = \frac{V_L^2}{V_0 - V_L^2} : \frac{V_L^1}{V_0 - V_L^1}.$$

Case 2. Now, let both elements R_L, R_i be changed. In this case, the $I - V$ characteristic family or a bunch of straight lines R_i is obtained with the centre G in Fig.2.

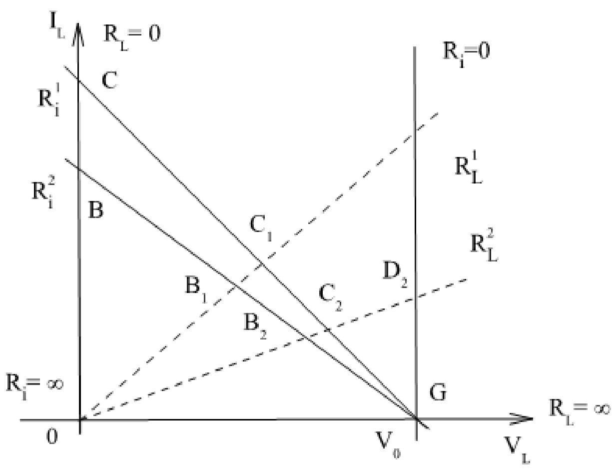

Fig.2. Bunches of straight lines R_L, R_i.

The coordinate of the centre G, corresponding V_0, does not depend on values R_i. Physically, it means that the current across this element is equal to zero. The element R_i can accept such characteristic or base values, as $0, \infty$. The third characteristic value or a scale one is not present for R_i .

Let the relative regimes be considered for this case. Let the internal resistance R_i be equal to R_i^1, and resistance of load varies from R_L^1 to R_L^2. In this case, the point of an initial regime $C_1 \to C_2$. If R_i is equal R_i^2, the initial regime point $B_1 \to B_2$. Therefore, the regime change is determined by the load

$$m_L^{21} = (C\, C_1\, C_2\, G) = (0\ V_L(C_1)\ V_L(C_2)\ V_0) = \\ = (0\ R_L^1\ R_L^2\ \infty) = (B\, B_1\, B_2\, G).$$

This determination of regime does not depend from R_i.

Similarly, the regime change is determined by the R_i

$$m_i^{21} = (\,0\,C_2\;B_2\;D_2\,) = (0\;\;V_L(C_2)\;\;V_L(B_2)\;\;V_0\,) = (\infty\;R_i^1\;\;R_i^2\;\;0\,)\,.$$

The geometrical interpretation helps to understand the choice of the characteristic points for the cross ratio. The above-mentioned arguments make it possible to correlate the regimes of the compared circuits and to give a basis for an analysis of the general case of a circuit.

Now, let us introduce a projective plane. For this purpose, we will consider the most simple active two-port network, which contains two independent loads Y_{L1}, Y_{L2} in Fig.3.

