# The Traveling Twins Paradox

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## Abstract

A modified version of the Twin Paradox (TP), termed the Traveling Twins Paradox (TTP), is described. In the TTP, the twins move towards each other, via two identical fast vehicles, from two starting points, located along the *x* axis at coordinates $+(l+\Delta l)$ and $-(l+\Delta l)$, respectively. The twins first accelerate up to velocity *v* along identical runways of length $\Delta l$ each, then they shut down engines and travel towards each other with constant equal velocities (*v*). After passing each other they use the same runways to decelerate and stop. Unlike the standard TP, the scenario of the TTP is completely symmetric. Moreover, in the range $(-l, +l)$ the TTP system is inertial. Thus, all solutions resorting to a preferred frame of reference (the "staying" twin in the standard TP), or to General Relativity (acceleration/deceleration effects) do not apply. This implies that the TTP poses an unsolvable paradox in Special Relativity.



**Keywords**: Twin Paradox, Relativity.


## Introduction

The Twin Paradox, first proposed by P. Langevin [1], is undoubtedly the most famous thought experiment in physics. The enormous literature about it renders any attempt to review it almost impossible. In the Twin Paradox, one of two twins stays on Earth while the other twin travels at near the speed of light to a distant star and returns to Earth. According to Special Relativity (SR), the twin who stayed on Earth should measure a time dilation of $\gamma(\frac{v}{c})$ relative to the time measured by the traveling twin, but due to SR's first axiom, the traveling twin should measure an equal time dilation relative to the staying twin,  hence the paradox.

A similar argument to the one presented above was proposed more than half a century ago by Herbert Dingle in a paper published in *Nature* [2], and in several subsequent papers and a book [3-5]. Dingle argued that the theory of Special Relativity leads to inconsistency. Specifically, he argued that "Einstein deduced, from the basic ideas of his theory that a moving clock works slower than a stationary one. By a similar line of reasoning I deduced from the same basic ideas that the same moving clock works faster than the same stationary one. Hence the theory, since it entails with equal validity two incompatible conclusions, must be false". ([3], p. 41). Dingle posited that the inconsistency of Special Relativity stems from Einstein's attempt to reconcile his theory with Lorentz's electrodynamics [3]. Dingle's critique was countered by many leading physicists [e.g., 6-10]. The prevailing solution of the paradox is one which prescribes that the "traveling" twin returns younger than the "staying" twin [11, 12]. A similar solution was reached by Albert Einstein himself, first within the framework of Special Relativity, and later within the framework of General Relativity. In his famous 1905 paper [13], although calling SR's answer a ‘peculiar consequence’ (*eigent ümliche Konsequenz*), Einstein stated that the traveling brother is the one to become younger. According to Einstein, this solution is independent of whether the travel-path is comprised of straight lines or of a closed curve of any shape. Einstein's argued that "If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity [$v$] until it has returned to A, which takes, say t seconds, then this clock will lag on its arrival at A by $\frac{1}{2}\ t\ (\frac{v}{c})^2$ seconds behind the clock that has not been moved" [14].

Other attempts to solve the Twin Paradox evoke the General Relativity of accelerating frames [12, 15, 16]. As mentioned before, Einstein himself, after developing General Relativity, resorted to this explanation in 1918, when he argued that since one of the clocks is in an accelerated frame of reference, the postulates of the Special Theory of Relativity do not apply to it and so "no contradictions in the foundations of the theory can be construed" [14, 17]. Solutions evoking the acceleration argument seem to be the prevalent ones. Other attempts to solve the paradox in the framework of circular [e.g., 7] or another closed-curve motion will not be reviewed.

Here I propose a modified version of the twin paradox, in which neither the argument that the frame of the "staying" twin is the preferred one, nor the argument of a non-inertial system, apply. The new paradox, termed the Traveling Twins Paradox (TTP) is as follows:

Consider the case of two twins, A and B. Assume that A is located at coordinate $x = +(l + \Delta l)$ along the $x$ axis, and B is located at coordinate $x = -(l + \Delta l)$ along the same axis. Assume that the twins start moving toward each other using identical vehicles. Assume that each uses a runway of internal length $\Delta l$ to accelerate to a desired velocity $v$, after which the power engines are shut down and the two vehicles complete travel with constant velocity v until they pass each other, after which they use the same runways to decelerate and stop. Assume that v is high enough to produce relativistic effects. Denote the internal times of the start and end of the uniform motion, of A and B, by $t_A^s$, $t_A^e$ , $t_B^s$, $t_B^e$ , respectively. We can set $t_A^s = t_B^s = 0$.

Two unique features distinguish the TTP from the classical TP. First, the acceleration and deceleration phases of the travel are accomplished prior to the start times, so in the path of interest, i.e., $-l \leq x \leq +l$, the motion is uniform. Second, the TTP is completely symmetric in the sense that there is no possible way to designate one frame of reference as a preferred frame. In the TTP both twins are travelers and the symmetry is complete. For example, the system will remain unchanged under the transformation $x' = x$.

From the perspective of SR, following the acceleration phase, each twin will observe that he is stationary while his twin brother is approaching him from a distance $x = 2l$ with constant velocity of $2v$. Thus, upon passing side by side, each twin is predicted to measure a time dilation of $\gamma(2\frac{v}{c})$ relative to the other twin. In other words each twin will observe that the other twin is younger than him, hence the paradox**.**

**Concluding Remarks**

Solutions to the classical Twin Paradox rely on considerations of a preferred internal frame within the context of Special Relativity, or on General Relativity considerations, related to the acceleration/deceleration phases of travel (or on both). In the Traveling Twin Paradox (TTP) described here, none of the aforementioned considerations apply. In the travel path of interest, the system is inertial, which dismisses the acceleration/deceleration argument. Moreover, the TTP is completely symmetric, in the sense that the positions of A and B are indistinguishable, leaving no way for preferring one frame over the other.

The complete symmetry of the Traveling Twins Paradox should lead to a symmetric solution, according to which the twins should pass each other at $x = 0$ to find that they have aged equally. The TTP poses an unsolvable problem within the framework of Special Relativity. We know that the twins approaching each other will meet sometime, somewhere, and compare clocks. The inability of Special Relativity to produce one prediction, instead of two contradictory predictions, should be highly disturbing to current physics.

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