

The Conservation of Information Principle

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Abstract- New formulas are proposed as a means to express the conservation of information principle. The first of them modifies the expression we presented in a previous work (viXra:1312.0060). The second one deals with the entropy of a particle in a circle. It is used to treat in a novel way the thermodynamics of a quantum gas. The wave function of this particle, when inserted in the conservation principle, leads to its entropy and the structure of energy levels to its internal energy.

1 – Some Preliminaries

In a paper dealing with the proton-electron mass ratio [1], the “Conservation of Information Principle” was proposed in order to evaluate a modified Planck’s momentum related to the curved space-time. The equation representing this principle was written as

$$S + I = S + \ln(\Psi^2) = C. \quad (1)$$

In equation (1), S is the Bekenstein-Hawking entropy [2,3], Ψ is a wave function associated to the curved space-time and C is a constant.

Indeed equation (1) seems not to be the best form to express the conservation of information principle. Instead of equation (1), we propose that the more appropriate form to write this principle is

$$S + I = S + \ln\Psi = C. \quad (1A)$$

If we consider the surface horizon of a black hole as a spherical surface of radius r , and by permitting its area or equivalently its radius to vary, we have with L_{Pl} being its Planck’s radius

$$S = (\pi r^2)/L_{Pl}^2. \quad (2)$$

Inserting (2) into (1A), we get

$$\Psi = \Psi_0 \exp(-S) = \Psi_0 \exp(-\pi r^2/L_{Pl}^2). \quad (3)$$

In reference [1] the wave function given by (3), was used in a Schrodinger equation as a means to find a modified Planck’s momentum in a curved space-time.

2 – Particle in a Circle: A Plane-wave-like Case

Perhaps relation (1A) applies essentially to the cases where the wave function is a real quantity (not complex). In more general cases, specifically for the plane-wave-like case, we can write

$$S + [i k_B/(2\pi)] \ln\Psi = S_0. \quad (4)$$

Let us write $\Psi(\theta)$, the wave function of a particle in a circle. A particle of mass m is constrained to move in a circle of radius R and its position on the circumference is conveniently specified by an angle θ (please see reference [4]). We have

$$\Psi_n(\theta) = \exp(i n \theta), \quad n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (5)$$

$$S(\theta) = S_0 + [k_B / (2\pi)] n \theta. \quad (6)$$

$$S_n \equiv S_{\max} = S_0 + k_B n. \quad (7)$$

In (5) to (7), n are the quantum numbers and S_{\max} is the maximum entropy related to a given quantum number.

The energy levels of this system are

$$E_n = \hbar^2 n^2 / (2mR^2). \quad (8)$$

3 – Connection with Thermodynamics

Let us consider the gas of particles described by (8), in contact with a thermal reservoir. In an isothermal process at a fixed size R , and taking in account the first law of thermodynamics we can write

$$\Delta Q = \Delta U. \quad (9)$$

Here we consider the internal energy, namely the kinetic energy, as being quantized. We have

$$U_N \equiv E_N = \hbar^2 N^2 / (2mR^2). \quad (10)$$

We put the system in thermal contact with a heat reservoir at the absolute temperature T slightly greater than T_N , so that the system makes a transition to the state $N+1$. This can be achieved in a quasi-static process if $N \gg 1$. In this process we have

$$\Delta U = E_{N+1} - E_N = \hbar^2 (2N + 1) / (2mR^2). \quad (11)$$

Now we evaluate the excess of entropy developed in this process

$$\Delta S = \Delta Q / T_N = [\hbar^2 / (2mR^2)] [(2N+1) / T_N]. \quad (12)$$

Meanwhile relation (7) implies that

$$\Delta S = S_{N+1} - S_N = k_B. \quad (13)$$

Putting (13) into (12) yields

$$k_B T_N = \hbar^2 (2N + 1) / (2mR^2). \quad (14)$$

For $N \gg 1$, we can neglect the term independent of N in (14) and we get

$$K_B T_N = \hbar^2 N / (mR^2), \quad N \gg 1. \quad (15)$$

On the other hand, if we extend the validity of (14) to small values of N , we see that the lowest temperature is given by $N=0$, namely

$$K_B T_0 = \hbar^2 / (2mR^2). \quad (16)$$

Therefore relation (14) is in accord with the third law of thermodynamics which states that the absolute zero is not accessible.

4 – The Fermi Gas Connection

In the case of $N \gg 1$, let us take R as the radius of gyration R_g of a polymer chain, having the monomer size equal to the reduced Fermi wavelength λ_F and with the number of monomers \mathcal{N} equal to $2N$. This leads to (please see reference [5])

$$R^2 = R_g^2 = \mathcal{N} \lambda_F^2 = 2N \hbar^2 / (m^2 v_F^2). \quad (17)$$

Inserting (17) into (15) we get

$$K_B T_N = \frac{1}{2} m v_F^2. \quad (18)$$

Then it seems that a Fermi gas can also be encompassed in the present structure. The interesting feature to be noted is the adjustment of the quantum number N to be equal to half of the number of monomers composing the polymer chain.

5 – Concluding Remarks

We observe that in the present model of a quantum gas all relevant quantities are quantized. Then, the internal energy U_N is quantized and also are quantized the temperature T_N and the entropy S_N . Temperature and entropy both grow linearly with N , while internal energy grows with N squared. It is interesting to compare the present case with the black hole thermodynamics. There the entropy goes with the mass squared, while the internal energy grows linearly with the mass and the temperature goes with the inverse of the mass (please see reference [6]).

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