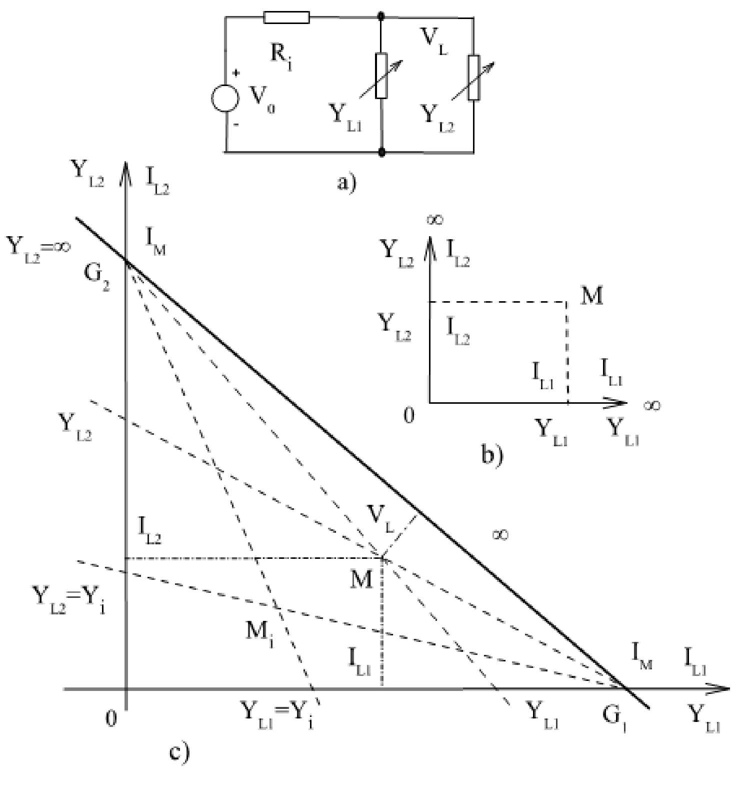

Fig.3. Simple active two-port network - a), Euclidean plane, $R_i = 0$ - b), projective plane, $R_i > 0$ - c).

If internal resistance R_i of a voltage source V_0 is equal to the zero, the independent change of currents of loads takes place at the independent change of conductivity of loads. The family of load characteristics (there are parallel lines) coincides with rectangular Cartesian system of coordinates on a Euclidean plane. Then, the calibrations of the coordinate axes, in values of a current and conductivity, coincide. It is obvious that the circuit does not possess the own scales. Therefore, it is possible to express the regime only in absolute or actual values of a current or conductivity.

Let the internal resistance R_i accepts a finite value. In this case, the dependent change of load currents takes place. There are two bunches of the load straight lines, which centers are defined by the SC current I_M. Then, the straight line of the maximum current I_M passes across these centers. This straight line is the line of infinity ∞. The obtained "deformed" coordinate grid defines a projective plane. The axes of coordinates can be calibrated in values of corresponding currents or conductivities of the loads. In this case, there is an internal scale, i.e. the value of conductivity $Y_i = 1/R_i$ or the current I_M.

It is possible to accept that values of the load conductivities are equal, for example, to internal conductivity Y_i and define the point M_i of a characteristic regime. The projective coordinates is uniquely set by four points; it is three points of the triangle of reference $G_1 0 G_2$ and the scale point M_i.

The point of a running regime M can be set by the values of load conductivities (non-uniform coordinates) $M(Y_{L1}, Y_{L2})$ or by the load currents (homogeneous coordinates) $M(I_{L1}, I_{L2}, V_L)$. The sense of homogeneous coordinates consists that they are proportional to the length of perpendiculars from a point to the sides of a triangle of reference. The homogeneous coordinates are used for uncertainty elimination, when the point is on line of infinity ∞. Presence of the fourth (characteristic) point allows to introduce the cross ratios m_{L1}, m_{L2} and to set a regime in a relative form.

3. Projective Coordinates of an Active Two-port

Let us give the necessary relationships for an active two-port network in Fig.4 with changeable conductivities of loads Y_{L1}, Y_{L2} [2].

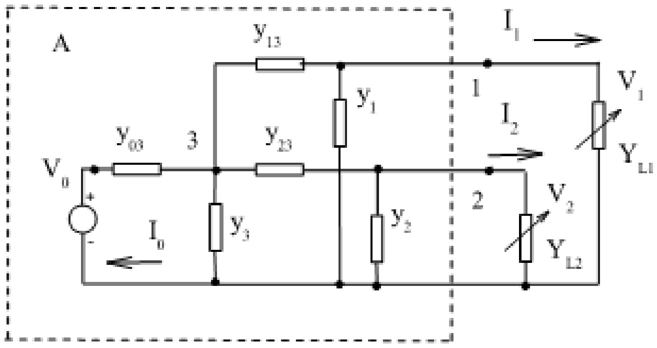

Fig.4. An active two-port network.

The circuit is described by the following system of the Y - parameters equations

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} \\ Y_{12} & -Y_{22} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \end{pmatrix}, \tag{1}$$

where I_1^{SC}, I_2^{SC} are the short circuit SC currents of both loads.

Taking into account the voltages $V_1 = I_1 / Y_{L1}$, $V_2 = I_2 / Y_{L2}$, two bunches of load straight lines $(I_1, I_2, Y_{L1}) = 0$, $(I_1, I_2, Y_{L2}) = 0$ with parameters Y_{L1}, Y_{L2} are shown in Fig.5.

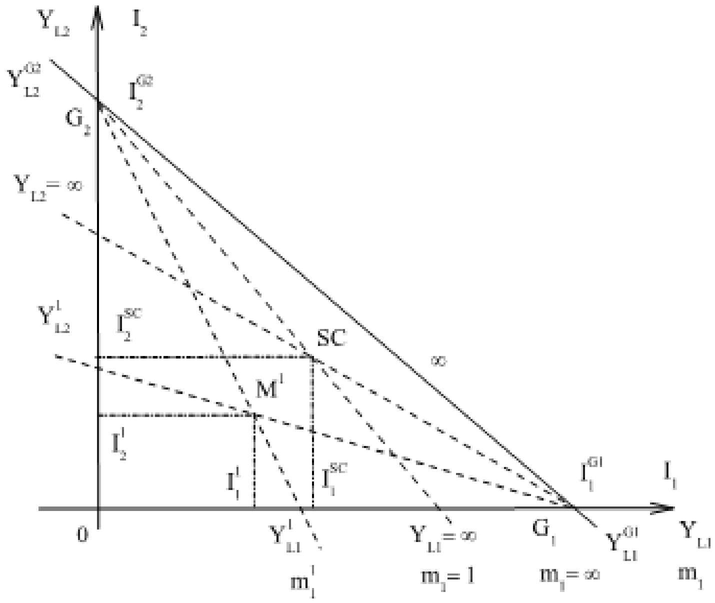

Fig.5. Two bunches of straight lines with parameters Y_{L1}, Y_{L2}.

The bunch center, a point G_2, corresponds to the bunch with the parameter Y_{L1} and corresponds to such a regime of the load Y_{L1} which does not depend on its values. It is carried out for the current and voltage $I_1 = 0, V_1 = 0$ at the expense of a choice of the regime parameters of the second load Y_{L2}

$$I_2 = I_2^{G2}, \quad Y_{L2} = Y_{L2}^{G2}. \tag{2}$$

The parameters of the center G_1 of the bunch Y_{L2} are expressed similarly.

Another form of characteristic regime is the short circuit regime of both loads, $Y_{L1} = \infty$, $Y_{L2} = \infty$, that is presented by the point SC. The open circuit regime of both loads is also characteristic and corresponds to the origin of coordinates.

Let the initial or running regime corresponds to the point M^1 which is set by the values of conductivities Y_{L1}^1, Y_{L2}^1 or currents I_1^1, I_2^1 of the loads. Also, this point is defined by the projective non-uniform m_1^1, m_2^1 and homogeneous $\xi_1^1, \xi_2^1, \xi_3^1$

coordinates which are set by a triangle of reference $G_1 \; 0 \, G_2$ and a unit point SC. The point 0 is the origin of coordinates and the straight line $G_1 \; G_2$ is the line of infinity ∞.

The non-uniform projective coordinate m_1^1 of the running regime is set by a cross ratio of four points

$$m_1^1 = (0 \, Y_{L1}^1 \, \infty \, Y_{L1}^{G1}) = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \div \frac{\infty - 0}{\infty - Y_{L1}^{G1}} = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} . \tag{3}$$

There, the points $Y_{L1} = 0,\; Y_{L1} = Y_{L1}^{G1}$ correspond to the extreme or base values. The point $Y_{L1} = \infty$ is the unit point. For the point $Y_{L1}^1 = Y_{L1}^{G1}$, the projective coordinate $m_1 = \infty$ defines the sense of a line of infinity $G_1 \; G_2$. The cross ratio for the projective coordinate m_2^1 is expressed similarly.

The homogeneous projective coordinates ξ_1, ξ_2, ξ_3 set the non-uniform coordinates as follows

$$m_1 = \frac{\xi_1}{\xi_3} = \frac{\rho\xi_1}{\rho\xi_3}, \quad m_2 = \frac{\xi_2}{\xi_3} = \frac{\rho\xi_2}{\rho\xi_3}, \tag{4}$$

where ρ is a coefficient of proportionality. The homogeneous coordinates are defined by the ratio of the distances of the points M^1, SC to the sides of the triangle of reference

$$\rho \, \xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}} = \frac{I_1^1}{I_1^{SC}}, \quad \rho \, \xi_2^1 = \frac{I_2^1}{I_2^{SC}}, \quad \rho \, \xi_3^1 = \frac{\delta_3^1}{\delta_3^{SC}} .$$

For finding the distances $\delta_3^1, \delta_3^{SC}$ to the straight line $G_1 \; G_2$, the equation of this straight line is used. Then

$$\left(\frac{I_1^1}{I_1^{G1}} + \frac{I_2^1}{I_2^{G2}} - 1 \right) = \mu_3 \delta_3^1, \left(\frac{I_1^{SC}}{I_1^{G1}} + \frac{I_2^{SC}}{I_2^{G2}} - 1 \right) = \mu_3 \delta_3^{SC},$$

where μ_3 is a normalizing factor.

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The homogeneous coordinates have a matrix form

$$\rho[\xi]=[C]\cdot[I], \tag{5}$$

where matrix

$$[C]=\begin{pmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G2}\mu_3\delta_3^{SC}} & -\frac{1}{\mu_3\delta_3^{SC}} \end{pmatrix},$$

and vectors

$$[I]=\begin{pmatrix} I_1 \\ I_2 \\ 1 \end{pmatrix},\ [\xi\ ]=\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}.$$

The inverse transformation is

$$\rho[I]=[C]^{-1}\cdot[\xi], \tag{6}$$

where matrix

$$[C]^{-1}=\begin{pmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G1}} & \frac{I_2^{SC}}{I_2^{G2}} & -\mu_3\delta_3^{SC} \end{pmatrix}.$$

From here, we pass to the currents

$$I_1 = \frac{\rho I_1}{\rho 1} = \frac{I_1^{SC} \cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2 - \mu_3 \delta_3^{SC}}, \quad I_2 = \frac{\rho I_2}{\rho 1}. \tag{7}$$

Obtained transformation allows finding the currents I_1, I_2 for the preset values of conductivities Y_{L1}, Y_{L2} by using the coordinates m_1, m_2.

3.1 Normalized form of equations

It is possible to present the equations (7) for current I_1 by the normalized or relative form

$$\frac{I_1}{I_1^{G1}} = \frac{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot (m_1 - 1) + \frac{I_2^{SC}}{I_2^{G2}} \cdot (m_2 - 1) + 1}, \tag{8}$$

These equations for current allow obtaining in a relative form the equations of load characteristics $I_1(V_1, V_2)$, $I_2(V_1, V_2)$ which correspond to the system (1). For this purpose, we express the non-uniform coordinates m_1, m_2 through the currents and voltages

$$m_1 = \frac{Y_{L1}}{Y_{L1} - Y_{L1}^{G1}} = \frac{I_1 / I_1^{G1}}{(I_1 / I_1^{G1}) - V_1 / V_1^{G1}}, \quad m_2 = \frac{I_2 / I_2^{G2}}{(I_2 / I_2^{G2}) - V_2 / V_2^{G2}}.$$

Having substituted these values in system (8), we receive the required equations

$$\begin{pmatrix} I_1 / I_1^{G1} \\ I_2 / I_2^{G2} \end{pmatrix} = \begin{pmatrix} Y_{11}^N & Y_{12}^N \\ Y_{21}^N & Y_{22}^N \end{pmatrix} \cdot \begin{pmatrix} V_1 / V_1^{G1} \\ V_2 / V_2^{G2} \end{pmatrix} + \begin{pmatrix} I_1^{SC} / I_1^{G1} \\ I_2^{SC} / I_2^{G2} \end{pmatrix}, \tag{9}$$

where normalized Y^N-parameters are

$$\begin{pmatrix} Y_{11}^N & Y_{12}^N \\ Y_{21}^N & Y_{22}^N \end{pmatrix} = \begin{pmatrix} (1 - I_1^{SC} / I_1^{G1}) & -I_1^{SC} / I_1^{G1} \\ -I_2^{SC} / I_2^{G2} & (1 - I_2^{SC} / I_2^{G2}) \end{pmatrix} .$$

These expressions allow estimating at once a qualitative condition of such circuit, how much the current regime is close to the characteristic values. Then, the currents I_1^{G1}, I_2^{G2} represent scales. In this sense, the initial system of the equations (1) does not give a direct representation about the qualitative characteristics of a circuit. The obtained results to the active multi-port networks are shown in [3]

3.2 Recalculation of the load currents.

Let a subsequent regime corresponds to the point M^2 in Fig.6 with the parameters of loads Y_{L1}^2, Y_{L2}^2, I_1^2, I_2^2.

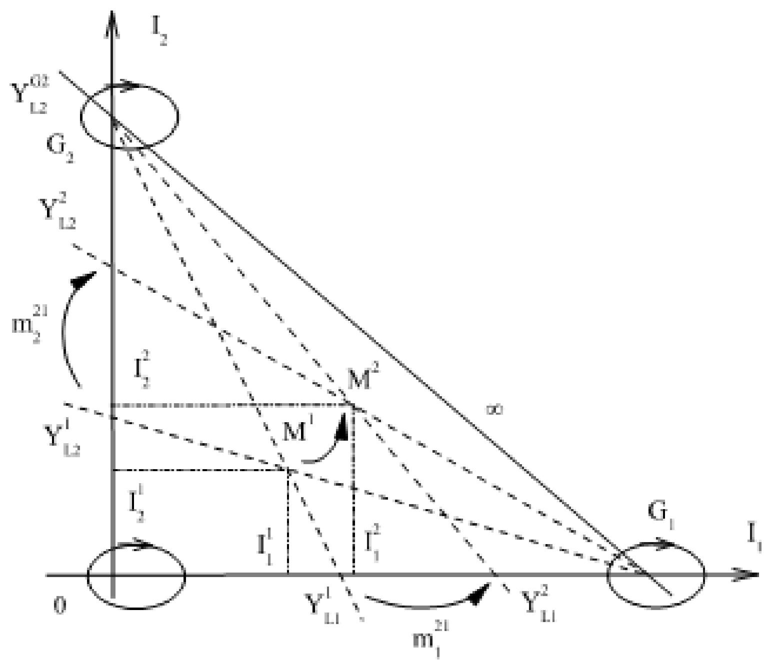

Fig. 6. Change of a regime at the expense of the load conductivities Y_{L1}, Y_{L2}.

The non-uniform m_1^2, m_2^2 coordinates are defined similarly to (3). Therefore, the changes of the regime m_1^{21} , m_2^{21} are naturally expressed through the cross ratio

$$m_1^{21} = (0\, Y_{L1}^2\, Y_{L1}^1\, Y_{L1}^{G1}) = m_1^2 : m_1^1,\; m_2^{21} = m_2^2 : m_2^1 .$$

We also define the homogeneous coordinates of the point M^2 and present the non-uniform coordinates, m_1^2 and m_2^2 in a form

$$m_1^2 = m_1^{21}\frac{\xi_1^1}{\xi_3^1},\quad m_2^2 = m_2^{21}\frac{\xi_2^1}{\xi_3^1}.$$

Using the transformations (7), we receive the current

$$I_1^2 = \frac{(I_1^{SC}\cdot m_1^{21})\cdot \xi_1^1/\xi_3^1}{\left(\dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1^{21}\right)\cdot\dfrac{\xi_1^1}{\xi_3^1} + \left(\dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2^{21}\right)\dfrac{\xi_2^1}{\xi_3^1} - \mu_3\delta_3^{SC}}.$$

Then, taking into account the expression (6), we obtain the transformation

$$\rho[I^2] = [C^{21}]^{-1}\cdot[\xi^1],$$

where matrix

$$[C^{21}]^{-1} = \begin{pmatrix} I_1^{SC}\cdot m_1^{21} & 0 & 0 \\ 0 & I_2^{SC}\cdot m_2^{21} & 0 \\ \dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1^{21} & \dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2^{21} & -\mu_3\delta_3^{SC} \end{pmatrix}.$$

Using (5), we receive the resultant transformation as a product of the two matrixes

$$\rho[I^2]=[C^{21}]^{-1}\cdot[C]\cdot[I^1]\ . \tag{10}$$

The obtained relationships carry out the recalculation of currents at respective change of load conductivities. These relations are the projective transformations and possess group properties.

3.3 Comparison of operating regimes and parameters of active two-ports

Let two active two-ports with various parameters are given. Bunches of the load characteristics and conformity of the characteristic and running regimes points are presented in Fig. 7.

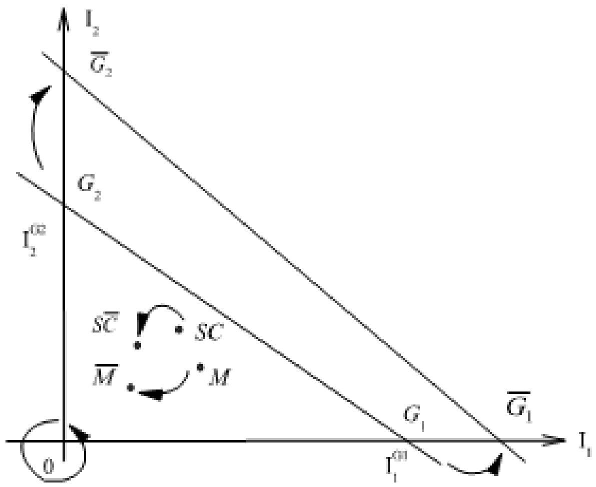

Fig.7. Conformity of the characteristic and running regimes points of two active two- ports.

The problem is to find such values of conductivities and currents of loads of the second two-port (a point $\overline{M}$) that its relative regimes would be equal to regimes of the first two-port (a point M has preset values of conductivities Y_{L1}, Y_{L2} and currents I_1, I_2). Recalculation of the load conductivities $\overline{Y}_{L1}$, $\overline{Y}_{L2}$ is carried out according to (3)

$$\frac{Y_{L1}}{Y_{L1}-Y_{L1}^{G1}}=\frac{\overline{Y}_{L1}}{\overline{Y}_{L1}-\overline{Y}_{L1}^{G1}},\ \frac{Y_{L2}}{Y_{L2}-Y_{L2}^{G2}}=\frac{\overline{Y}_{L2}}{\overline{Y}_{L2}-\overline{Y}_{L2}^{G2}}. \tag{12}$$

Therefore, it is necessary to find a transformation which provides a recalculation of currents. For this purpose, we consider the systems of projective coordinates of these two-ports. Systems of coordinates are set by triangles of reference $G_1\ 0\ G_2$, $\overline{G}_1\ 0\ \overline{G}_2$ and unit points SC, $\overline{SC}$ accordingly. Conditions of equality of regimes lead to equality of projective coordinates $[\overline{\rho}\overline{\xi}]=[\xi]$ of points M $\overline{M}$. According to (5), for these two-ports

$$[\rho\xi]=[C]\cdot[I],\ [\overline{\rho}\overline{\xi}]=[\overline{C}]\cdot[\overline{I}].$$

Therefore, a necessary transformation is the product of two matrixes [3]

$$[\overline{\rho}\overline{I}]=[\overline{C}]^{-1}\cdot[C]\cdot[I].$$

4. Conclusions

1. Projective geometry adequately interprets "kinematics" of a circuit with changeable parameters of loads, allows performing the deeper analysis and to obtain the relationships useful in practice.
2. The obtained formulas of the recalculation of the currents possess the group properties at change of conductivity of the loads. It allows expressing the final values of the currents through intermediate changes of the currents and conductivities.
3. The derived normalized equations of the network define the scales for the currents and conductivities and allow comparing regimes of different circuits.
4. From the methodological point of view, the presented approach is applied for a long time in other scientific domains, as mechanics (the principles of special relativity), biology (the principles of age changes of plants and organisms).

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